

Fuzzy Optimization Method for Wind Load Monitoring of Fixed Offshore Wind Turbines

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Abstract

Fixed offshore wind turbines generally use statistical parameters of sea conditions in or near the target sea area as design benchmarks during the design phase. However, these sea state statistical parameters vary substantially from the actual sea state parameters experienced by offshore wind turbines. This situation brings uncertainty to the structural safety of offshore wind turbines. Online monitoring of wind loads exerted on the wind tower topside, which uses monitoring the structural response of the wind tower to determine wind load online, provides another technical approach for accurately measuring the wind loads exerted on the wind tower topside. However, online monitoring of wind loads can be affected by factors such as the ill-posedness of mathematical models and errors in monitoring data. The mathematical model for wind load monitoring is usually optimized only with the ill-posedness index as the objective function to reduce or overcome model ill-posedness. The error level of monitoring data is not considered in optimization, which likely leads to a high proportion of monitoring data error and then reduces the quality of monitoring data. Hence, a fuzzy optimization method was developed in this study to reduce the ill-posedness index of mathematical models and the influence of monitoring data errors on the monitoring results, in which the ill-posedness index of the mathematical models is weighted and unified with the error index of the monitoring data. In addition, the effectiveness and reliability of the method developed were demonstrated through numerical examples of jacket wind turbines, and structural damage can be accurately analyzed using the monitored wind loads.

Keywords Wind load monitoring; Fixed offshore wind turbines; Fuzzy optimization; Ill-posedness index of mathematical models; Error index of monitoring data

1 Introduction

Many design ideas have been proposed, and various specifications and processes have been developed during the design and construction of wind turbine support structures to ensure adequate strength against environmental loads. However, structural safety incidents still occur due to the numerous uncertainties in the actual operation of offshore

wind turbines. The loads encountered by these structures are highly diverse and random, making it nearly impossible to account for all of them during the design phase. Recent studies have emphasized the importance of understanding the dynamic behavior of wind turbines under different excitations. For example, Bozyigit et al. (2023) proposed an analytical approach for seismic analysis of onshore wind turbines, considering soil-structure interaction, which provides critical insights into their dynamic response under seismic conditions. Similarly, Bozyigit et al. (2025) developed a rapid assessment method for the seismic performance of large monopile-supported offshore wind turbines under scour and highlighted the need for advanced dynamic load analysis to ensure structural integrity. These studies accentuate the importance of developing more scientific structural analysis methods and robust design standards to increase safety. Moreover, online monitoring techniques, such as real-time load identification through indirect methods, offer practical solutions for enhancing structural safety by addressing the challenges caused by complex, unpredictable operational conditions.

Dynamic load identification is the process of reversing the external loads of a structure based on its dynamic response. This technique belongs to the typical inverse problems within the field of structural dynamics (Zhi et al.,

Article Highlights

- A method for monitoring wind loads on a jacket-type wind turbine is introduced.
- A multiobjective optimization method for sensor position considering the ill-posedness of the mathematical model and monitoring noise is proposed.
- The relative error between the fatigue damage values computed using the identified wind loads and the actual values is less than 13.8%.

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2016; Liu et al., 2022a; Liu et al., 2022b). Load identification is principally categorized into two, namely, load identification in the frequency domain and time domain (Wang et al., 2020; He et al., 2020; Miao et al., 2020).

The dynamic load identification in the time domain is extremely sensitive to initial values, and cumulative errors are inevitable. Moreover, the ill-posedness of the kernel function matrix has a substantial influence on the identification results. For the identification of random loads, a deterministic time function cannot be used to describe the random process, which hinders applying traditional load identification methods that establish forward models through transfer functions. Currently, almost no method can be used for identifying random loads in the time domain, especially for long-term online identification.

The frequency domain method performs Fourier transform on the time-domain signal to acquire the amplitude–frequency characteristic curve. This method has high requirements for sample length and is appropriate for identifying long-term or stationary loads. The direct inverse method of frequency response began in the aviation field, and Bartlett and Flannelly (1979) identified the dynamic load at the center of the helicopter hub with the input of acceleration response. Giansante et al. (1982) identified the dynamic loads on the main shaft and tail rotor of a helicopter during flight using acceleration response and transfer function matrix. The direct inverse method of the frequency response function is simple and easy to implement, but Starkey and Merrill (1989) discovered that this method has poor anti-interference ability, especially in identifying loads near the natural frequency. Karlsson (1996) established that the inverse operation of the frequency response function matrix is greatly sensitive to response measurement noise and model errors, which can seriously affect the stability of load identification results.

Two strategies are commonly employed to address problems of ill-posedness:

1) Excellent reverse engineering algorithms are developed to obtain high-precision, high-stability identification results in the case of an ill-posed model.

2) The ill-posedness of mathematical models is reduced or completely eliminated via optimizing sensor locations.

Regularization methods, including the Tikhonov method (Zhang and Xu, 2016; Jiang et al., 2020), Truncated Singular Value Decomposition (Chen et al., 2020), Damped Singular Value Decomposition (Tian et al., 2019), and iterative regularization methods such as the Landweber iteration (Wang et al., 2011), Conjugate Gradient iteration (Xiao et al., 2013), and Least Squares QR iteration (Kong et al., 2020) have become fundamental for resolving problems of ill-posedness. However, these methods cannot always generate satisfactory identification results.

Gupta and Dhingra (2013) introduced a D-optimal design method to optimize sensor locations to establish a mathe-

matical model with low ill-posedness, in which a Sequential Exchange Algorithm is employed to perform the D-optimal design process. Several numerical examples of dynamic load identification demonstrate the superiority of the D-optimal design method, especially in cases where the mathematical model dimension is low. However, when the dimensions of the mathematical model are large, D-optimal design fails to reduce model ill-posedness effectively. To address this issue, Zhang et al. (2019a; 2019b) proposed a C-optimal design method to decrease model ill-posedness by optimizing sensor locations, in which a Successive Reduction Algorithm is used to perform the C-optimal design process. The C-optimal design method is superior to the D-optimal design method, especially when the mathematical model dimensions are large, but the computational efficiency of the C-optimal design method is lower than that of the D-optimal design method (Zhang et al., 2019b; Zhang et al., 2019a). To improve computational efficiency when the mathematical model dimensions are large, a block C-optimal algorithm (Ma et al., 2024b) and an improved simulated annealing algorithm (Ma et al., 2025) were proposed.

Current research on wind load identification for offshore wind turbines is fairly limited, and most studies focus on support structure damage (Jeong et al., 2020), load conditions (Wang et al., 2022), and structural responses under external excitations (Xu et al., 2022). In frequency-domain identification methods, the simultaneous consideration of monitoring data error and model ill-posedness has not been sufficiently addressed because previous research often tackles these issues separately. This paper integrates frequency-domain random load identification theory, sensor placement optimization algorithms, and inverse techniques for linear equations to propose a fuzzy optimization method for wind load identification at the top of a jacket-supported offshore wind turbine tower. The fuzzy optimization method improves accuracy and stability by simultaneously addressing monitoring data error and model ill-posedness.

2 Wind load identification method for jacket wind turbines

2.1 Dynamic load identification in the frequency domain

Assuming n points of stationary random excitation and m points of stationary random response on a linear time-invariant system (He et al., 2019; Ma et al., 2024a), the relationship between input and output can be expressed as Eq. (1).

$$\mathbf{S}_{yy}(\omega) = \mathbf{H}(\omega) \times \mathbf{S}_{xx}(\omega) \times \mathbf{H}(\omega)^H \quad (1)$$

where $\mathbf{S}_{yy}(\omega) \in \mathbf{C}^{m \times m}$ is the power spectral density matrix of the structural response; $\mathbf{S}_{xx}(\omega) \in \mathbf{C}^{n \times n}$ is the power spectral density matrix of the structural excitation; $\omega = (\omega_1, \omega_2, \dots, \omega_N)$ denotes the frequency; Superscript (H) is the conjugate transpose operator; $\mathbf{H}(\omega) \in \mathbf{C}^{m \times n}$ is the frequency response function matrix of the physical relationship between the excitation points to be identified and the response measurement points, and this matrix is complex-valued.

This equation forms the foundation of frequency-domain dynamic load identification. It fundamentally states that the response at any given frequency depends on the excitation force and the system's frequency response characteristics. In practical applications, load identification aims to determine $\mathbf{S}_{xx}(\omega)$ based on measured $\mathbf{S}_{yy}(\omega)$ and a known or estimated $\mathbf{H}(\omega)$. However, numerous challenges arise in this process.

In the above equation, the $\mathbf{H}(\omega)$ matrix is commonly obtained through numerical analysis, and it differs from the real frequency response function matrix; that is, uncertainty exists in the mathematical model of load identification, which will have a negative effect on the load identification results. Moreover, the ill-posedness of the $\mathbf{H}(\omega)$ matrix can reduce the accuracy and stability of load identification. Finally, the errors in monitoring data are represented by the $\mathbf{S}_{yy}(\omega)$ matrix in the mathematical model. These errors will reduce the accuracy and stability of load identification. To address this and the other two issues mentioned above, it is recommended to apply the theory of dynamic load identification to practical engineering.

The uncertainty of the mathematical model for load identification principally depends on the calculation accuracy of the $\mathbf{H}(\omega)$ matrix. The higher the accuracy of the $\mathbf{H}(\omega)$ matrix, the smaller the uncertainty of the mathematical model. In finite element analysis, the calculation accuracy of structural response at the locations of structural changes and welds is usually low. Thus, the selection of measurement point positions should avoid these locations with low calculation accuracy of structural response, which can indirectly reduce the uncertainty of the mathematical model.

The degree of ill-posedness of the load identification mathematical model can be represented by the condition number of the $\mathbf{H}(\omega)$ matrix, and the larger the condition number, the higher the ill-posed degree of the mathematical model. When the condition number is less than 100, the mathematical model is not ill-posed. Two strategies are generally employed to address ill-posed problems, as mentioned above.

The error in monitoring data cannot be removed, which principally depends on the accuracy and stability of the monitoring sensors. However, in load identification, the influence of monitoring data errors on the load identifica-

tion results can be reduced by installing sensors at locations with larger structural response amplitudes, which thereby reduces the proportion of monitoring data errors.

In summary, the following three points are worth considering to improve the accuracy of load identification:

- 1) Positions with low structural response calculation accuracy should be avoided when considering the placement of sensors.
- 2) Mathematical model ill-posedness can be reduced or completely eliminated by optimizing sensor arrangement.
- 3) The adverse effect of monitoring data error on load identification results can be reduced by optimizing sensor arrangement.

Among the above three points, 2) and 3) may conflict, which needs optimization to reduce model ill-posedness and the adverse effect of monitoring data error on the load identification results simultaneously.

2.2 Establishment of an initial mathematical model

Jacket wind turbines are subject to wind and wave loads, where wind loads encompass those acting on the turbine blades (aerodynamic load) and the tower structure. In practical engineering, the structural response monitored by sensors placed on the inner wall of the tower results from the combined effects of these loads.

An offshore wind turbine in the South China Sea is used as a case study. The foundation stands at a height of 84.3 m, while the tower structure reaches a height of 72 m. Additionally, the pile legs extend to a depth of 15.6 m below the mud surface. The jacket foundation and tower structure were simulated using Shell181 elements, a four-node finite element type available in ANSYS, which is appropriate for analyzing thin to moderately thick shell structures. The material properties used in the simulation are as follows:

Jacket foundation: Density = 7849 kg/m³, Elastic modulus = 2.06×10^{11} N/m², and Poisson's ratio = 0.3;

Tower structure: Density = 8100 kg/m³, Elastic modulus = 2.1×10^{11} N/m², and Poisson's ratio = 0.3.

The element size is approximately 0.5 m, the K- and X-type pipe connection positions are densified, and the grid size is approximately 0.1 m. The meshing length was determined based on a convergence study, where mesh size was progressively refined until the results (e.g., natural frequencies and stress distributions) showed minimal changes (less than 2%) between successive refinements. This approach ensures that the mesh is adequately fine to capture the structural behavior accurately while maintaining computational efficiency. The final model consists of 48 588 elements. The structural finite element model is illustrated in Figure 1.

The wind turbine tower can be simplified as a thin-walled cylindrical structure. Structural stress analysis reveals that when torque is applied to the tower's topside, the microelements on its surface are in a pure shear state,

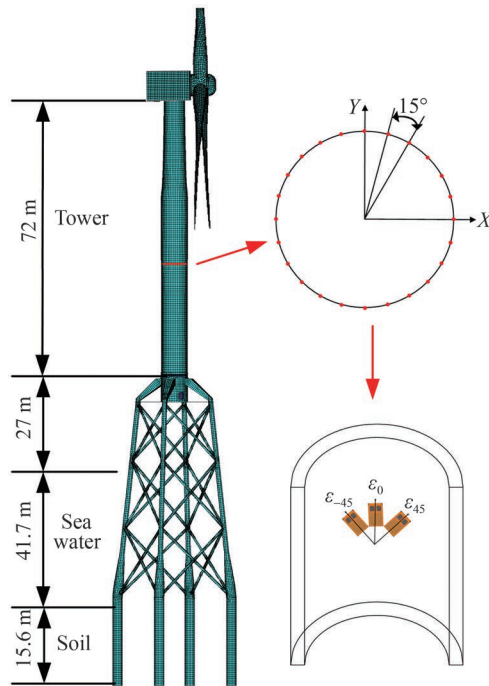


Figure 1 Finite element model, candidate sensor locations, and directions of an offshore wind turbine structure

and the principal stresses at the tower's surface manifest at angles of $\pm 45^\circ$ relative to the tower's axis. Therefore, three-way sensors were initially positioned at each monitoring location at 0° , 45° , and -45° relative to the axis (Figure 1) to capture the structural response to aerodynamic loading comprehensively. Sensors are arranged in a circular pattern every 3 m along the tower's vertical direction, along the entire height of 72 m of the tower. Each circle of sensors is spaced at 15° intervals, with 24 sensors per circle. This configuration results in 23 tiers with 1 656 sensors.

The final mathematical model can be expressed as Eq. (2), considering the candidate sensor locations and directions.

$$S_{yy}(\omega)_{1656 \times 1656} = H(\omega)_{1656 \times 6} \cdot S_{xx}(\omega)_{6 \times 6} \cdot H(\omega)_{1656 \times 6}^H \quad (2)$$

2.3 Wave-induced structural response

The irregular wave load was analyzed using AQWA software, in which the sea conditions of $H_s = 2.25$ m and $T = 4.75$ s, $H_s = 2.25$ m and $T = 5.25$ s, and $H_s = 4.75$ m and $T = 6.75$ s were selected as the study case. The joint probability density distribution of wind and waves in the specified sea area is shown in Figure 2.

Subsequently, the wave loads were mapped onto the finite element model, and transient dynamic analysis was conducted to acquire the stress response history of the entire structure under the wave loads. The calculation time is 200 s, and the highest stress distribution under each sea

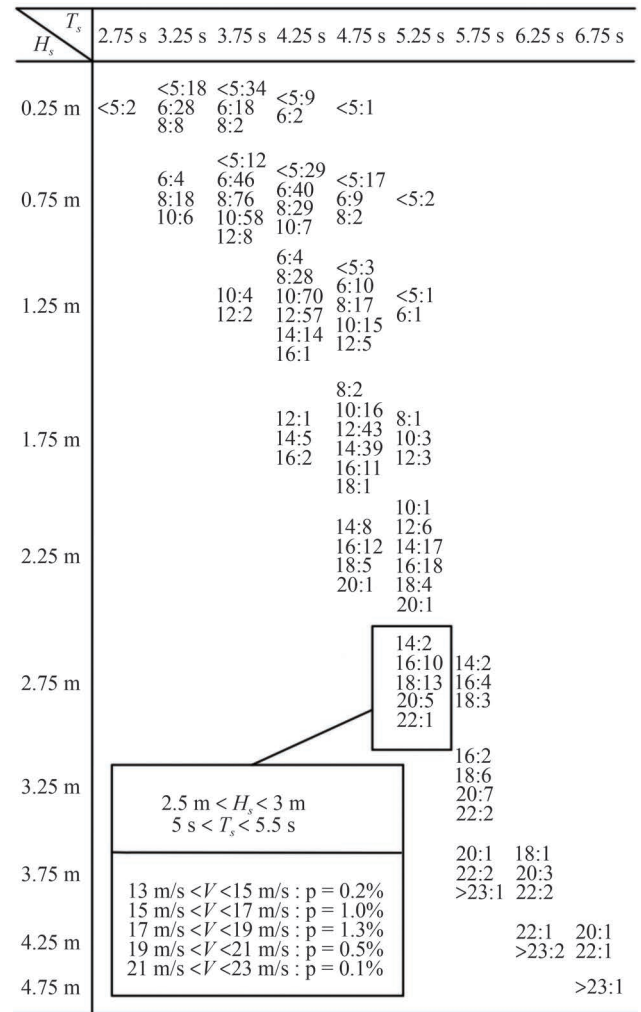


Figure 2 Joint probability density distribution of wind and waves (Zhu et al., 2024)

state is shown in Figure 3.

Figure 3 reveals that the stress of the tower is approximately 0.21 MPa. Hence, the effect of wave loads on the wind turbine tower structure is small and can be ignored.

2.4 Wind-induced structural response

The influence of near-surface natural winds on structures within the atmospheric boundary layer mainly involves the effects of mean wind and turbulent wind. Within the boundary layer of the near-Earth atmosphere, the change pattern of mean wind speed with height above the ground is called mean wind speed profile or wind speed gradient. This profile is largely described by logarithmic and exponential laws (Choi, 2004; Tamura et al., 2007). The variation in shear stress can be considered negligible within the surface layer extending up to 300 m above the ground.

The wind speed profile under these conditions can be mathematically expressed as Eq. (3).

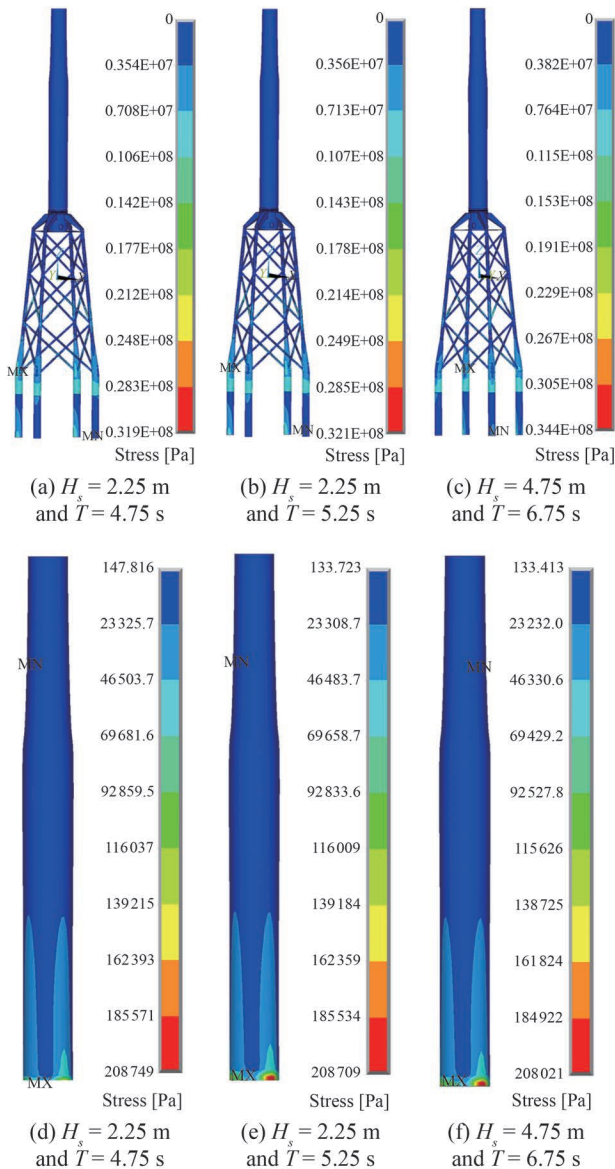


Figure 3 Structural stress under three dissimilar sea conditions

$$U(z) = \frac{u_*}{k} \left[\ln \left(\frac{z}{z_0} \right) + 5.75 \frac{z}{z_g} \right] \quad (3)$$

where $U(z)$ represents the mean wind speed at height z above the ground; u_* is the friction velocity, and $u_* = 0.775 \text{ m/s}$; k is the von Kármán constant, and $k \approx 0.4$; z_0 is the surface roughness, and $z_0 = 0.01$; z_g is the gradient height, and $z_g = 0.175u_*/f_c$; f_c is for the Coriolis constant, and $f_c = 2\omega \sin \Phi$; ω is the angular velocity of the Earth's rotation, and $\omega = 7.27 \times 10^{-5} \text{ rad/s}$; Φ is the latitude.

Spectral methods are generally employed to simulate temporal evolution, considering the time-varying nature of wind speed. The variation of wind speed over time is simulated using horizontal turbulent wind speed spectra, as shown in Eq. (4), considering the measured wind field data

in coastal areas of China (Liu et al., 2012).

$$\frac{nS_u(n)}{\sigma_u^2} = \frac{23.41f}{(1 + 30.63f)^{5/3}} \quad (4)$$

where $S_u(n)$ is the power spectral density of horizontal wind speed; n is the frequency; σ_u is the root mean square of horizontal turbulent wind speed, and $\sigma_u^2 = 0.05 + 1.26u_*^2$; $f = nz/U$ is the dimensionless frequency, also known as Monin similarity coordinate; and U is the average wind speed.

GH Bladed software is generally used in offshore wind turbine engineering to determine the aerodynamic load of wind turbines. In GH Bladed software, the time history of equivalent concentrated aerodynamic loads can be acquired with the input of wind spectrum wind turbine parameters. The time history of equivalent concentrated aerodynamic loads exerted on the topside of the wind turbine tower for wind speeds of 8, 12, 16, 20, and 24 m/s is shown in Figures 4–8, respectively.

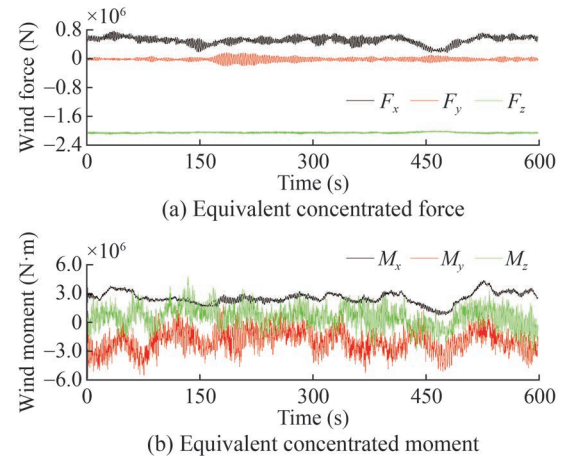


Figure 4 Time history of equivalent concentrated aerodynamic loads corresponding to 8 m/s

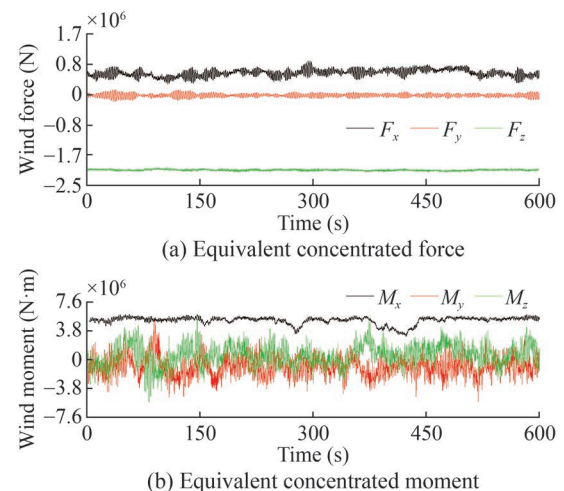


Figure 5 Time history of equivalent concentrated aerodynamic loads corresponding to 12 m/s

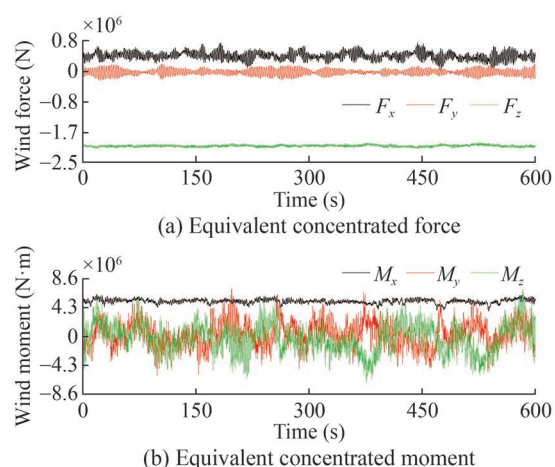


Figure 6 Time history of equivalent concentrated aerodynamic loads corresponding to 16 m/s

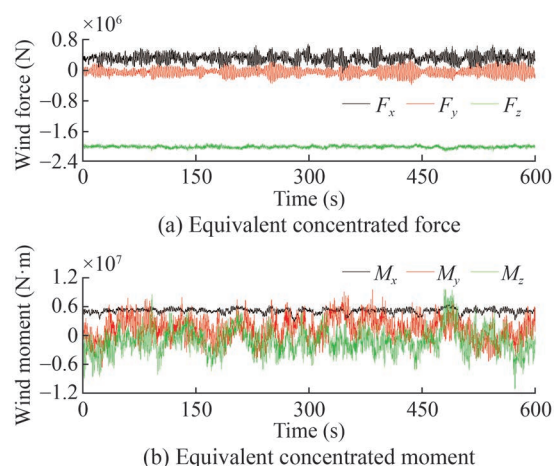


Figure 7 Time history of equivalent concentrated aerodynamic loads corresponding to 20 m/s

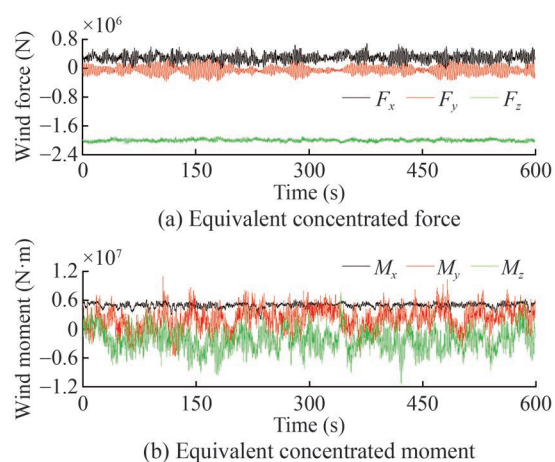


Figure 8 Time history of equivalent concentrated aerodynamic loads corresponding to 24 m/s

Transient analysis was performed using the finite element method with a linear elastic material model. The

equivalent load time history was applied at the top of the tower, and the structural response in the time domain was computed using the full method. A time step of 0.02 s was adopted to guarantee the accuracy of the dynamic analysis. Structural stress can be determined by performing transient dynamic calculations, and the stress distributions of the tower at 100, 300, and 500 s for wind speeds of 8, 12, 16, 20, and 24 m/s are shown in Figures 9–13, respectively.

The equivalent concentrated load of a wind turbine has six components, namely, F_x , F_y , F_z , M_x , M_y , and M_z . Loading these loads onto the topside of the tower results in the minimum structural response at the top and the maximum at the bottom, as shown in Figures 9–13. Thus, the sensor arranged at the top of the tower has the highest proportion of monitoring data error, whereas the sensor arranged at the bottom of the tower has the smallest proportion of monitoring data error when the error of the monitoring sensor is $\pm 10 \mu\epsilon$.

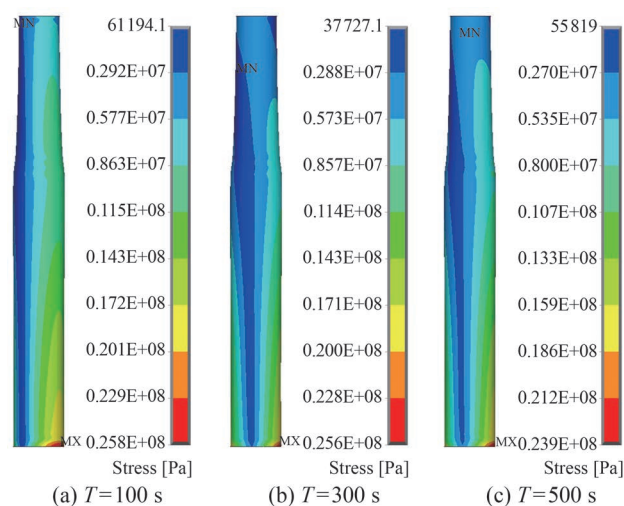


Figure 9 Stress distributions of the tower, wind speed = 8 m/s

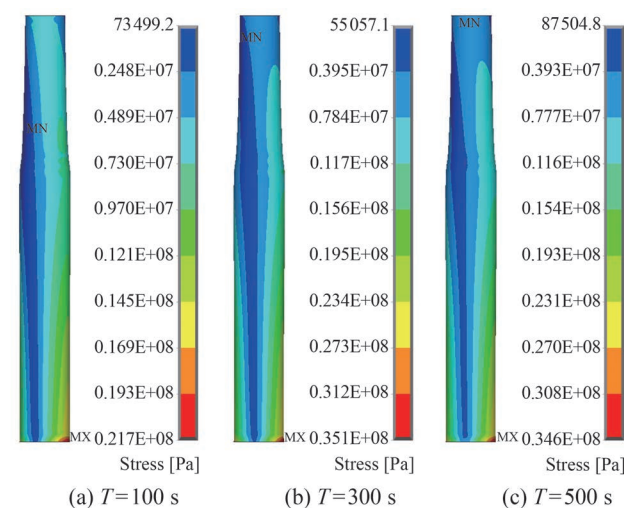


Figure 10 Stress distributions of the tower, wind speed = 12 m/s

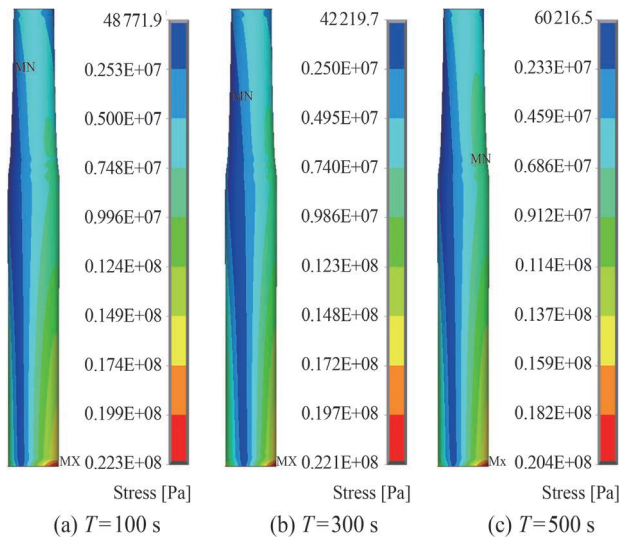


Figure 11 Stress distributions of the tower, wind speed = 16 m/s

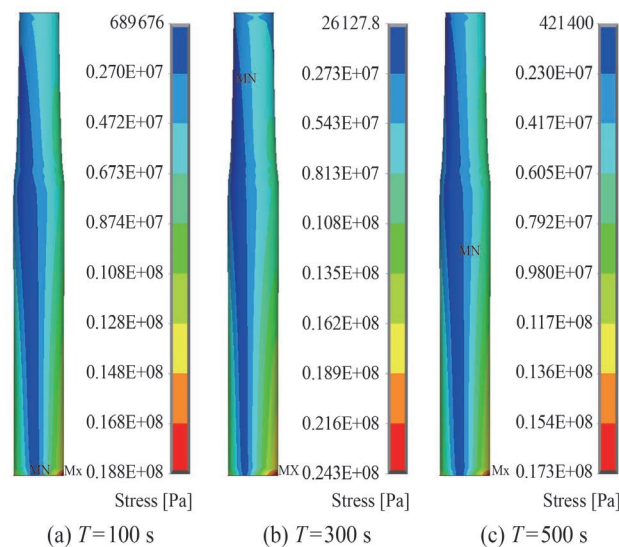


Figure 12 Stress distributions of the tower, wind speed = 20 m/s

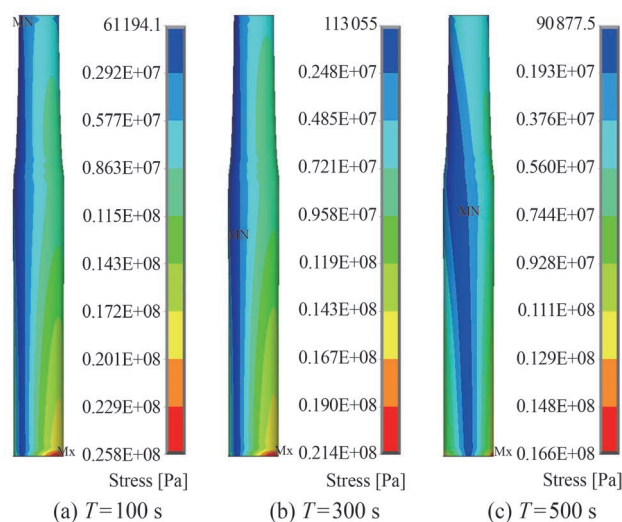


Figure 13 Stress distributions of the tower, wind speed = 24 m/s

3 Analysis of interference factors in dynamic load identification

The tower structure is divided into 10 equally spaced zones along the height direction, as shown in Figure 14.

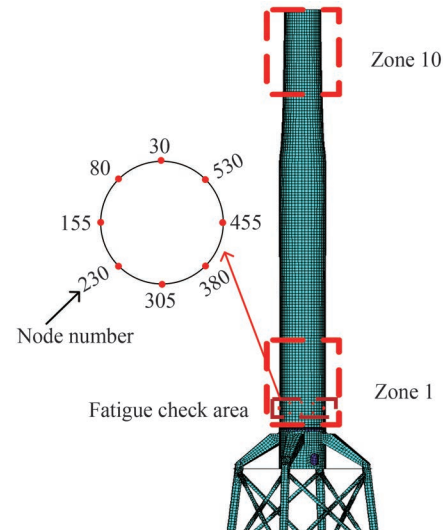


Figure 14 Division of monitoring zones

3.1 Changing trend of model ill-posedness index

When the positions of sensors are limited to a certain zone, the initial mathematical model for load identification can be optimized through C- and D-optimal design methods to reduce the ill-posedness index of the mathematical model. Moreover, an approximate relationship between the ill-posedness index of the optimized mathematical model and the installation height of the sensor can be acquired, as shown in Figure 15.

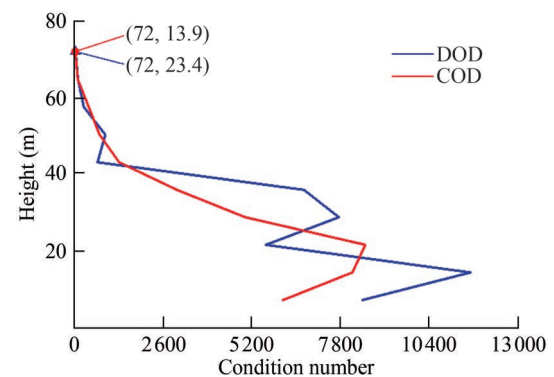


Figure 15 Approximate relationship between model ill-posedness index and sensor height

Figure 15 shows that when the sensor is confined to the top zone of the tower, i.e., Zone 10, the ill-posedness of the mathematical model can be completely removed through

the C- or D-optimal design method. At this point, the condition number of the mathematical model does not exceed 50. Thus, if only the ill-posedness index of the mathematical model is used as the optimization objective, restricting the sensor locations to Zone 10 can fully meet the requirements. In this case, high-precision load identification results will be obtained.

The time history of equivalent concentrated aerodynamic loads corresponding to 8 m/s, as shown in Figure 4, is used as a case to validate the load identification accuracy in the frequency domain, in which the monitoring data error is not considered. The Moore–Penrose pseudo inverse method is used to solve the mathematical model for load identification in the frequency domain, and the identification results are shown in Figure 16.

Figure 16 shows that the accuracy of load identification

results is very high, with almost no errors. When monitoring data error is considered, i.e., $\pm 10 \mu\epsilon$, the identification results are shown in Figure 17.

Figure 17 shows that the monitoring data error of only $\pm 10 \mu\epsilon$ has a substantial adverse effect on the identification results. Thus, in the identification of aerodynamic loads exerted on the topside of the tower, only pursuing low or non-ill-posed mathematical models is unreasonable.

3.2 Changing trend of error index

The tower structure is divided into 10 equally spaced zones along the height direction, as shown in Figure 14, and the stress distributions of the tower at 100, 300, and 500 s for wind speeds of 8, 12, 16, 20, and 24 m/s are displayed in Figures 9–13, respectively. The average strain $\overline{\mu\epsilon}$

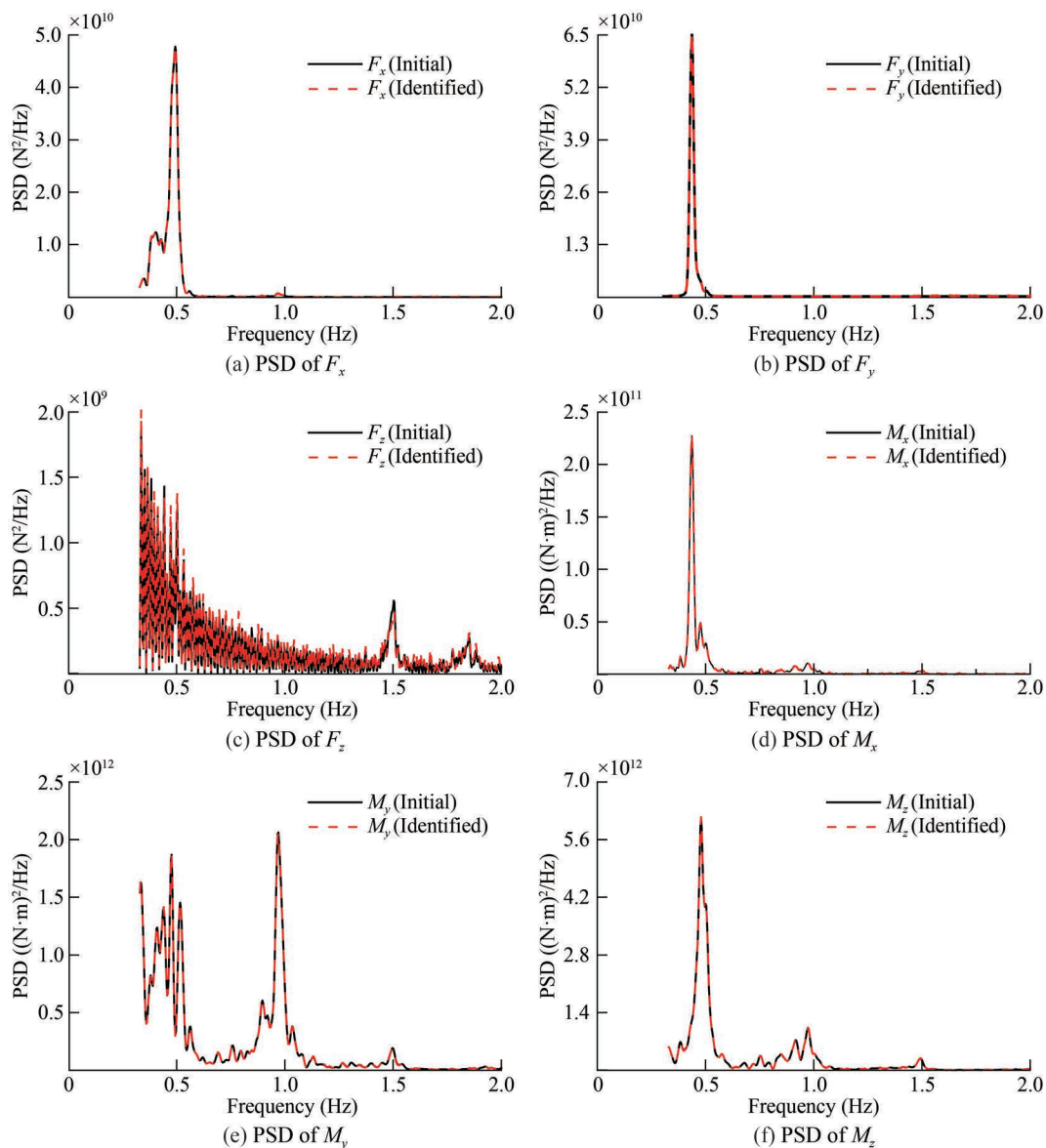


Figure 16 Load identification at a wind speed of 8 m/s without monitoring data error

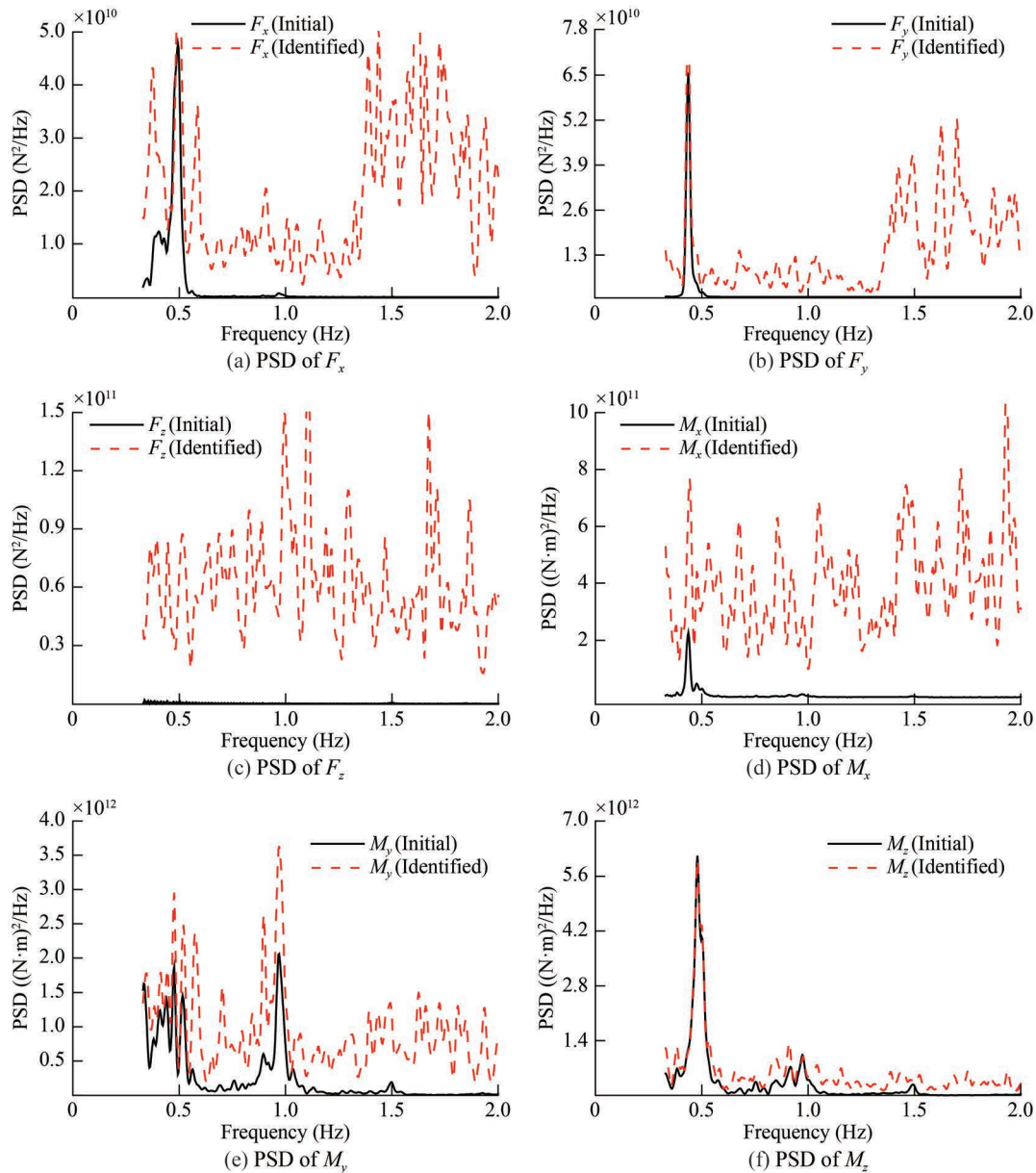


Figure 17 Load identification at a wind speed of 8 m/s with monitoring data error

of the structure within each zone, where the direction of the structural strain is consistent with the installation direction of the strain sensor, is computed. Then, the approximate average error ratio is acquired using Eq. (5), as shown in Figure 18.

$$E_{\text{ratio}} = \frac{10 \mu \varepsilon}{\mu \varepsilon} \quad (5)$$

Figure 18 presents that the error ratio of monitoring data is highest at the top of the tower and lowest at the bottom. Therefore, if only the error ratio of monitoring data is used as the optimization objective, restricting the sensor position to Zone 1 can fully meet the requirements. However, in this case, the load identification results will be influ-

enced by model ill-posedness.

The time history of equivalent concentrated aerodynamic loads corresponding to 8 m/s, as shown in Figure 4, is also used as a case to confirm the load identification accuracy in the frequency domain. The Tikhonov regularization method is used to solve the ill-posed mathematical model for load identification in the frequency domain, in which the regularization parameters are obtained by the L-Curve algorithm. The identification results when the monitoring data error is not considered are shown in Figure 19.

The identification results when the monitoring data error is considered are shown in Figure 20.

Figures 19 and 20 reveal that the accuracy of load identification is extremely low, and only a few load components, such as F_x and, are accurately identified. In this case,

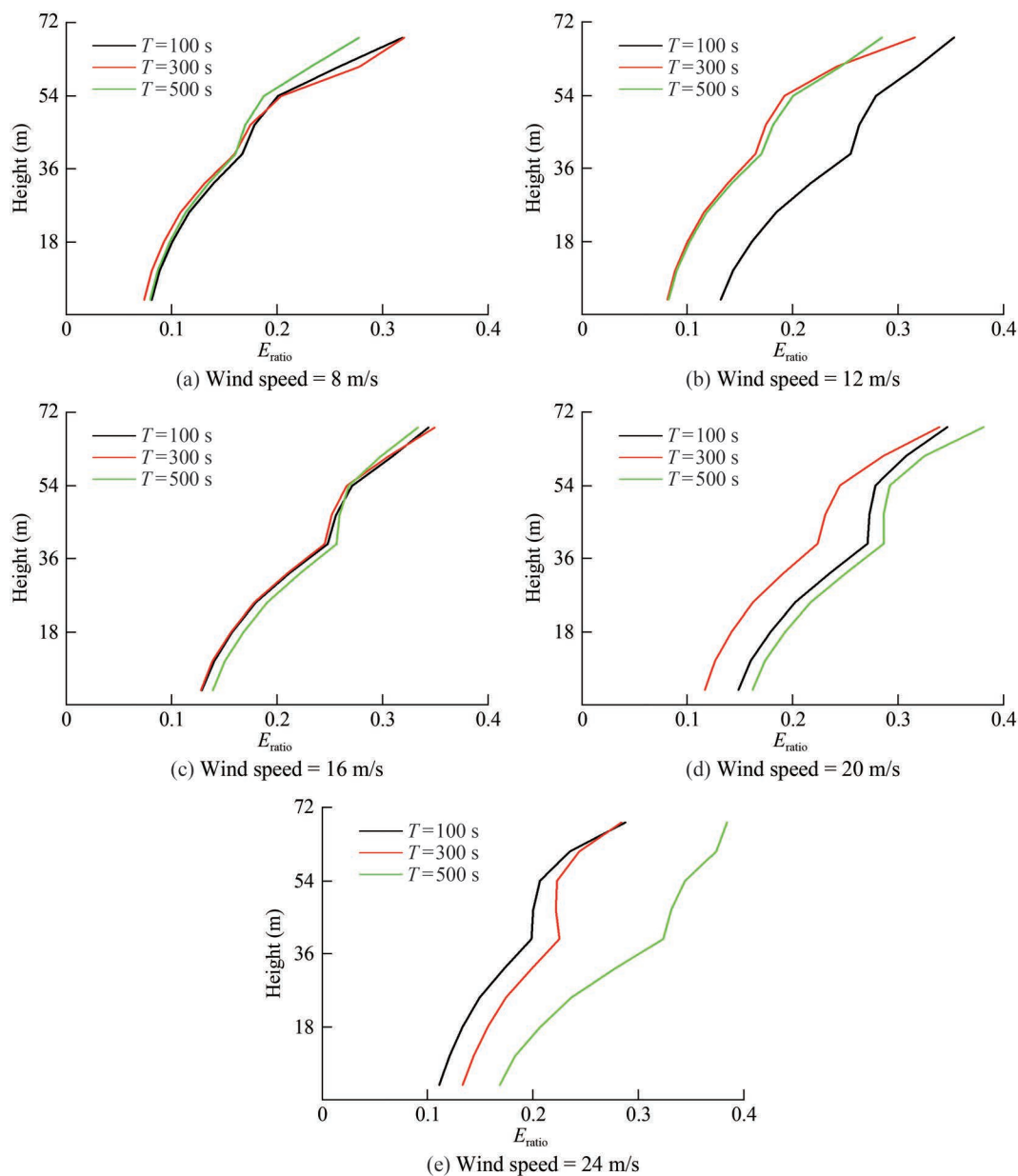


Figure 18 Approximate relationship between monitoring data error ratio and sensor height

model ill-posedness has the greatest influence on the identification results, whereas the influence of the monitoring data error can be neglected.

4 Fuzzy optimization method for strain gauge locations and directions

In summary, the ill-posedness of the mathematical model and the ratio of monitoring data errors must be fully considered to obtain accurate load identification results. The ill-posedness of the mathematical model and the proportion of errors in monitoring data are related to the height of sensor placement. The result of load identification includes

six components, and defining a comprehensive, unified index to verify the accuracy of recognition results is difficult. Moreover, the results of load identification are intended to be used for structural damage analysis to acquire structural damage that is close to reality. Therefore, structural damage is used to validate the accuracy of load identification.

4.1 Frequency domain fatigue analysis of a jacket wind turbine

The power spectral density function of stress under wind loading naturally exhibits broadband characteristics. However, the probability density function of stress range from the theoretical solution does not exist for broadband spectra.

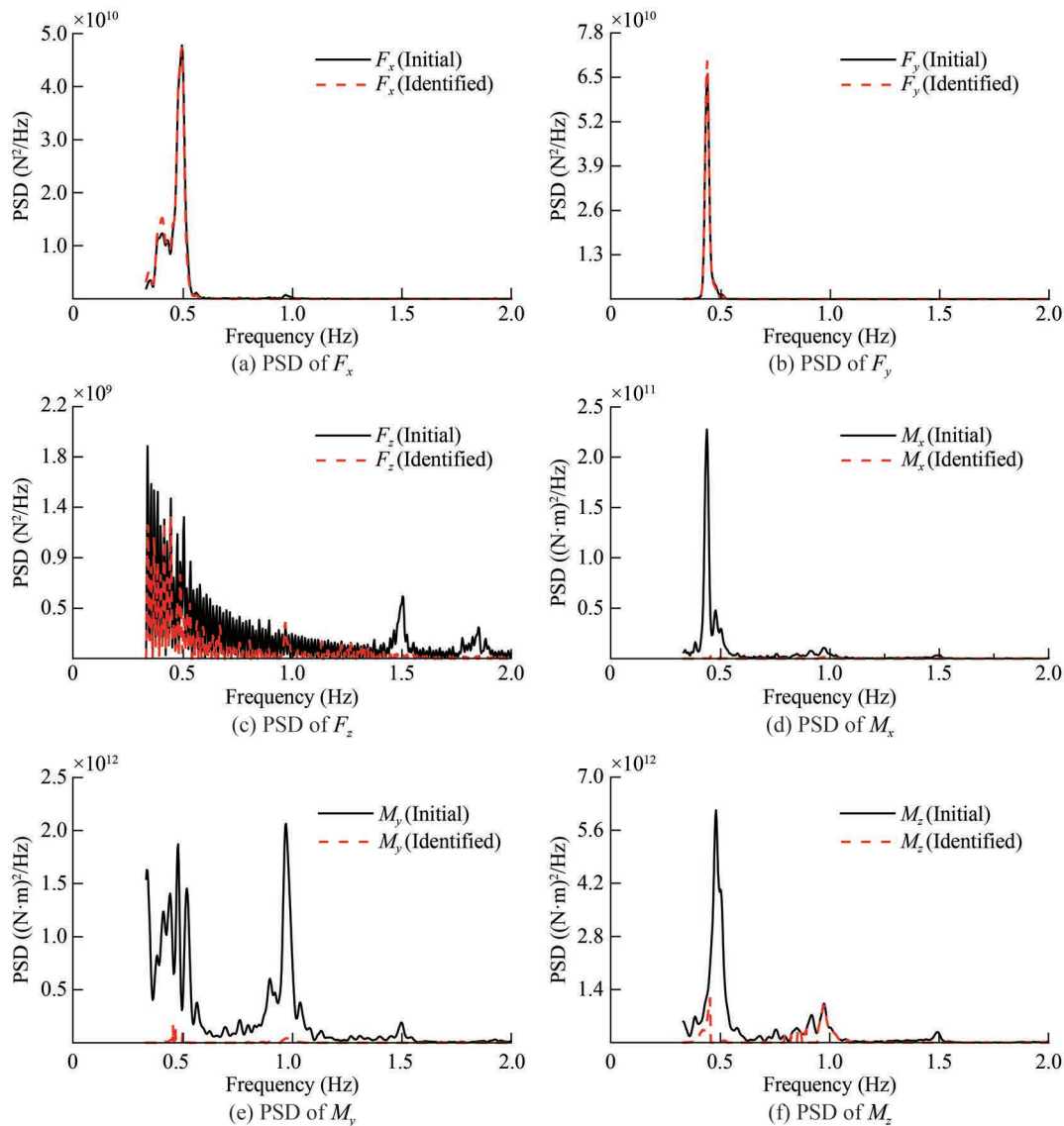


Figure 19 Load identification at a wind speed of 8 m/s without monitoring data error

Currently, approximate methods are commonly employed to compute fatigue damage for broadband spectra. The following methods are primarily used in ship and ocean engineering.

4.1.1 Narrow-band approximation method

The narrow-band approximation method, which utilizes Eq. (6) to approximate damage, represents a more conservative methodology.

$$D_{NB} = \frac{TV_0}{C} \left(2\sqrt{2\lambda_0} \right)^m \Gamma \left(\frac{m}{2} + 1 \right) \quad (6)$$

where T is the service life of structural design, C and m are parameters of the S - N curve, v_0 is the mean crossing rate, λ_0 is the zero-order spectral moment of the power spectral density function, and $\Gamma(\cdot)$ is the gamma function.

4.1.2 Bandwidth correction method

The bandwidth correction method refers to applying bandwidth correction to Eq. (6) and results in Eq. (7).

$$D_{WB} = \rho_{WB} \cdot D_{NB} \quad (7)$$

where ρ_{WB} is the bandwidth correction factor.

The methods for computing the bandwidth correction factor ρ_{WB} primarily include the Wirsching–Light method (Wirsching and Light, 1980), the Ortiz–Chen method (Ortiz and Chen, 1987), and the Lutes–Larsen method (Lutes and Larsen, 1990).

4.1.3 Dirlik method

Fatigue damage is computed using an approximate probability density function formula, which is extensively known as the approximate formula proposed by Dirlik (1985).

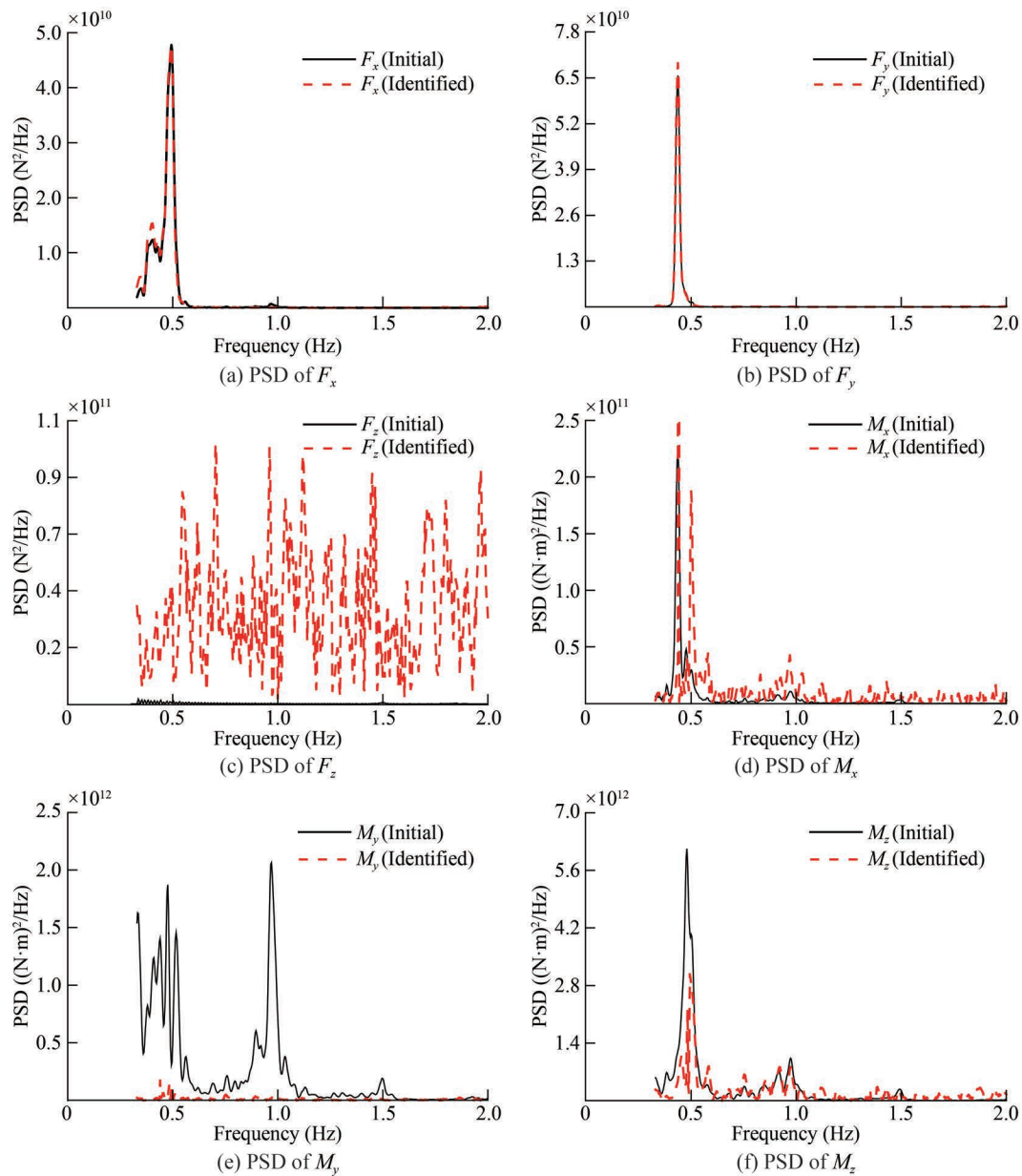


Figure 20 Load identification at a wind speed of 8 m/s with monitoring data error

The semi-empirical model proposed by Dirlik combines a Rayleigh distribution with two exponential distributions. The distribution pattern of this model closely resembles that of the stress range after rainflow counting. Accordingly, the fatigue damage incurred by broadband stochastic processes can be accurately determined using this model. The mathematical representation by Dirlik is displayed in Eq. (8), and the stress range distribution is depicted according to Dirlik, as shown in Eq. (9).

$$D_{\text{Dirlik}} = \frac{v_p \cdot T}{C} \int_0^\infty [S^m \cdot p_{\text{RFC}}^{\text{DK}}(S)] dS \quad (8)$$

$$p_{\text{RFC}}^{\text{DK}}(S) = \frac{1}{(\lambda_0)^{\frac{1}{2}}} \frac{G_1}{Q} \exp\left(-\frac{z}{Q}\right) + \frac{G_2 z}{R^2} \exp\left(-\frac{z^2}{2R^2}\right) + G_3 z \exp\left(-\frac{z^2}{2}\right) \quad (9)$$

where $z = S/\sqrt{\lambda_0}$, $x_m = \alpha_1 \cdot \alpha_2$, $Q = 1.25(\alpha_2 - G_3 - G_2\gamma)/G_1$, $\gamma = (\alpha_2 - x_m - G_1^2)/(1 - \alpha_2 - G_1 + G_1^2)$, $G_1 = 2(x_m - \alpha_2^2)/(1 + \alpha_2^2)$, $G_2 = (1 - \alpha_2 - G_1 + G_1^2)/(1 - \gamma)$, $G_3 = 1 - G_1 - G_2$, $\lambda_i = \sum_0^\infty f^i S(f) df$, $\alpha_i = \lambda_i/\sqrt{\lambda_0 \lambda_{2i}}$, and S is the stress amplitude.

In this paper, the Dirlik method is used to examine fatigue damage.

4.2 Approximate distribution of monitoring data error on the tower structure

Error ratio is used to evaluate the level of error in the monitoring data. It is the proportion of the error in the monitored data relative to the total strain value and indicates the extent of error influence on the monitoring signals.

In practical monitoring, strain error can reach up to $\pm 10 \mu\epsilon$. Strain values along the height direction under numerous operating conditions were simulated to examine the error level, and the vertical, $+45^\circ$, and -45° strain values at each layer of nodes were extracted. The maximum strain value for each layer was then selected, and variation curves of these maximum strain values under diverse conditions were plotted. Subsequently, an envelope of the maximum strain values under all conditions was derived. By dividing $10 \mu\epsilon$ by the envelope value, the error ratio distribution along the height direction was determined, as shown in Figure 21.

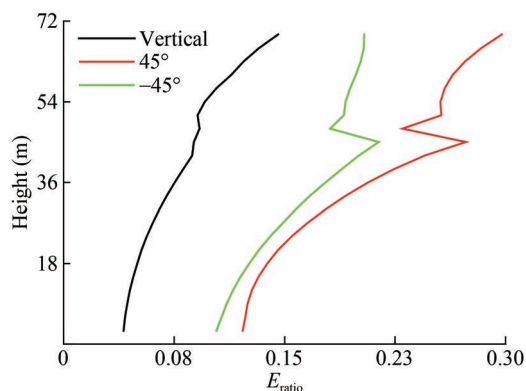


Figure 21 Error ratio distribution of monitoring signals along the height of the tower structure

Figure 21 presents that the error ratio mostly ranges from 0.04 to 0.3, and the general trend indicates that the error ratio is lowest at the bottom of the tower and highest at the top.

4.3 Fuzzy optimization method

The wind turbine tower structure largely withstands wind loads from the top, with fairly low stress levels in the upper sections and higher stress levels in the lower sections. The monitoring data from the upper structure are more vulnerable to monitoring data error, whereas the lower structure is less affected. A larger condition number reduces the accuracy of load identification, whereas a higher data error level also lowers identification effectiveness. Therefore, the weighted sum of the condition number

and the proportion of sensor error is used as the objective function for sensor optimization, which takes the form of a convex combination (Bertsekas, 2009), to consider the influences of condition number and monitoring data error on load identification comprehensively, as expressed in Eq. (10).

$$\text{Minimize: } f(m) = (1 - K) \cdot \text{cond}(m) + K \cdot \varepsilon_{\text{ratio}}(m) \quad (10)$$

where $f(m)$ is the objective function of fuzzy optimization; m is the sensor locations during optimization, acting as decision variables, with a target number of six, and $m \in M$; M represents all candidate sensor positions; $\text{cond}(m)$ is the maximum condition number of the frequency response function matrix for the current sensor positions; $\varepsilon_{\text{ratio}}(m)$ is the error ratio for the current sensor signals; K is the weighting factor.

Because the condition number and monitoring data error affect the accuracy of load identification and subsequent fatigue analysis, but the extent of their individual effects on the results is difficult to quantify, weighting factor K is introduced in the objective function to provide the optimization an element of uncertainty and thereby define it as a fuzzy optimization method. The detailed procedure is as follows:

1) The value of K is set from 0 to 1, with increments of 0.04. When $K = 0$, the objective function only considers the condition number. When K is between 0 and 1, the objective function considers both the condition number and error ratio. When $K = 1$, the objective function only considers the error ratio. Smaller intervals of K values increase the chances of finding the optimal solution.

2) When $K = 0$, all candidate sensor locations are treated as the current sensor positions.

3) One sensor is removed from the current positions in sequence, all possible remaining combinations m' are obtained, $f(m')$ is calculated for each combination, and the combination with the minimum value is retained.

4) Step 3) is repeated until the number of remaining sensors equals the target number, the final optimized sensor combination m'' is obtained, and the optimized sensor positions are determined. Then, $K = K + 0.04$.

5) Steps 2), 3), and 4) are repeated until K reaches 1, and optimized sensor positions for different K values are obtained.

Using a wind speed of 8 m/s with a wind direction of 0° as an example, the fatigue analysis period is 25 years, fatigue damage is determined based on the identified load, and the effectiveness of the sensor placement optimization is assessed using the relative error of fatigue damage. Eight nodes at the base of Zone 1 were selected for fatigue damage calculations, as shown in Figure 14. The optimized sensor positions for different K values were determined, and the strain error of $\pm 10 \mu\epsilon$ was added to the mon-

itoring signals. Wind load was identified with the input of the simulated monitoring data, and fatigue damage values at the bottom nodes were analyzed using the Dirlik method. These values were compared with the damage values under the original wind load to study the relative error. Smaller fatigue damage errors indicate that the optimized sensor arrangement can identify the wind load more effectively, and the structural damage computed according to the identification load is more accurate. The trend of damage values at the fatigue checkpoints as K varies is shown in Figure 22.

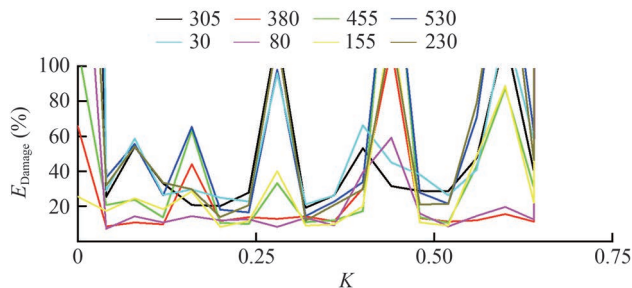


Figure 22 Trend of damage values at the fatigue checkpoints as K varies

Figure 22 shows that when K is extremely small (0–0.04) or extremely large (0.6–1), the identification results are suboptimal. The best results are acquired within a reasonable range, which necessitates multiple attempts to determine the optimal weighting factor. When $K = 0.2$, the total relative error of node damage values is minimized, as shown in Table 1.

Table 1 Relative error of fatigue damage

Node	Fatigue damage (original)	Fatigue damage (identified)	Error (%)
305	0.00124	0.00150	20.40
380	0.00217	0.00247	14.00
455	0.00337	0.00385	14.30
530	0.00227	0.00275	20.70
30	0.00125	0.00156	25.40
80	0.00217	0.00249	14.50
155	0.00337	0.00378	12.10
230	0.00227	0.00265	16.50

The optimized sensor locations and directions corresponding to $K = 0.2$ are shown in Figure 23.

The change in the condition number of the frequency response function matrix corresponding to the optimized sensor locations and directions is shown in Figure 24.

The maximum condition number for the frequency response function matrix corresponding to the optimized

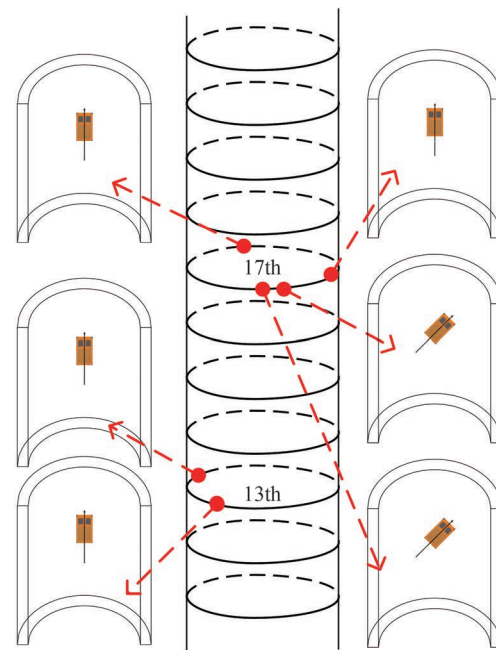


Figure 23 Optimized sensor locations and directions

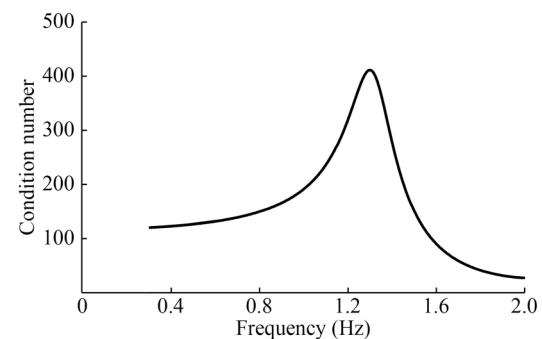


Figure 24 Condition number changes of the frequency response function matrix

sensor positions is 400, whereas the average proportion of errors in the optimized sensor signals is approximately 0.1568. The effectiveness of using the optimized sensors to identify wind loads is illustrated in Figure 25.

Figure 25 shows that the identification results of F_x , F_y , M_y , and M_z are satisfactory, and the accuracy of the identification results of F_z and M_x is very low. The relative error of structural damage at the eight bottom nodes is displayed in Table 1, and the maximum relative error is approximately 25.40%.

Wind turbines are subjected to continuous dynamic loading due to fluctuating wind conditions, which leads to fatigue accumulation over time. Accurately identifying wind loads in real time facilitates estimating the progressive accumulation of fatigue damage and anticipating potential structural failures before they occur. This information can be integrated into predictive maintenance strategies, where maintenance activities are scheduled based on actual

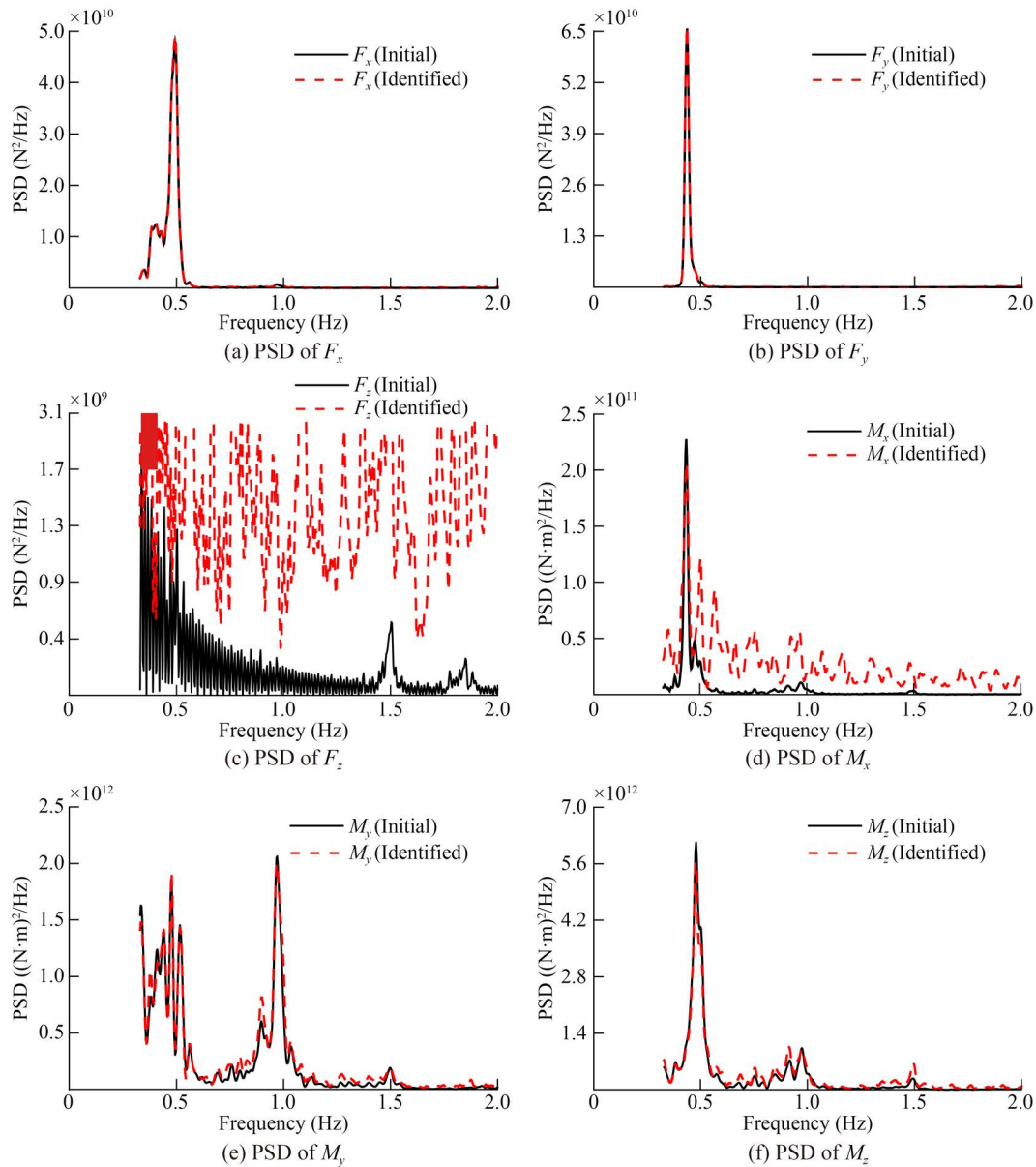


Figure 25 Wind load identification results

load conditions rather than fixed time intervals, which thereby improves cost efficiency and turbine availability.

4.4 Expansion of optimized sensor locations and directions

Wind turbines are equipped with an active yaw system, which adjusts the direction of the turbine based on real-time changes in wind direction and ensures the blades are always aligned with the wind. The previously discussed optimized sensor placement scheme was obtained under the assumption of a constant wind direction. However, wind direction continuously changes and potentially ranges from 0° to 360° . When the wind direction changes, the load identification and fatigue damage estimation using

the original monitoring scheme may not be optimal. Hence, the optimized sensor arrangement needs to be expanded to accommodate varying wind directions.

From a top-down perspective, the tower structure is symmetric about the x-axis and y-axis. Sensor arrangement can be expanded by rotating and copying the optimized sensor arrangement (Figure 23) by 45° , 90° , and 135° . These expanded sensor locations enable effective monitoring of wind loads from any wind direction. The mathematical model corresponding to the expanded sensor arrangement can be stated as Eq. (11).

$$\mathbf{S}_{yy}(\omega)_{24 \times 24} = \mathbf{H}(\omega)_{24 \times 6} \cdot \mathbf{S}_{xx}(\omega)_{6 \times 6} \cdot \mathbf{H}(\omega)_{24 \times 6}^H \quad (11)$$

Eq. (11) is an indeterminate equation, which means 6

unknown quantities are solved by 24 known quantities, which is beneficial to improving the accuracy and stability of the identification results. The wind load identification results corresponding to the expanded sensor arrangement are shown in Figure 26.

Figure 26 reveals that the accuracy of identification results corresponding to the expanded model is similar to those corresponding to the nonexpanded model. The fatigue damage calculated using the identified wind loads is presented in Table 2.

The relative error of structural damage at the eight bottom nodes is listed in Table 2, and the maximum relative error is approximately 13.8%, which is less than the relative error corresponding to the nonexpanded model and indicates that the expanded model is beneficial to enhancing the accuracy and stability of the identification results.

Although the expanded sensor arrangement can effec-

Table 2 Fatigue damage calculated using identified load spectra

Node	Fatigue damage (original)	Fatigue damage (identified)	Error (%)
305	0.00124	0.00142	13.80
380	0.00217	0.00235	8.20
455	0.00337	0.00361	7.10
530	0.00227	0.00251	10.40
30	0.00125	0.00141	13.50
80	0.00217	0.00234	7.70
155	0.00337	0.00360	6.70
230	0.00227	0.00250	10.10

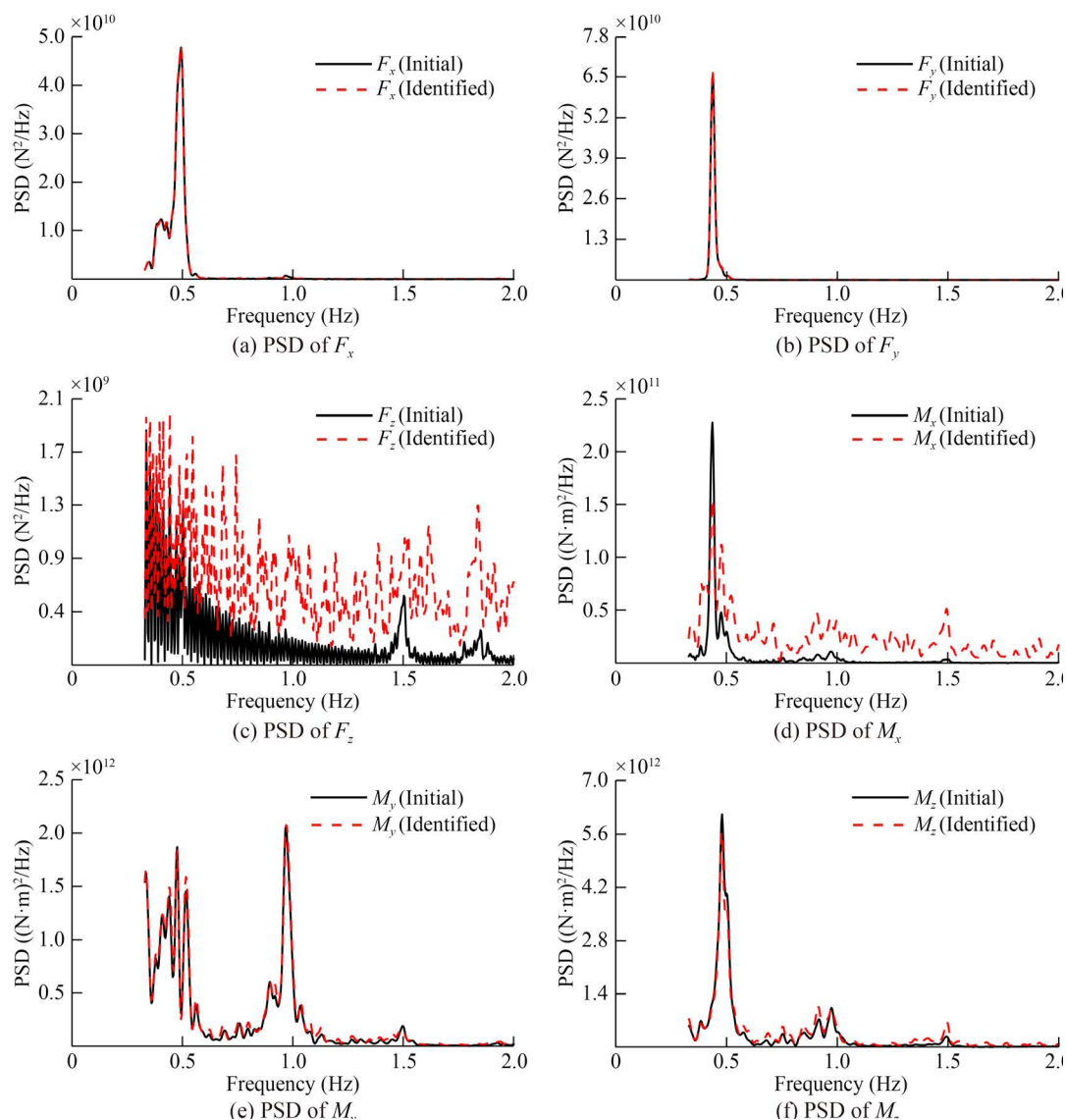


Figure 26 Wind load identification results using expanded sensor arrangement

tively improve wind load monitoring under varying wind directions, it also requires processing a larger volume of sensor data and increases the demands on data storage, processing, and transmission. However, modern wind turbines are generally equipped with advanced monitoring and control systems, such as Supervisory Control and Data Acquisition systems, which can efficiently manage data streams from multiple sensors. Hence, despite the increased number of monitoring points, the implementation of the expanded sensor arrangement remains feasible, given advancements in current monitoring technologies.

5 Numerical examples

5.1 Jacket wind turbine with 30° wind direction and 12 m/s wind speed

Fuzzy optimization of strain sensor locations and directions considering model ill-posedness and monitoring data error has been explained in detail above, and the load identification results are satisfactory at 0° wind direction. The load identification results with 30° wind direction and 12 m/s average speed are shown in Figure 27, in which the monitoring data error of $\pm 10 \mu\epsilon$ is considered.

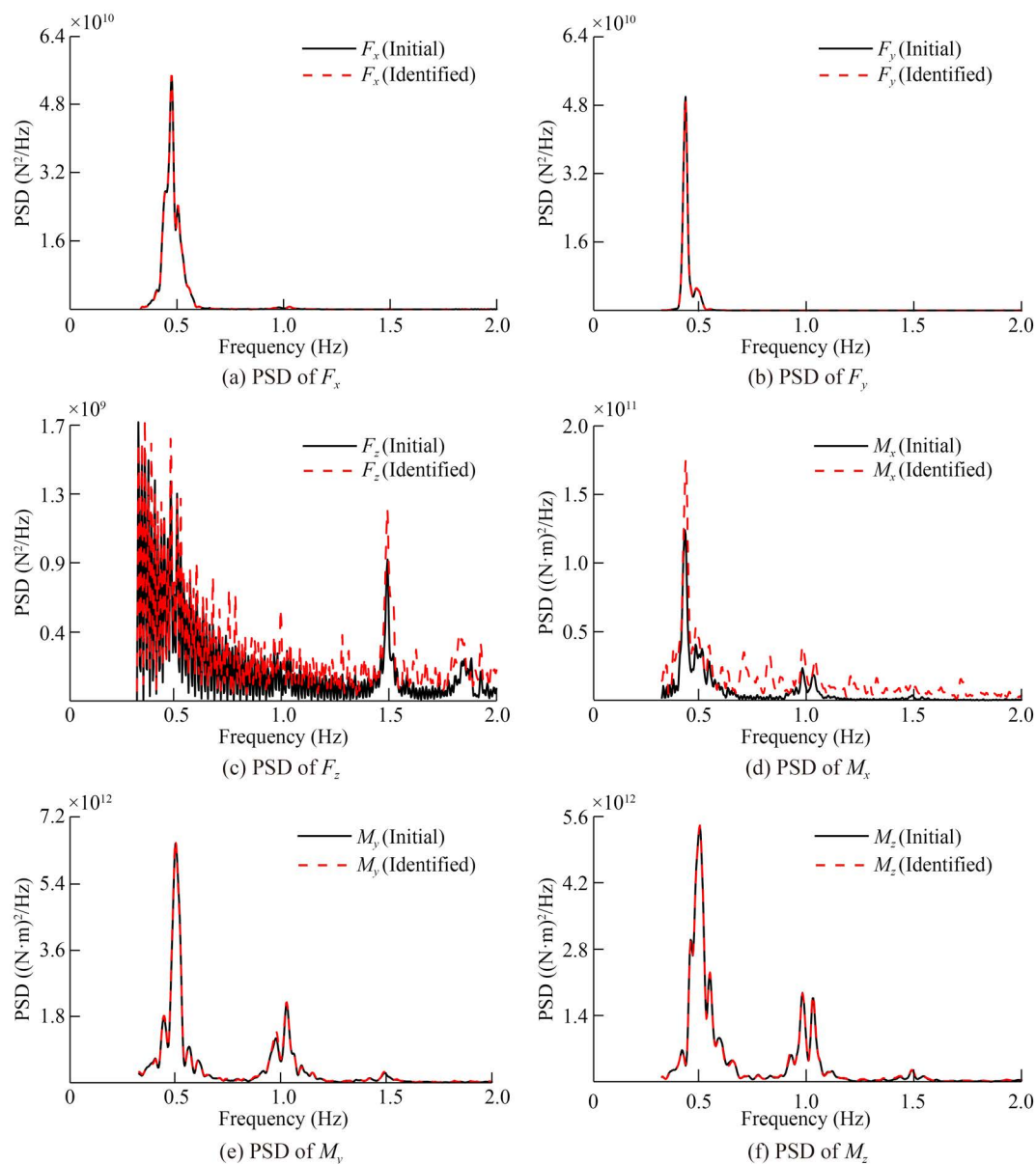


Figure 27 Load identification results with 30° wind direction and 12 m/s average speed

The fatigue damages corresponding to the identification results and the original wind load are presented in Table 3.

Table 3 reveals that the errors between the fatigue damages corresponding to the identification results and the

Table 3 Comparison of fatigue damage values

Node	Fatigue damage (original)	Fatigue damage (identified)	Error (%)
305	0.00107	0.00120	12.10
380	0.00246	0.00264	7.30
455	0.00373	0.00390	4.60
530	0.00208	0.00221	6.20
30	0.00107	0.00120	11.50
80	0.00246	0.00263	6.90
155	0.00373	0.00389	4.40
230	0.00208	0.00221	6.10

original wind load do not exceed 12.1%. Hence, the identification result of wind load is accurate when considering the model's ill-posedness and the monitoring data error of $\pm 10 \mu\epsilon$ simultaneously.

5.2 Jacket wind turbine with 60° wind direction and 16 m/s wind speed

The load identification results with 60° wind direction and 16 m/s average speed are shown in Figure 28, in which the monitoring data error of $\pm 10 \mu\epsilon$ is considered.

The fatigue damages corresponding to the identification results and the original wind load are presented in Table 4.

Table 4 reveals that the errors between the fatigue damages corresponding to the identification results and the

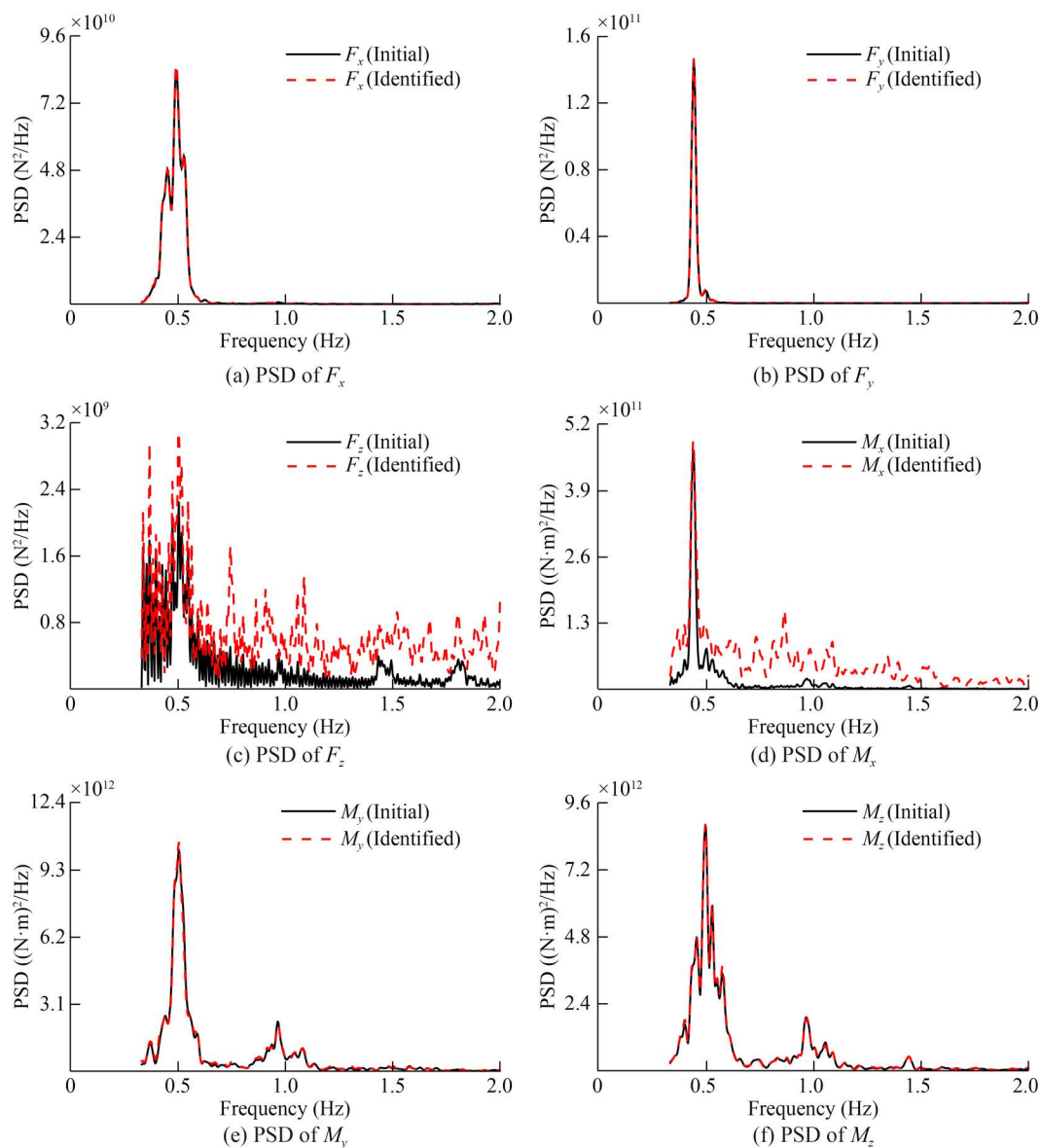
**Figure 28** Load identification results with 60° wind direction and 16 m/s average speed

Table 4 Comparison of fatigue damage values

Node	Fatigue damage (original)	Fatigue damage (identified)	Error (%)
305	0.00526	0.00541	2.80
380	0.00876	0.00884	1.00
455	0.01231	0.01252	1.70
530	0.00832	0.00861	3.50
30	0.00526	0.00543	3.10
80	0.00880	0.00888	1.00
155	0.01237	0.01257	1.60
230	0.00836	0.00863	3.20

original wind load are less than 4%. Thus, the identification result of wind load is accurate when considering model ill-posedness and monitoring data error of $\pm 10 \mu\epsilon$ simultaneously.

6 Conclusions

In this study, a fuzzy optimization method was developed to reduce the ill-posedness of mathematical models and the influence of monitoring data errors on the wind load monitoring results. The ill-posedness index of the mathematical models and the error index of the monitoring data were weighted and unified to form a comprehensive objective function. Several numerical examples were used to validate the effectiveness and reliability of the developed method. The following conclusions are drawn:

1) Easing the ill-posedness of the frequency response function matrix at the structural resonance frequency facilitates effective reduction in ill-conditioning across the entire frequency range.

2) Fuzzy optimization aims to minimize the error of fatigue analysis results with the input of wind load identification results, and the weighted sum of the ill-posedness index of the model and the error index of the monitoring data is appropriate as the objective function of the fuzzy optimization method.

3) The final optimized mathematical model is an indeterminate equation; that is, 6 unknown quantities are solved by 24 known quantities, which is helpful in enhancing the accuracy and stability of the identification results.

4) The relative error in computing the structural fatigue damage using the power spectral density of the identified load does not exceed 13.8% when considering model ill-posedness and monitoring data error of $\pm 10 \mu\epsilon$.

5) The proposed method cannot identify wind pressure loads on the tower structure or wave loads on the foundation. The main challenges are as follows: Wind pressure loads exhibit a nonlinear distribution along the height of the tower. Identifying the detailed spatial distribution of

wind pressure loads requires an impractically large number of sensors. Wave loads mainly affect the stress distribution in the splash zone of the foundation. However, this region is also influenced by upper wind turbine loads. Hence, sensors must be deployed on the tower structure and the foundation structure to distinguish wind and wave loads accurately.

6) Future research should focus on developing an equivalent load model that represents distributed wind pressure as discrete horizontal forces at different tower heights and expanding the sensor network to include the tower structure and the foundation splash zone to enable simultaneous identification of wind and wave loads.

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