

Numerical Investigation of Damage to Typical Plate Structures Induced by Sequential Underwater Explosions

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Abstract

As a critical structural component in marine engineering, the air-backed stiffened plate frame plays a vital role in large-scale offshore installations. Underwater explosions can cause severe damage to nearby structures. This paper investigates the cumulative damage characteristics of the air-backed stiffened plate frame under underwater explosion loads. Compared to a single explosion, secondary explosions induce more complex structural damage due to material hardening after initial loading and substantial changes in load distribution on deformed surfaces. The arbitrary Lagrangian–Eulerian method is adopted to simulate the entire dynamic response of the air-backed stiffened plate frames subjected to sequential secondary underwater explosions. Additionally, the damage characteristics of plate frame structures with varying materials, plate thicknesses, and stiffener configurations under sequential secondary underwater explosion loading are further examined. Results indicate that, under the same charge weight and standoff distance, sequential secondary explosions markedly exacerbate localized structural damage, with extreme cases exhibiting notable differences in damage models. In contrast, a single explosion leads to more pronounced overall structural damage, characterized by larger deformation amplitudes of the plate structure and greater energy absorption. The influence of the primary shock factor on the damage enhancement effect of structures under secondary blast loading is substantial. A smaller primary shock factor leads to larger damage amplification in subsequent explosions. Stiffened plate structures with varying material types, plate thicknesses, and stiffener configurations exhibit consistent patterns.

Keywords Stiffened plates; Structural response; Cumulative damage; Sequential secondary load; Underwater explosion

1 Introduction

The air-backed stiffened plate frame is a widely used support structure and serves as a vital component in marine facilities such as naval vessels and offshore platforms. Research on the effects of sequential secondary underwater explosion loads on marine structures largely focuses on

their interaction with air-backed stiffened plate frames. Underwater explosion loads primarily comprise shock waves and bubble loads (Choi et al., 2024; Didoszak and Kwon, 2025; Zhang et al., 2023; Zhang et al., 2024a; Feng et al., 2025; Xu et al., 2025). The structural damage caused by underwater explosions represents a highly complex fluid–structure interaction problem (Bardiani et al., 2025; Mannacio et al., 2024; Xu et al., 2025). Unlike the initial damage, the structural response during secondary damage is considerably more intricate. Residual stress fields remaining after the initial loading, along with plastic-hardening materials, alter the mechanical behavior of the structure. During secondary damage, the deformed structure modifies the propagation paths and reflection patterns of loads on its surface, resulting in altered load distribution. These coupled factors substantially complicate the analysis of structural responses to secondary damage. A substantial amount of research has been conducted by scholars both domestically and internationally on the structural damage caused by underwater explosions. The primary research methods include theoretical derivation, experimental studies, and numerical analysis. Theoretical derivation mainly involves using energy the-

Article Highlights

- The sequential secondary explosion markedly exacerbates localized structural damage.
- The shock factor of the initial explosion notably affects the damage effectiveness of subsequent underwater explosions.
- Stiffened plate structures with varying material types, plate thicknesses, and stiffener configurations exhibit consistent dynamic response patterns.

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ory to calculate the maximum deformation of plate structures under explosive loads (Mostofi et al., 2018; Yao et al., 2024), which provides accurate predictions for simple plate structures. The energy method is based on the principle of energy conservation, establishing a structural damage prediction model where the energy exerted by the explosive load is assumed to equal the plastic deformation energy of the structure. For simple structures such as stiffened plates, this energy method enables rapid deformation predictions, with results typically demonstrating a good agreement with the numerical calculations, with an error margin of 15%. However, the energy method overlooks the complex dynamic processes involved; thus, it cannot provide a detailed reflection of the structural response process. In experimental research, underwater explosion tests conducted in water tanks have been used to investigate the damage characteristics of ordinary steel plates and composite stiffened plates (Liu et al., 2022). Zhu et al. (2010) summarized the damage modes of stiffened plates subjected to underwater explosions, while Suresh and Ramajeyathilagam (2021) investigated the deformation behaviors and key influencing factors of plate structures under various loading conditions. Additionally, spark discharge bubble tests have emerged as an alternative research method (Chen et al., 2023). Various numerical methods have been employed in these investigations, including the Runge-Kutta discontinuous Galerkin method (RKDG) (Qin et al., 2021), Runge-Kutta discontinuous Galerkin infinite element method (RKDG-FEM) coupling method (Ge et al., 2020), Smoothed Particle Hydrodynamics Reproducing Kernel Particle Method (SPHRKPM) method (Peng et al., 2021), EFEM method (Xu et al., 2021), and ALE method (Cui et al., 2015). Each method offers distinct strengths and limitations, and the selected method typically depends on the specific conditions when addressing plate structure damage. Apart from the above methods, Weighted compact nonlinear schemes and machine learning methods are emerging research approaches that are gradually being applied in the field of underwater explosions (Chen and Deng, 2024; Zhao et al., 2025).

Extensive research has been conducted domestically and internationally on the cumulative damage of plate structures caused by multiple underwater explosions. Compared to single explosions, multiple underwater explosions present more complex load distributions due to coupled interactions among multiple bubbles, including phenomena such as multi-bubble mutual coupling (Rungsiyaphornrat et al., 2003). During the pulsation of dual bubbles, their interactions primarily manifest in four distinct behaviors: mutually distant liquid jets caused by bubble collapse, mutually directed liquid jets, bubble coalescence, and ejection effects (Chew et al., 2013; Fong et al., 2009). In out-of-phase dual bubble explosions, additional coupling effects between shockwaves and bubbles are observed during

their dynamic evolution (Choi et al., 2024). The presence of nearby walls substantially affects bubble pulsation, often inducing jets directed toward the wall; this influence is strongly dependent on the standoff distance (Chew et al., 2011). When bubbles interact with elastoplastic structures, the standoff distance notably affects bubble morphology, migration velocity, and jetting behavior. Additionally, the structural frequency sensitivity must be considered during bubble–structure interactions (Javier et al., 2021). Underwater multi-point initiation leads to a superposition effect of shock waves. Studies indicate that the peak overpressure of shock waves is significantly influenced by the standoff distance and structural size. Specifically, the superposition effect intensifies with increasing standoff distance and decreasing structural dimensions (Li et al., 2018). For cylindrical shells subjected to underwater explosion, dual explosive sources cause more severe damage to the outer plates than a single source with equivalent yield, whereas the single source inflicts greater damage to the internal structure (Chen et al., 2024).

The structural response under multiple loadings is more complex than that under a single loading. Research has shown that residual stresses existing in the grillage after the initial blast can inhibit central deflection during subsequent loadings (Henchie et al., 2014). This inhibitory effect becomes markedly pronounced as the number of explosions increases, primarily due to the accumulation of residual stresses and material hardening mechanisms (Rezasefat et al., 2019). Under the same total charge mass, plate structures subjected to multiple small equivalent explosions exhibit smoother deformation profiles (Nasiri et al., 2022). When the deformation of stiffened plates is small, welds considerably strengthen the structure. However, as deformation increases, membrane effects begin to dominate the structural resistance, diminishing the reinforcing role of the welds (Behtaj et al., 2022).

Huang et al. (2023) and Zhang et al. (2019) studied the cumulative damage characteristics of air-backed stiffened plate structures subjected to multiple underwater explosions. Their study revealed that repeated explosive load application leads to a pseudo-shakedown phenomenon. This phenomenon refers to a dynamic response where a structure initially experiences plastic deformation under repeated impacts. However, the intensity of this impact drops below a critical threshold, plastic strain accumulation ceases, and the structural response stabilizes over time. The concept of pseudo-shakedown was first introduced by Jones in 1973. Additionally, the central deflection of the plate was found to have a linear correlation with the thinning rate and a quadratic correlation with the shock factor. Li et al. (2020) investigated the damage characteristics of steel circular plates subjected to multiple underwater explosions, finding that the thinning rate at the center of the steel plate increases linearly with the center

deflection. Lin et al. (2021) studied the damage effects of multipoint synchronized underwater explosion shock waves on circular plates, analyzing the destructive effects of these synchronous explosions. Zhang et al. (2024b) employed the ALE method to investigate the damage and deformation characteristics of ring-stiffened cylindrical shells under double charge underwater explosions, demonstrating that double charges cause considerably larger deformations in the cylindrical shell compared to a single explosion of the same total mass. Considering cumulative damage in complex structures, Cheng et al. (2024) explored the damage characteristics of complex multi-compartment structures subjected to simultaneous and delayed underwater explosions. Their findings showed that two underwater explosions led to cumulative damage effects, resulting in an increase in the plastic deformation of the structure. Li et al. (2024) conducted experimental studies on the cumulative damage of ship hull beams under repeated underwater explosions, noting that the cumulative deflection of the hull beam increases approximately linearly with the number of explosions, while the residual strength of the hull beam continuously decreases as the number of explosions increases.

Overall, while existing research on the damage caused by single underwater explosions to plate frame structures is relatively comprehensive, offering valuable insights into damage mechanisms and influencing factors, studies on the cumulative effects of multiple underwater explosions are still limited. Particularly, the underlying cumulative damage mechanisms driving cumulative damage remain underexplored. Moreover, the differences and interrelationships between structural responses to multiple versus single damage events have not been comprehensively discussed. The present study aims to address these research gaps by conducting a detailed analysis of the cumulative damage characteristics of plate frame structures under sequential underwater explosion loads. The analysis focuses on local responses, energy absorption, and load distribution on wall surfaces and compares these with the characteristics observed under single damage events. The findings provide novel insights into the effective mitigation of damage in plate frame structures.

2 Numerical methods

2.1 ALE method

The behavior of solid materials is typically modeled using the Lagrangian method, which offers computational efficiency. However, considerable deformations in metallic materials can severely compromise the accuracy and efficiency of this method when applied to near-field underwater explosion problems, rendering it impractical for cal-

culations. Conversely, while the Eulerian method is well-suited to modeling fluid behavior, it does not allow for grid movement. Aiming to overcome this limitation, complex techniques such as grid smoothing and transport are often employed, thereby reducing the accuracy of Eulerian computations. The arbitrary Lagrange–Eulerian (ALE) method has been proposed as a means of addressing the strengths and weaknesses of the aforementioned methods. By extending the Lagrangian algorithm, the ALE method allows grid points to either move with the physical quantities or remain fixed. The ALE method combines the advantages of Lagrangian and Eulerian techniques, which substantially enhances grid flexibility, making it particularly effective for simulations involving fluid–structure interactions.

In the ALE method, the continuous medium at the initial time t_0 and the deformation time t are denoted as Δ_x and Δ_x , respectively. A reference configuration, which is independent of the initial time and the current configurations, is introduced and denoted as Δ_κ . A reference coordinate system $O\kappa_1\kappa_2\kappa_3$ is defined to specify the positions of each point in the reference configuration. The positions of each point in the reference configuration are determined by its position vector κ in the reference coordinates. Therefore, in the ALE method (Belytschko et al., 2014), the derivative of any function f relative to the moving body is denoted as follows:

$$\left(\frac{\partial f}{\partial t}\right)_x = \left[\frac{\partial f(\kappa, t)}{\partial t}\right]_\kappa + c_\alpha \frac{\partial f}{\partial x_\alpha} \quad (1)$$

where $c_\alpha = u_\alpha - \omega_\alpha$ represents the convective velocity described by the ALE method, u_α is the material velocity of particle X , and ω_α is the material velocity at the reference point κ , which is also the grid velocity. Thus, when converting the basic equations of continuum mechanics into the governing equations in the current configuration as described by the ALE method (Belytschko et al., 2014), the specific expression is as follows:

Conservation of mass equation:

$$\rho_{,t[\chi]} + \rho_{,i}c_i + \rho v_{k,k} = 0 \quad (2)$$

Conservation of momentum equation:

$$\rho(v_{i,t[\chi]} + v_{i,j}c_j) = \sigma_{ji,j} + \rho b_i \quad (3)$$

Conservation of energy equation:

$$\rho(E_{,t[\chi]} + E_{,i}c_i) = \sigma_{ij}D_{ij} + b_i v_i + (k_{ij}\theta_{,j})_{,i} + \rho s \quad (4)$$

where ρ represents density, t represents time, χ represents the reference coordinate system, c_i represents the transmission speed, v represents the material velocity, σ_{ij} is the Cauchy stress tensor, b_i denotes the tension per unit mass, E is the internal energy per unit mass, D_{ij} represents the deformation rate tensor, k_{ij} represents the heat conduction

matrix, θ represents temperature, and ρs represents the heat source per unit volume.

The fluid–structure interaction is determined by the interaction forces between ALE and Lagrangian elements. Initially, the process begins by identifying the contact behavior between the Lagrangian and ALE components. When a coupled Lagrangian surface is detected to intersect with an ALE element, the coupled nodes are marked at the current time step, and the motion trajectories of the two materials are then tracked. Subsequently, the penetration distance of the ALE material into the Lagrangian surface is computed. Based on this penetration, the coupling force between the materials is finally calculated. The algorithm for calculating the coupling force is established by defining master and slave coupling points on the contact surface (Shyue, 2001). A linear spring relationship is established between the detected master–slave coupling points to calculate the coupling force, with a parallel damper incorporated to eliminate high-frequency oscillatory behaviors.

Figure 1 shows the specific situation of the interaction between the coupling points in the ALE method. The expression for the contact force algorithm is as follows:

$$F_s = -F_c = k \cdot d + C_i \cdot \dot{d} \quad (5)$$

where F_c denotes the contact force acting on the Lagrangian nodes, F_s represents the contact force acting on the ALE grid nodes, k is the spring stiffness, $\dot{d}$ is the penetration vector of the contact surface, and C_i indicates the damping coefficient.

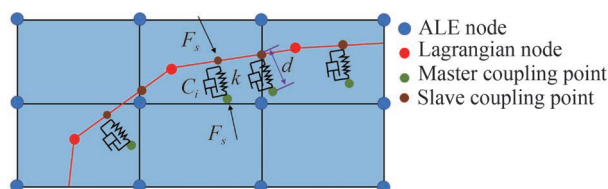


Figure 1 Arbitrary Lagrangian–Eulerian (ALE) dynamic coupling interface algorithm

This paper investigates the cumulative damage characteristics of air-backed stiffened plate frames subjected to sequential underwater explosions, with a primary focus on accurately modeling damage accumulation. A critical aspect of this study is the effective consideration of cumulative damage effects. Before simulating the second explosion, retaining the computational results of key physical quantities such as stress, strain, and displacement from the initial explosion is crucial. This strategy ensures that the influence of the initial explosion on the structure is fully captured in the subsequent analysis. In this work, the full restart technique is employed to simulate the secondary damage process. During the initial simulation, a restart file is generated to store the computational results. In the secondary analysis, the initial calculation data is retrieved by invoking the restart file. The stress, strain, and displace-

ment of the structure are then initialized using the STRESS_INITIALIZATION keyword, setting the initial conditions for the subsequent calculation phase. The simulation allows for the successful completion of the secondary cumulative damage.

2.2 Equation of state

In simulating near-field underwater explosions, the equation of state for the fluid medium is used to control pressure fluctuations based on temperature, density, energy, and other variables.

1) Eos of water

The Mie–Grüneisen equation of state is adopted for the water medium (Zhang and Yang, 2012), and its specific expression is as follows:

$$P = \frac{\rho_0 c^2 \mu \left[1 + \left(1 - \frac{\gamma_0}{2} \right) \mu - \frac{a}{2} \mu^2 \right]}{\left[1 - (S_1 - 1) \mu - S_2 \frac{\mu^2}{1 + \mu} - S_3 \frac{\mu^3}{(1 + \mu)^2} \right]} + (\gamma_0 + a \mu) E \quad (6)$$

When the fluid is in a negative pressure state, the state equation is as follows:

$$P = \rho_0 c^2 \mu + (\gamma_0 + a \mu) E \quad (7)$$

where μ represents the relative volume change, defined as $\mu = 1/V - 1$; P stands for pressure; E represents the internal energy per unit volume of water; V represents the relative volume of water; and ρ_0 and c correspond to the density of water and the speed of sound in water, respectively. $S_1 - S_3$ are constants, and the specific numerical values are listed in Table 1.

Table 1 Parameters of the water state equation

ρ_0 (kg/m ³)	c (m/s)	S_1	S_2
1000	1480	2.56	1.986
S_3	γ_0	a	E (J·m ³)
0.226	0.5	2.67	2.02×10^5

2) Eos of explosive

The detonation behavior of explosives is modeled using the Jones–Wilkins–Lee equation of state. In this study, the high-energy explosive TNT is considered, and the form of the state equation is as follows (Zhang et al., 2024c):

$$P = \rho_0 c^2 \mu + (\gamma_0 + a \mu) E \quad (8)$$

where $\eta = \rho/\rho_0$; ρ_0 represents the initial density; ρ represents the reference density; A , B , R_1 , R_2 , and ω are material constants; and E is the internal energy per unit mass. The specific values for these parameters are shown in Table 2.

Table 2 Parameters of TNT explosive materials

ρ_0 (kg/m ³)	D (m/s)	A (GPa)	B (GPa)
1630	6930	371	3.23
R_1	R_2	A	E (J·m ³)
4.15	0.95	0.3	7×10^9

3) Eos of air

The state equation for the air medium is described by the *LINEAR_POLYNOMIALMODEL (Zhang et al., 2024c), and is given by the following expression:

$$P = C_0 + C_1 v + C_2 v^2 + C_3 v^3 + (C_4 + C_5 v + C_6 v^2) E \quad (9)$$

where $v = \rho/\rho_0 - 1$, E represents the energy per unit volume, ρ_0 represents the initial density, ρ represents the reference density, and C_{0-6} are material constants. The specific values for these parameters are shown in Table 3.

Table 3 Parameters of the air state equation

C_0	C_1	C_2	C_3	C_4	C_5	C_6	V_0	E (J·m ³)
0	0	0	0	0.4	0.4	0	1	2.53×10^5

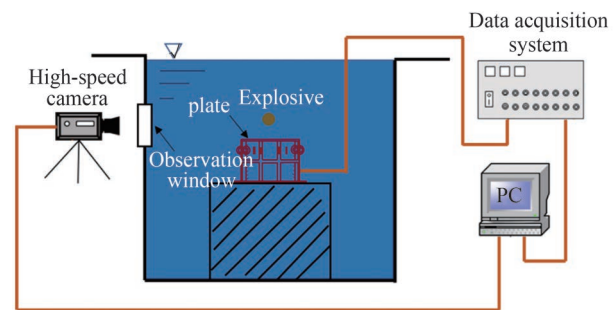
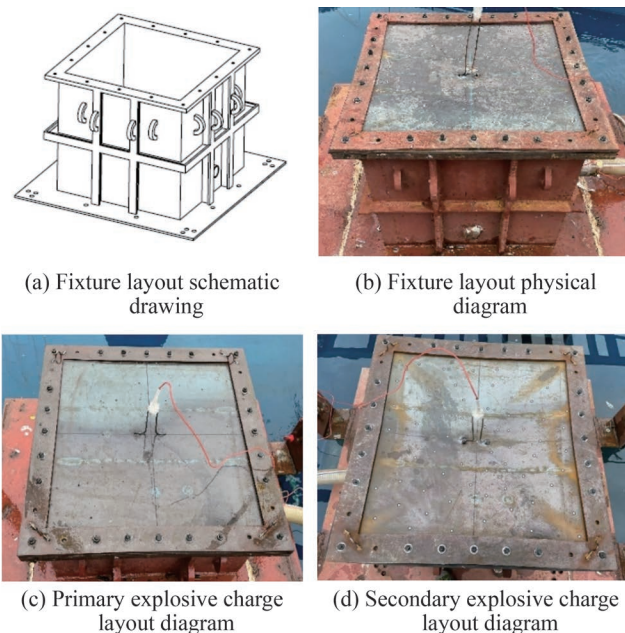
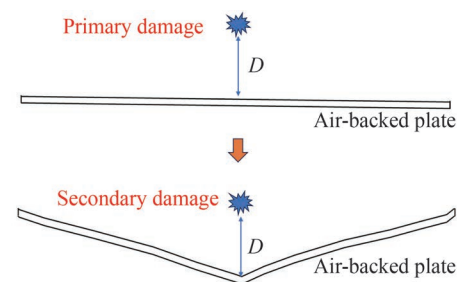
3 Cumulative damage characteristics of air-backed stiffened plate

3.1 Numerical model and verification

The reliability of the research method directly impacts the accuracy of the numerical simulation calculations. Before applying the ALE method to investigate cumulative damage mechanisms in plate frame structures subjected to sequential underwater explosions, experimental validation was conducted through a series of air-backed stiffened plate tests under secondary underwater explosions. The sequential underwater explosion experiment was conducted in a cubic water tank with a 4 m edge length. The stiffened plate was securely mounted on a custom-designed fixture to maintain air-backed conditions. The fixture comprised a 760 mm × 760 mm × 4 mm Q235 steel base plate, reinforced with T-shaped profiles (cross-sectional dimensions: 3 mm × 25 mm/3 mm × 40 mm). The explosive charge comprised 10 g of RDX, placed at a standoff distance of 100 mm from the structure.

Figure 4 illustrates a schematic defining the secondary explosion standoff distance. In this study, the secondary explosion standoff distance is defined as the actual distance between the explosive charge and the deformed structure.

A scaled finite element model was developed to maintain geometric consistency with the experimental speci-

**Figure 2** Schematic of the working principle of the experimental equipment**Figure 3** Schematic of the experimental setup and charge arrangement**Figure 4** Schematic of the standoff distance for sequential secondary explosions

men. The structural configuration mirrored the dimensions of the physical test model, featuring a 760 mm × 760 mm × 4 mm plate frame fabricated from Q235 steel. A structural mesh with a 5 mm element size was applied throughout the model. The computational domain covered a three-dimensional space measuring 4.1 m × 4.0 m × 4.0 m, comprising a 4 m water column and a 0.1 m air layer. A gradually varying grid division technique was employed, achieving a

minimum element size of 4 mm, resulting in approximately 8.61 million finite elements in total. Rigid wall boundary conditions were applied to the fluid domain boundaries to simulate the effects of tank confinement. The specific flow field configuration is shown in Figure 5(a), and the finite element model of the air-backed stiffened plate is presented in Figure 5(b).

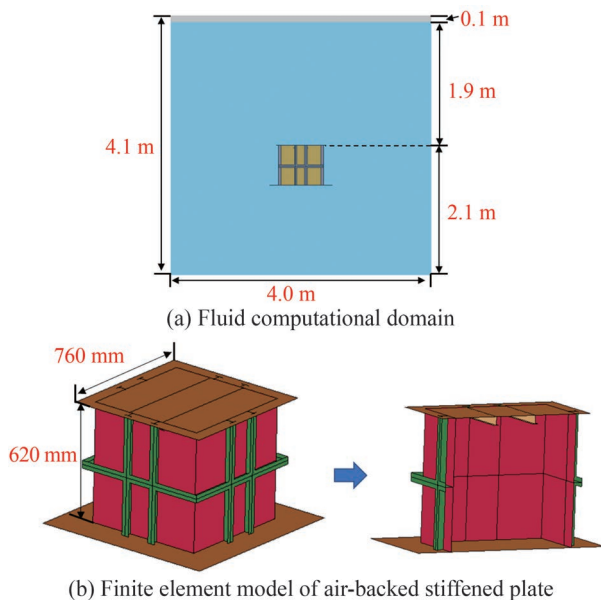


Figure 5 Finite element computational model

The plate structure material is Q235 steel, and its constitutive behavior is defined using the Cowper–Symonds (C–S) model. The C–S model is a strain-rate sensitive material model, with specific expression given as follows:

$$\sigma_d = \sigma_y \left[1 + \left(\frac{\dot{\epsilon}}{C} \right)^{\frac{1}{p}} \right] \quad (10)$$

where σ_d represents the dynamic yield stress, σ_y represents the static yield stress, $\dot{\epsilon}$ represents the equivalent strain rate, and C and p are strain rate parameters.

Table 4 Main parameters of Q235 steel material (Chen et al., 2012)

ρ (kg/m ³)	E (GPa)	μ	σ_y (MPa)
7800	210	0.3	235
C	p	ϵ_f	
40.4	5	0.28	

The equations of state for air and water were implemented as previously described. Based on the findings of Tong (2024), a conversion factor of 1.3 between RDX and TNT was adopted, and the simulation used TNT equivalence with this factor for modeling. This section investigates the damage to the plate frame under conditions involv-

ing a 10 g charge and a standoff distance of 100 mm. In previous research on structural damage caused by underwater explosions, the distinction between the shock wave and the bubble load stages has not been clearly established. The shock wave load generally lasts in the millisecond range, while the bubble load persists for a longer time in the second range. In this paper, based on previous preliminary divisions of the shock wave and bubble load stages, and considering the contact time between the detonation products and the deformed structure, the shock wave load action stage occurs before 1 ms, and the bubble load action stage occurs after 1 ms. Figure 6 shows the diffusion process of the detonation products between 0 ms and 1 ms. At 0.3 ms, the detonation products have not yet reached the structure, whereas those at 1 ms come into contact with it.

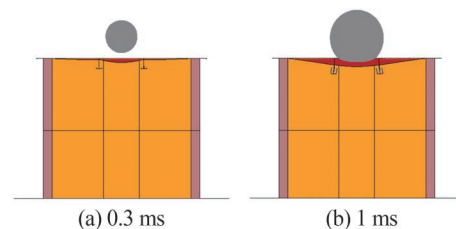


Figure 6 Expansion process of detonation products in water

Figure 7 illustrates the propagation of shock wave pressure and the resulting structural deformation. As shown in the figure, the shock wave propagates outward in a nearly spherical waveform. At 0.04 ms, the pressure wave reaches the structure, and the shock wave continues to diffuse outward. Reflection occurs at the structural surface, generating a compressive wave that converges with the incident wave, thereby forming a localized high-pressure zone. Over time, the reflected wave diffuses outward in a spherical pattern. Between 0.3 ms and 0.7 ms, the reflected wave rapidly attenuates with a sharp pressure drop, while the incident wave continues to propagate outward at higher pressure. During this phase, the structure undergoes inward depression under the pressure loading, reaching a maximum deflection of 56.7 mm induced by the shock wave. The final maximum deformation of the structure is 62.8 mm, with the shock wave-induced deformation contributing 90.3% of the total deformation.

Figure 8 illustrates the pulsation dynamics of an underwater explosion bubble and the resulting structural deformation. The left panel depicts experimental observations, while the right panel presents numerical simulations. Notably, the bubble remains in the expansion phase until 28 ms after detonation. Considering the influence of the underlying structure, the bottom of the bubble flattens during expansion, while the top remains unaffected. At 28 ms, the bubble reaches its maximum volume, after which contraction initiates. During this phase, the upper portion of the bubble retains a quasi-spherical geometry, while localized

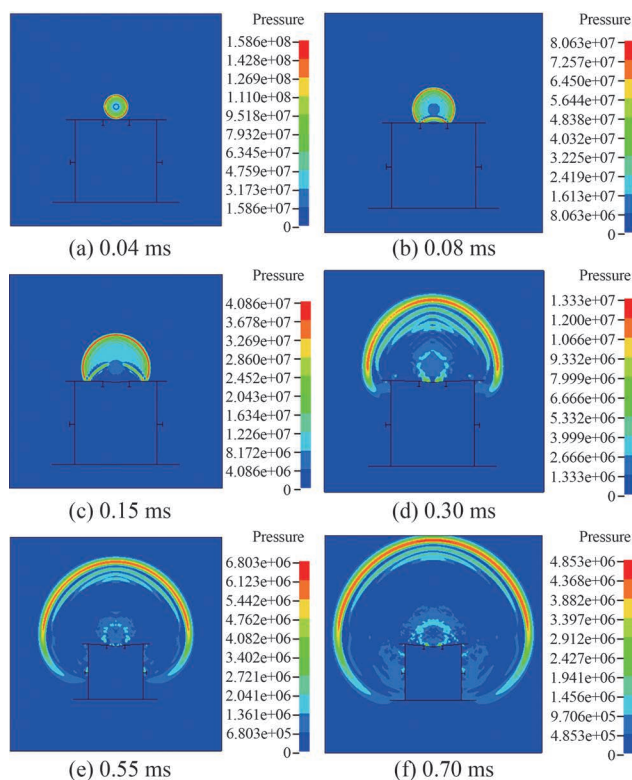


Figure 7 Propagation process of the shock wave during primary underwater explosions

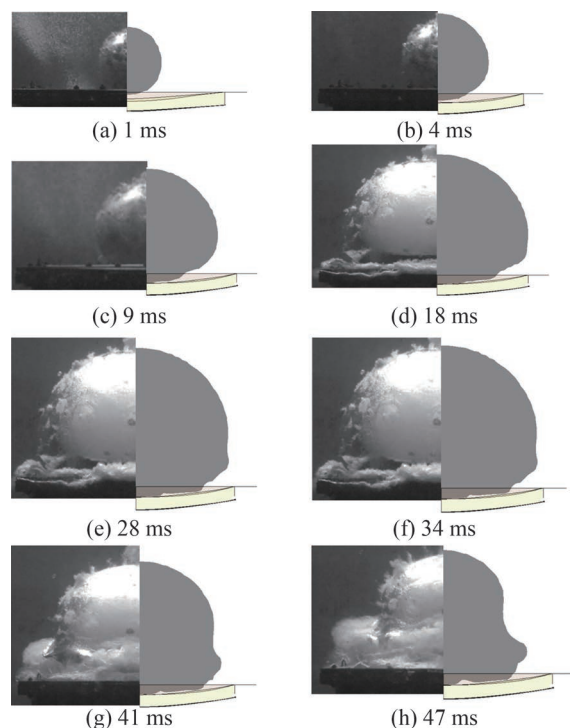


Figure 8 Bubble pulsation and structural damage evolution under primary underwater explosion

implosion near the structure leads to the formation of smaller bubbles. Throughout the process, the structure undergoes indentation deformation due to the interaction

between the shockwave and the bubble. The structural deformation induced during the bubble loading phase was 6.1 mm, accounting for 9.7% of the total structural deformation. The simulated bubble pulsation closely aligns with the experimental results, validating the numerical approach.

Similar to the primary explosion, the shock wave phase of the secondary explosion is defined by the time of contact between detonation products and the structure. The period before 1 ms is determined to correspond to the shock wave loading phase, while the period after 1 ms is identified as the bubble loading phase.

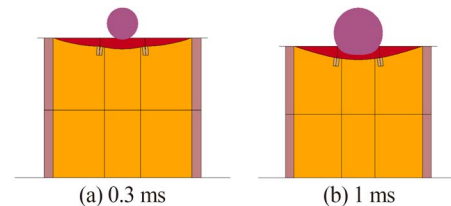


Figure 9 Expansion process of detonation products in secondary underwater explosion

Figure 10 illustrates the propagation of the shockwave from the secondary underwater explosion. Similar to the primary explosion, the shockwave initially propagates outward in a spherical pattern. Reflection occurs upon reaching the structural surface, generating a compression wave. This compression wave converges with the incident wave at the structural surface, forming a high-pressure zone that continues to propagate outward. Over time, the pressure rapidly attenuates, while the incident wave persists, maintaining relatively higher pressure. The maximum deformation induced by the secondary explosion on the structure is 21.7 mm, with the shockwave contributing 18.4 mm of this deflection, accounting for 84.8% of the total deformation.

Figure 11 shows the bubble pulsation and the evolution of structural damage during the secondary underwater explosion. Unlike the initial explosion phase, the pre-existing structural deformation leads to a more pronounced spherical expansion of the bubble during the early detonation stage. As the bubble continues to pulsate, the top maintains a nearly spherical shape, while the bottom remains flattened due to boundary effects of the structure. Compared to the initial explosion, the bubble exhibits a more prominent outward expansion, reaching its maximum volume at 26 ms. Subsequently, the bubble enters the contraction phase, resulting in progressively increasing structural indentation due to sustained loading. The structural deformation induced by the bubble load is 3.3 mm, which accounts for 15.2% of the total structural deformation.

Figure 12 shows a comparison between the 3D scanning results from sequential secondary underwater explosion tests and the corresponding numerical simulation results. Under the initial loading conditions, considerable deformation is predominantly concentrated at the geometric center of the plate structure, exhibiting a relatively uniform distri-

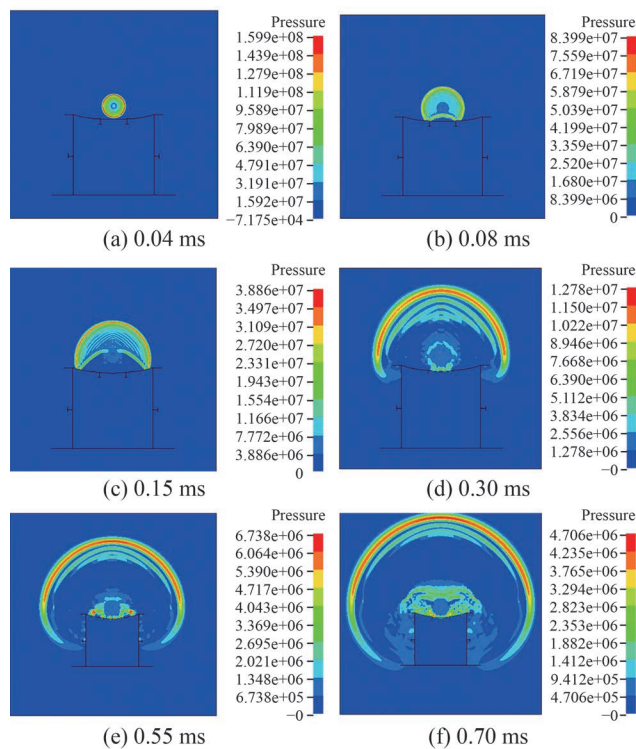


Figure 10 Propagation process of the shock wave during secondary underwater explosions

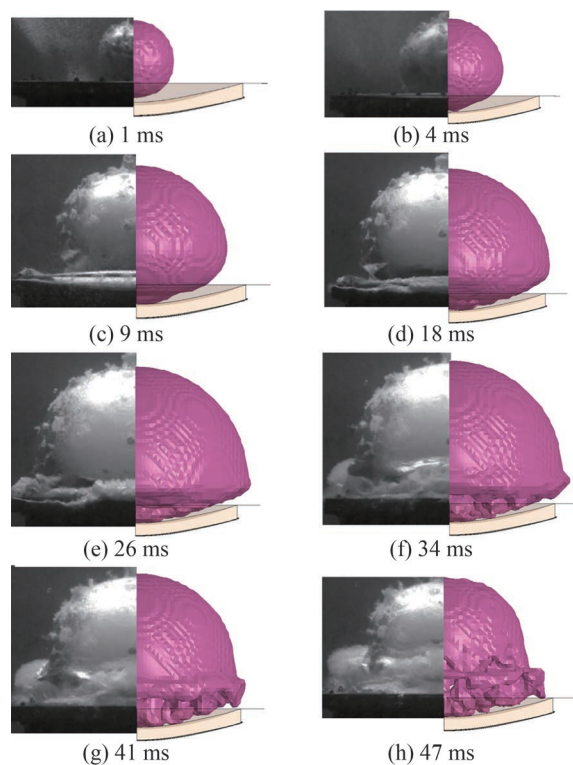


Figure 11 Bubble pulsation and structural damage process during secondary underwater explosion

bution pattern that is unaffected by the lateral reinforcement ribs. The central deformation region takes on an

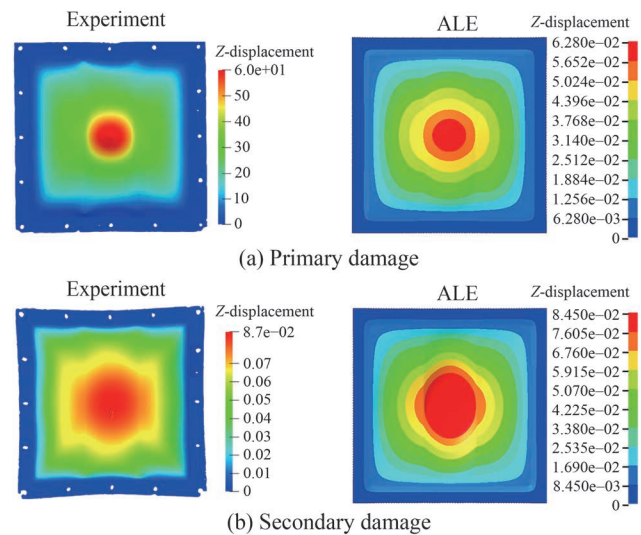


Figure 12 Comparison of normal displacement in the structure under sequential secondary underwater explosions

approximately circular shape, with experimental measurements recording a maximum deflection of 60.1 mm, compared to 62.8 mm in the numerical simulations. During secondary loading, deformation propagates radially outward. The reinforcement ribs enhance lateral structural stiffness, inducing elliptical distortion in the central deformation area. The maximum deflections in the experiments and simulations are 87.2 and 84.5 mm, respectively, demonstrating a discrepancy of less than 6%. This small difference validates the accuracy of the numerical model in simulating underwater explosions. The numerical simulation results of plate frame deflection show a positive deviation (overestimation) from experimental measurements in the first instance, whereas a negative deviation (underestimation) occurs in the second instance. This discrepancy is mainly attributed to the difference in the initial deformation of the plate frame between the numerical calculations and experimental results after the first explosion. This initial deformation change alters the boundary conditions for the secondary explosion, ultimately resulting in a negative deviation in the numerical calculation of deformation during the secondary explosion.

3.2 Simulation cases

The numerical simulations were performed under reference test conditions, with TNT serving as an equivalent substitute for RDX using an equivalence factor of 1.3 (Tong, 2024). The simulation employed charge weights of 5 g, 10 g, and 20 g to model different explosive scenarios. The secondary damage standoff distance ranged from 0 to 300 mm, with each primary damage condition incorporating five discrete secondary standoff intervals. The detailed configurations of these parameters are tabulated in Table 5.

Table 5 Condition arrangement table

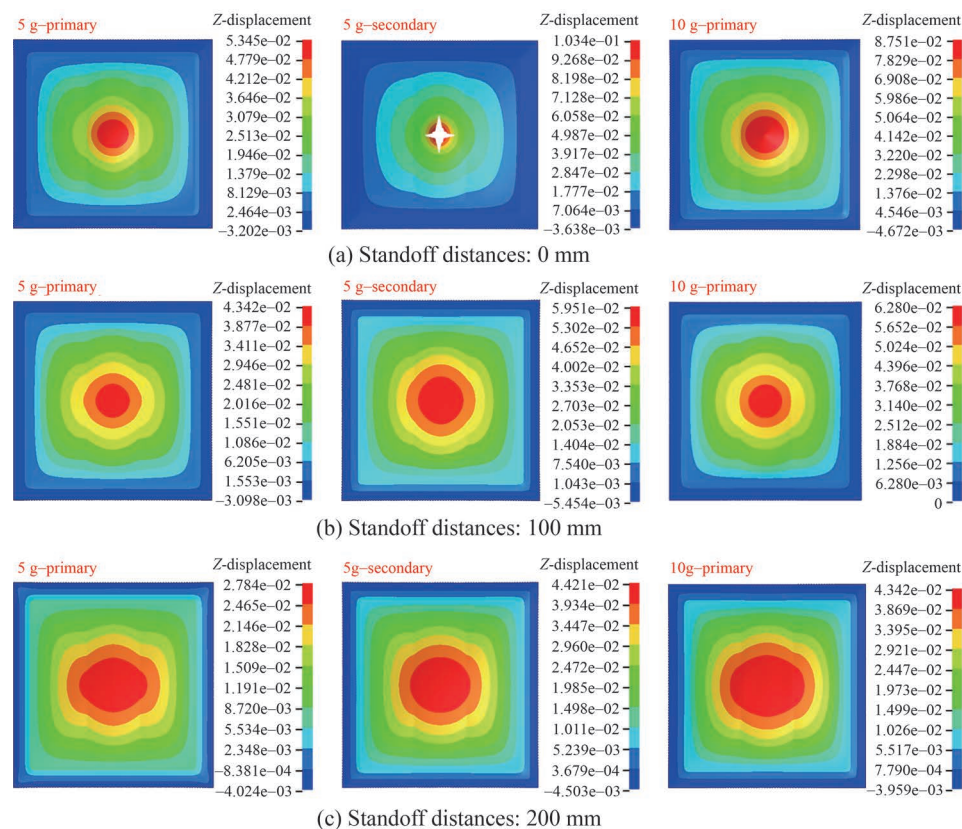
Primary explosion			Secondary explosion		
Charge (g)	Distance (mm)	Distance (mm)	Charge (g)	Distance (mm)	Distance (mm)
5	0	0	5	100	50
		50			75
		75			100
		100			150
		150			200
10	100	50	10	200	100
		75			150
		100			200
		150			250
		200			300
5	200	100	20	200	—
		150			—
		200	20	100	—
		250			—
		300			—

3.3 Analysis of damage results of air-backed stiffened plate

Figure 13 presents a comparative analysis of the maximum normal deflection in stiffened plate structures sub-

jected to sequential secondary explosions (5 g charge) at standoff distances of 0, 100, and 200 mm, compared with single explosion scenarios (10 g charge). The findings reveal distinct damage mechanisms between cumulative secondary explosions and equivalent single explosions. Specifically, under contact detonation conditions (0 mm standoff), secondary explosions induce structural cracking through cumulative damage effects, whereas single explosions mainly result in plastic deformation. At intermediate standoff distances (100 mm), the final deflection from cumulative secondary explosions is marginally smaller than that from single explosions. Conversely, at extended standoff distances (200 mm), the deflection induced by secondary explosions slightly exceeds that from the single explosion. This phenomenon indicates that, under specific spatial configurations, the cumulative damage from low-equivalence secondary underwater explosions can surpass the destructive efficiency of single high-equivalence detonations, demonstrating enhanced energy utilization efficiency in specific underwater explosion conditions.

Aiming to investigate the cumulative damage effects on stiffened plate deflection subjected to sequential underwater explosions, bar charts were used to present the maximum deflection values under various loading conditions. The results demonstrate that, when the standoff distance and charge mass remain constant, secondary explosions generally induce greater maximum deflection compared to

**Figure 13** Comparison of stiffened plate deflection between primary and secondary explosions

single explosion scenarios due to cumulative damage effects. This deflection amplification becomes notably pronounced as the shock factor increases. Particularly, in the 5 g contact explosion case, secondary explosions result in an 18.2% increment in deflection compared to the primary explosion alone. The subsequent five sets of comparisons of the maximum displacement data demonstrate that optimizing shock factors for sequential explosions can effectively amplify structural damage to the target structure. The shock factor is expressed as $SF = \sqrt{W}/R$ (Li et al., 2015a), where W represents the mass of the explosive charge, and R represents the standoff distance.

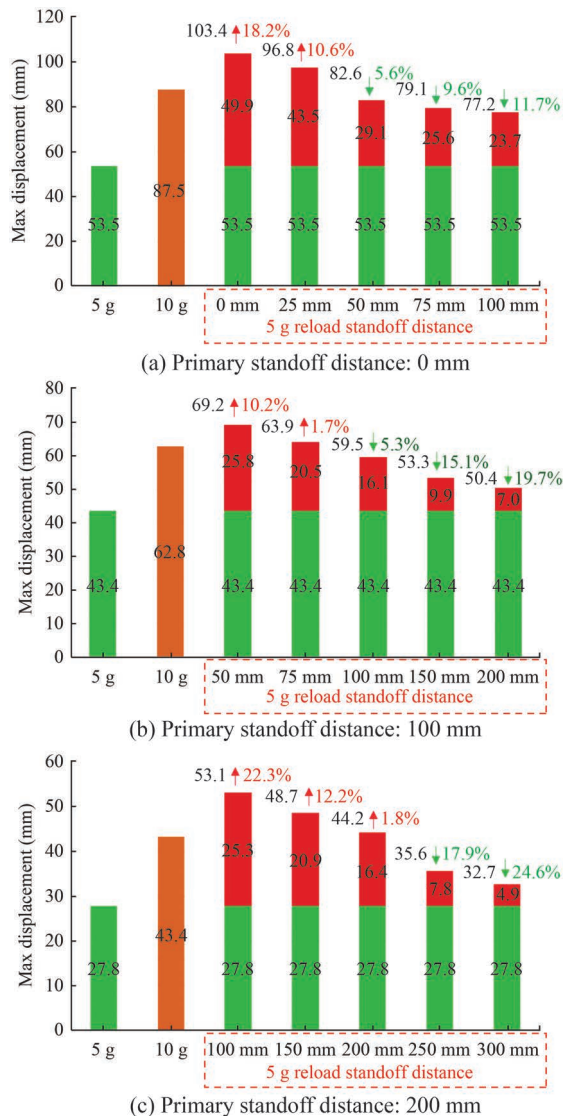


Figure 14 Comparison of maximum displacement in the stiffened plate between 5 g cumulative damage and 10 g single explosion

The incremental deformation of the plate frame under sequential secondary explosions is closely correlated with the shock factor of the primary explosion. Figure 16 illustrates the comparative curves showing the damage amplification effects of secondary explosions with varying shock

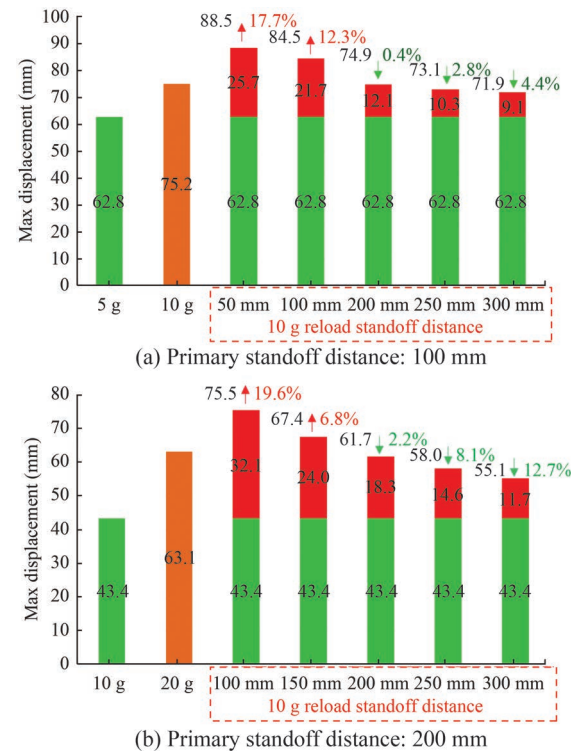


Figure 15 Comparison of maximum displacement in the stiffened plate between 10 g cumulative damage and 20 g single explosion

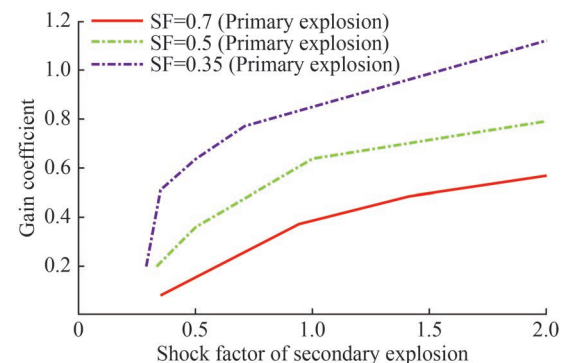


Figure 16 Displacement gain coefficient curve for secondary explosion with shock factor

factors. The gain coefficient is defined as the ratio of the maximum deformation induced by secondary damage to that caused by primary damage. The curve reveals that the maximum gain coefficient for secondary damage occurs when the shock factor of the primary explosion is 0.35. As the shock factor of the primary explosion increases, the gain coefficient for secondary damage gradually decreases. This finding indicates that smaller primary shock factors lead to more pronounced secondary damage amplification effects. When the primary explosion conditions are held constant, the gain coefficient initially rises rapidly and then stabilizes as the shock factor of the secondary explosion increases. This behavior can be attributed to the observation that structures subjected to large primary shock factors experience severe initial damage and considerable plas-

tic hardening effects, thereby reducing their susceptibility to secondary deformation. Conversely, smaller primary shock factors result in weaker plastic hardening, allowing the development of more substantial secondary deformation.

The study first analyzed the distribution pattern of the maximum deflection in the stiffened plate, followed by an investigation of the overall damage deformation characteristics, with particular focus on the deflection in its internal sections. Figures 17–20 compare the deformation of the stiffened plate structure under varying standoff distances and charge weights. The results indicate that when large plastic deformation dominates the primary damage mode, a single large-equivalent underwater explosion induces notably severe overall structural damage, resulting in extensive deformation across the entire plate. In contrast, secondary small-equivalent cumulative damage exhibits distinct characteristics: it primarily causes localized severe damage in specific regions of the stiffened plate, while the overall deformation remains smaller than that caused by a single explosion. This phenomenon becomes increasingly pronounced with higher shock factors. The findings indicate that secondary small-equivalent explosions are more effective for targeting localized areas, whereas single large-equivalent explosions are preferable for comprehensive structural destruction.

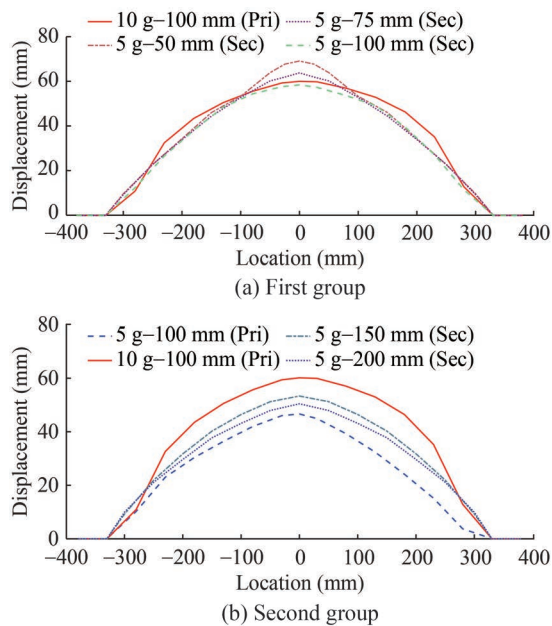


Figure 17 Comparison of deformation in stiffened plate under 5 g secondary explosion and 10 g single explosion at 100 mm standoff distance

3.4 Analysis of energy absorption by air-backed stiffened plate

Figure 21 compares the energy absorption of stiffened plate structures under primary and secondary explosion damage conditions. The results demonstrate that, under

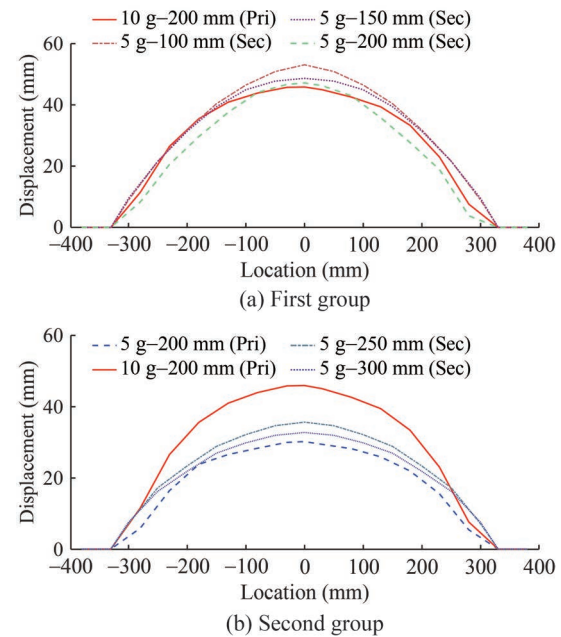


Figure 18 Comparison of deformation in stiffened plate under 5 g secondary explosion and 10 g single explosion at 200 mm standoff distance

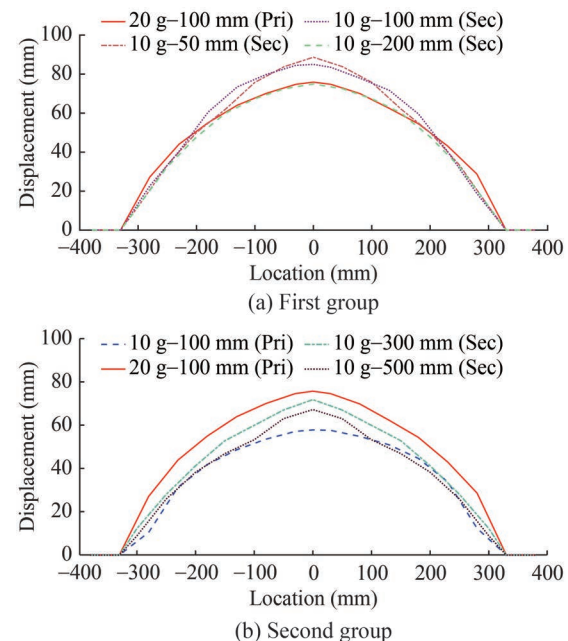


Figure 19 Comparison of deformation in stiffened plate under 10 g secondary explosion and 20 g single explosion at 100 mm standoff distance

identical total charge weight and standoff distance conditions, the total energy absorbed by the structure during secondary explosions is substantially lower than that in single explosions. This discrepancy becomes more pronounced as the standoff distance increases. A comparison of the maximum deflection of the frame structure (Figures 14 and 15) reveals that secondary explosions cause a more localized cumulative damage effect, with energy absorp-

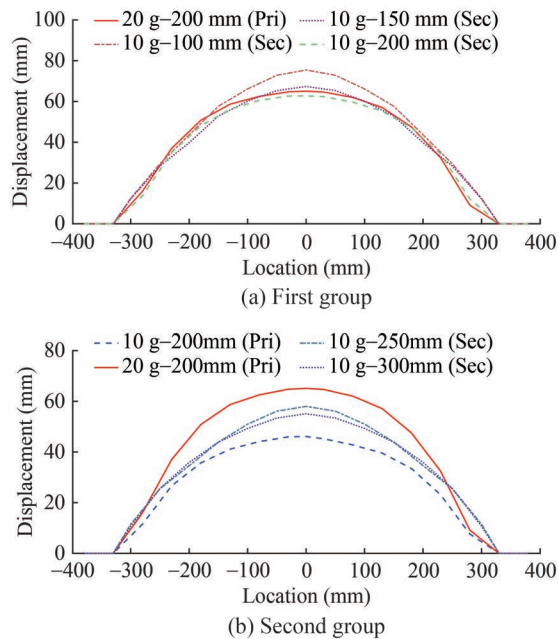


Figure 20 Comparison of deformation in stiffened plate under 10 g secondary explosion and 20 g single explosion at 200 mm standoff distance

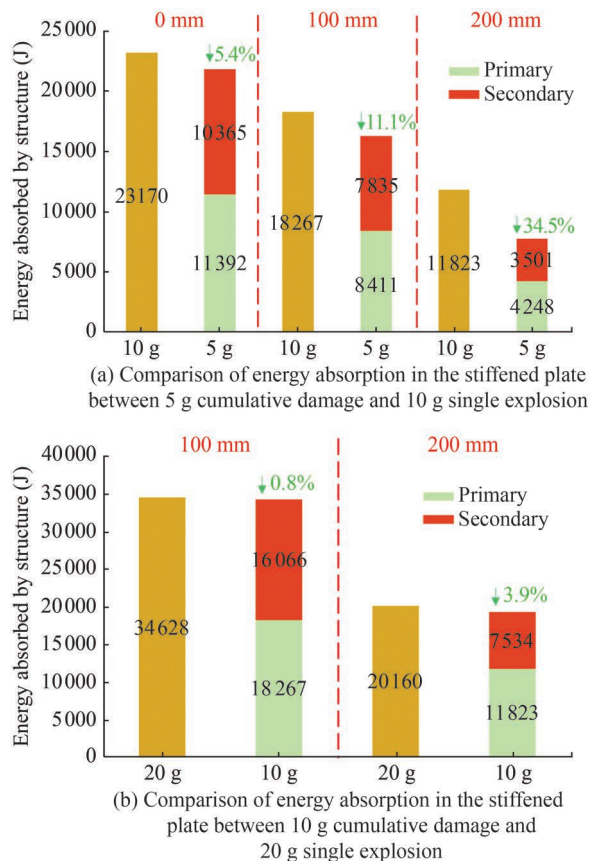


Figure 21 Comparison of energy absorption in the stiffened plate between primary and secondary damage

tion concentrated in the central area. In contrast, a single explosion leads to more extensive global structural dam-

age and higher total energy absorption by the structure.

Table 6 presents the energy absorption characteristics of the plate frame structure under primary and secondary explosions at identical standoff distances. The data indicate that the energy absorbed by the plate frame structure during secondary explosions is consistently lower than that absorbed during primary explosions. This observation reveals that the structure experiences strengthening after the initial loading, which reduces its susceptibility to subsequent deformation. The results demonstrate that the energy absorbed during secondary explosions ranges from 63.7% to 95.6% of the energy absorbed during the primary explosions.

Table 6 Comparison of energy absorption in the stiffened plate under sequential secondary explosions

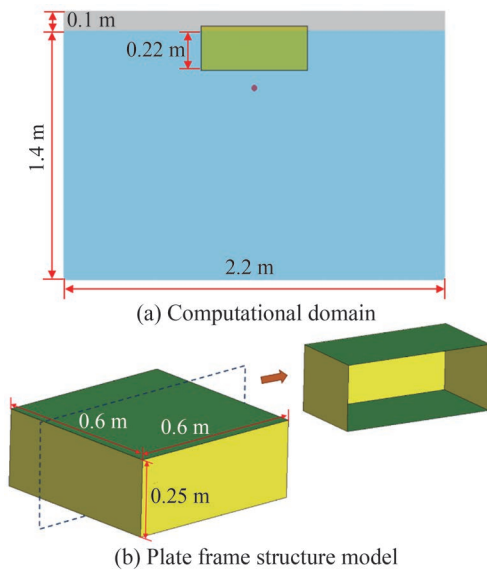
Case	Primary	Secondary	Total	Secondary/Primary
5 g-0 mm	11392	10365	21757	91.0%
5 g-100 mm	8411	7835	16246	93.1%
5 g-200 mm	4248	3501	7749	82.4%
10 g-100 mm	18267	17466	35733	95.6%
10 g-200 mm	11823	7835	19357	63.7%

4 Cumulative damage characteristics of air-backed plate

4.1 Finite element model and condition arrangement

Aiming to validate the generality of the previous conclusions, a simplified air-backed structure was further investigated. The target structure comprises a box-shaped compartment with dimensions of 0.6 m × 0.6 m × 0.25 m, fabricated from 907A material and featuring a plate thickness of 6 mm. The numerical model employs a uniformly distributed grid with a resolution of 3 mm. The structure is submerged to a depth of 0.22 m in water, which has a total depth of 1.4 m, while the air domain height above the water surface is 0.1 m. A spherical TNT charge was used for the underwater explosion simulation. Aiming to ensure a balance between computational accuracy and efficiency, the fluid domain was configured with dimensions of 2.2 m × 2.2 m × 1.5 m, incorporating a gradually refined grid with a minimum element size of 4 mm. Gravity effects were applied across the computational domain. Non-reflective boundary conditions were applied to the fluid domain to mitigate spurious wave reflections that might affect the structural response. The configurations of the filling materials within the compartment are illustrated in Figure 22.

The constitutive relationship of 907A steel is defined using the C-S model, with the specific parameters listed in Table 7.

**Figure 22** Finite element computational model**Table 7** Main parameters of 907A steel material (Li et al., 2015b)

ρ (kg/m ³)	E (GPa)	μ	σ_y (MPa)
7800	210	0.3	439.4
C	p	ε_f	
6180	1.56	0.28	

The calculation conditions are shown in Table 8. A secondary cumulative damage calculation was conducted using

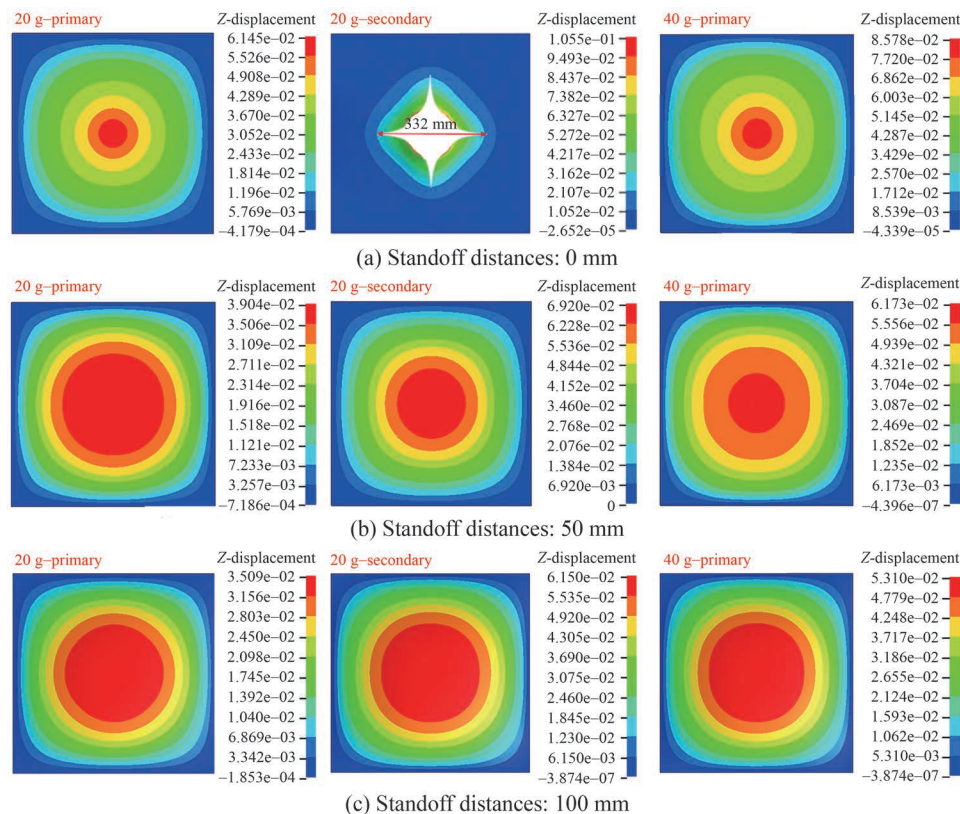
an equivalent of 20 g of TNT explosive, and the results were compared with those obtained with 40 g.

Table 8 Condition arrangement table

Primary explosion		Secondary explosion	Primary explosion		Secondary explosion
Charge (g)	Distance (mm)	Distance (mm)	Charge (g)	Distance (mm)	Distance (mm)
20	0	15	20	50	50
		50			88
		61			100
		75			120
		100			150
		150			180
20	100	50	40	0	—
		100			—
		150	40	50	—
		175			—
		200	40	100	—
		240			—

4.2 Analysis of damage results of the air-backed plate

Figure 23 presents a comparative analysis of the maximum displacement in air-backed plate structures subjected

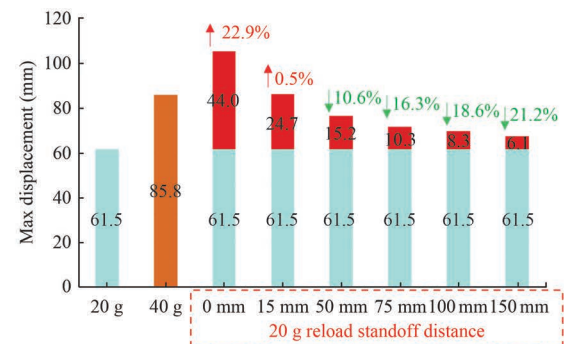
**Figure 23** Comparison of maximum displacement in the air-backed plate under single and sequential secondary explosions

to two sequential explosions with 20 g charges, versus a single explosion with a 40 g charge, both at identical standoff distances. Under contact explosion conditions, distinct damage modes emerge between the single and cumulative secondary explosions. In the 40 g contact explosion scenario, the structure undergoes substantial plastic deformation without rupture. In contrast, the cumulative damage from two 20 g explosions induces a petal-shaped central crack with a tear length of 332 mm. At a 50 mm standoff distance, the maximum displacement for the single explosion case is 61.7 mm, while the sequential explosions cause a displacement of 69.2 mm, representing a 12.2% increase. At a 100 mm standoff distance, the maximum deflection of the plate frame in the single explosion case is 53.1 mm, while under secondary cumulative damage, the maximum displacement increases to 61.5 mm, a 15.8% increase compared to the single explosion. These results indicate that, under identical charging conditions, sequential secondary explosions lead to more substantial localized damage in the structure.

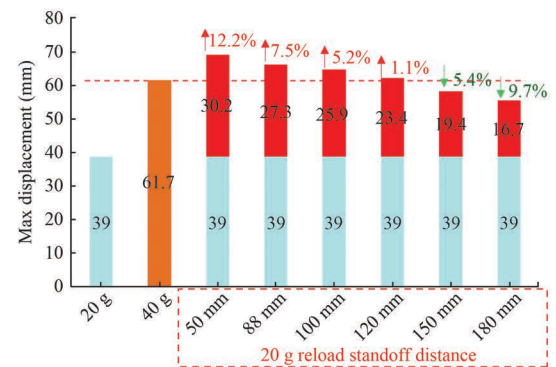
Figure 24 compares the maximum displacement of the air-backed plate under different charge weights and standoff distances. The bar chart reveals that secondary explosions have a considerable influence on the structural damage. Under identical standoff distances and charge weights, the structural deformation induced by secondary explosions exceeds that from a single explosion. This observation aligns with previous findings on stiffened plates, indicating that modifying the charge distribution can substantially amplify the damage effect.

Aiming to provide a highly intuitive comparison of the deformation in the air-backed plate, deflections were measured at various sections along the plate. The measurement setup is shown in Figure 25, where the center is positioned at 0 mm, and the two ends correspond to -300 and 300 mm.

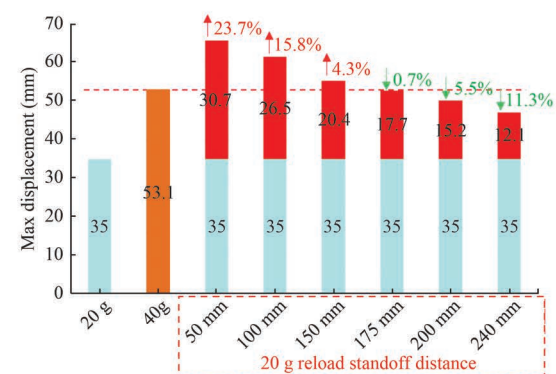
Figures 26–28 present a comparative analysis of the air-backed plate deflection under primary and secondary damage for different conditions. The results demonstrate that secondary explosions induce more substantial structural damage within certain standoff distances compared to primary explosions. For instance, under contact explosion conditions, the maximum deformation reaches 85.8 mm with a 40 g charge and 61.5 mm with a 20 g charge. In the secondary damage scenarios, at a 0 mm standoff distance, visible cracks appear on the plate, while at a 15 mm standoff distance, the maximum deflection increases to 86.2 mm, both exceeding the damage caused by the single 40 g charge explosion. Furthermore, at a 50 mm standoff distance, secondary damage results in greater deformation than a single 40 g explosion. These observations indicate that secondary explosions predominantly cause localized deformation with deeper indentations, whereas single explosions cause more extensive global deformation. This finding reveals that sec-



(a) Max displacement of the plate frame: 20 g–0 mm



(b) Max displacement of the plate frame: 20 g–50 mm



(c) Max displacement of the plate frame: 20 g–100 mm

Figure 24 Comparison of maximum deflection in the plate frame under single and sequential secondary explosions

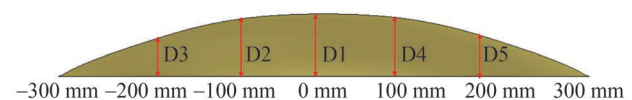
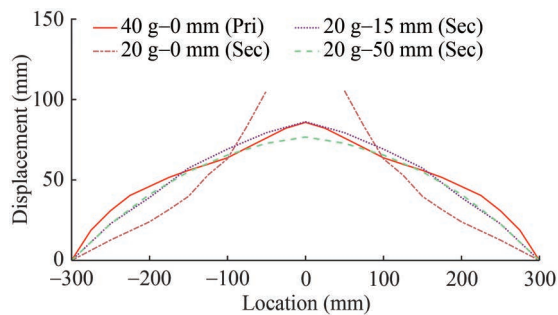


Figure 25 Schematic of air-backed plate deflection measurement

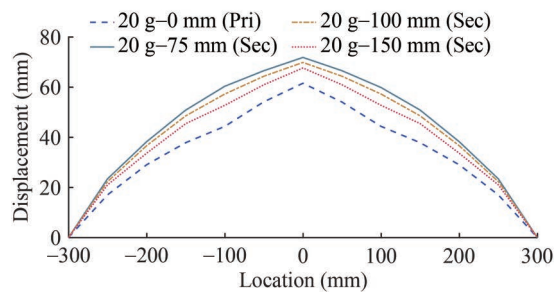
ondary explosions have a remarkable impact on the damage to specific areas of the structure.

4.3 Analysis of energy absorption by air-backed plate

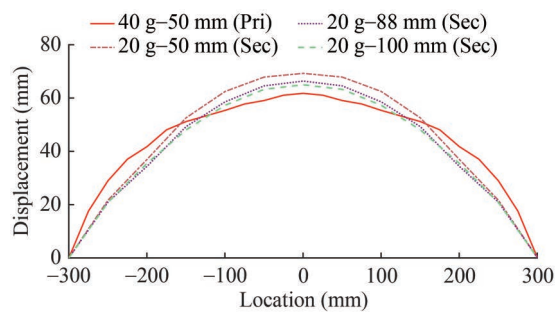
Figure 29 presents a comparison of energy absorption by the air-backed plate structure under primary and secondary damage conditions. The results demonstrate that,



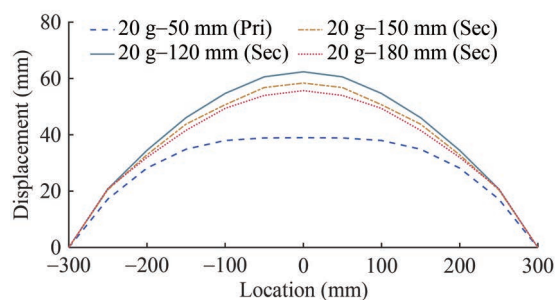
(a) First group



(b) Second group

Figure 26 Deflection comparison curve of the air-backed plate at 0 mm standoff distance

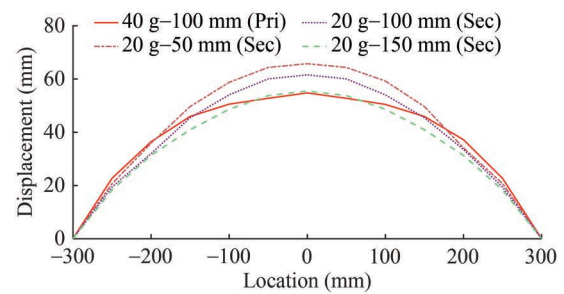
(a) First group



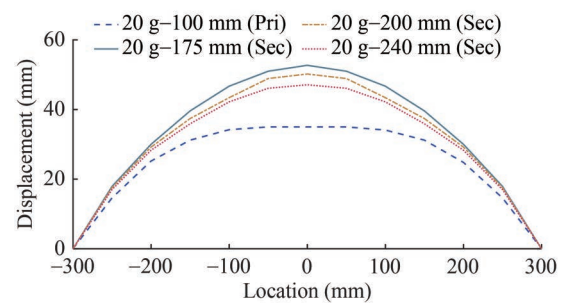
(b) Second group

Figure 27 Deflection comparison curve of the air-backed plate at 50 mm standoff distance

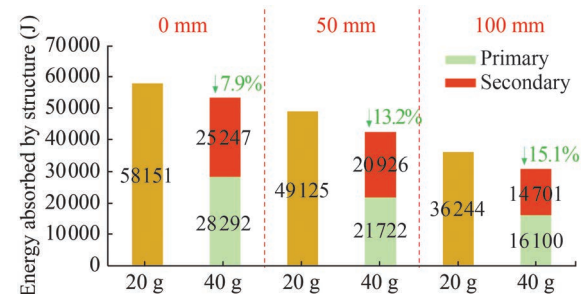
for identical total charge mass and standoff distance, the cumulative energy absorbed during secondary explosions is substantially lower than that absorbed during a single explosion. This discrepancy becomes highly pronounced as the standoff distance increases. Specifically, at a 0 mm standoff distance, the cumulative energy absorption in sec-



(a) First group



(b) Second group

Figure 28 Deflection comparison curve of the air-backed plate at 100 mm standoff distance**Figure 29** Comparison of energy absorption by the air-backed plate under a single explosion and secondary cumulative damage

ondary explosions is 7.9% lower than that of a single explosion, while at a 100 mm standoff distance, the difference increases to 15.1%. These findings indicate that single explosions cause extensive structural damage and exhibit remarkably high energy utilization efficiency. This conclusion aligns with the results observed for stiffened plates in previous analyses.

5 Conclusions

This paper investigates the mechanisms of cumulative structural damage caused by secondary underwater explosions, focusing on the evolution of damage in stiffened plate frames and conventional plate frame structures under sequential explosion loads. Through numerical simulation analysis, the study compares maximum structural deflection and energy absorption to assess the similarities and

differences in structural damage caused by primary and sequential secondary explosions, with equivalent mass and standoff distance. The key findings are summarized as follows:

1) The ALE method effectively simulates the progressive damage mechanisms in plate frames under sequential underwater explosions. Numerical simulations reveal that the maximum deflection of the plate frame, under primary and secondary underwater explosions, deviates from experimental measurements by less than 6% error, confirming the computational accuracy and reliability of this approach.

2) Under constant charge quantity and standoff distance conditions, the damage effects of sequential secondary explosions notably differ from those of a single explosion on the air-backed plate structure. Sequential secondary explosions notably enhance localized damage, with the maximum deflection at the central area of the plate exceeding that caused by a single explosion. In extreme cases, differences in the damage modes may also occur. Under the cumulative load from secondary explosions, the structure develops petal-shaped breaches, whereas a single explosion typically results in considerable plastic deformation. These trends were consistently observed across various materials, plate thicknesses, and structural configurations, demonstrating the universal applicability of the patterns derived in this study.

3) The magnitude of the initial shock factor plays a crucial role in influencing the accumulation of damage under subsequent loads. Small initial shock factors generally lead to highly pronounced cumulative damage effects. The careful selection of sequential shock factors can substantially improve the damage efficiency of the target structure.

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