

Nonlinear Control Allocation for Drilling Rigs with an Online Actuator Selection Method

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Abstract

Dynamic positioning systems (DPS) on marine vessels exhibit actuator redundancy, with more actuators than degrees of freedom. A control allocation unit is employed to address this redundancy. Practical systems often feature time-varying elements in the effectiveness matrix due to factors such as changing operating conditions, nonlinearity, and disturbances. Additionally, not all thrusters require engagement at each step to counteract disturbances and maintain position. Control efforts can be generated by selecting some thrusters based on their instant effectiveness, while others can remain on standby. Therefore, introducing a control allocation method that calculates the effectiveness matrix online and selects the most efficient thrusters could be effective. This paper introduces a fault-tolerant control allocation strategy for DPS with a varying effectiveness matrix. Specifically, the investigation focuses on a case study featuring eight azimuth thrusters used on a drilling rig. At each time step, the effective matrix is calculated online, followed by the selection of the four most effective thrusters based on the actuator effectiveness index, with the four serving as backups in case of a fault. The proposed strategy has been validated through simulation results, demonstrating advantages such as robustness against changes in the effectiveness matrix and reduced energy usage by the thrusters.

Keywords Marine vessels; Drilling rigs; Dynamic position systems; Control allocation; Actuator selection; Fault-tolerant systems

1 Introduction

Designed with advanced technology, offshore oil platforms float on water without relying on traditional support structures or foundations, enabling them to withstand external forces such as wind and sea waves. To address this challenge, these platforms are equipped with dynamic positioning systems (DPS) that autonomously maintain their position by regulating actuators to counteract environmental forces (Sørensen, 2011).

Dynamic positioning (DP) systems are inherently com-

plex and require seamless communication among various components to function effectively. Each DP supplier follows its own design philosophy, resulting in a variety of system configurations. However, certain components are universally essential across all DP systems, including position and heading reference systems, sensors, and propulsion systems. These elements are crucial to ensuring reliable performance and should not be overlooked. Traditionally, DP systems have been used for low-speed operations, where the basic DP functionality is to either maintain a fixed position and heading, or to move slowly from one location to another. A comprehensive history of DPS is presented in Faÿ (1990). Various control strategies, including model predictive control (Chen et al., 2012; Øveraas et al., 2023; Liu et al., 2023; Tang et al., 2022), robust adaptive control (Chen et al., 2012; Zhang et al., 2023; Li et al., 2022), neural fuzzy methods (Shen et al., 2023; Wang et al., 2022), and sliding mode control (Tannuri et al., 2010; Li and Lin, 2022; Septanto et al., 2023), can be employed to control these systems. Typically, DPS have more actuators than degrees of freedom, leading to actuator redundancy within the system. This redundancy provides greater flexibility in designing a suitable controller, thereby improving the accuracy, flexibility, maneuverability, and safety of the system. Applying a control allocation approach that manages control commands is necessary to implement DPS in practical applications. Control allocation involves distributing the control signals among the thrusters while satisfy-

Article Highlights

- Developing a fault tolerant control for drilling vessels based on a novel control allocation approach.
- Employing actuator effectiveness matrix to assess the efficacy of thrusters.
- Using fewer thrusters in the control allocation problem rather than all of them.
- Achieving significant reduction in power consumption as well as computational effort.

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ing the respective constraints. Several methods, such as least squares (Bordignon, 1996), quadratic programming (Enns, 1998), daisy chain (Buffington and Enns, 1996), neural fuzzy methods (Tohidi and Sedigh, 2013), feasible control allocation (Naderi et al., 2019), and corrected pseudo inverse along the null space (Tohidi et al., 2016), are commonly used to solve the control allocation problem. In a DPS, the control allocation unit plays a crucial role in distributing the required commands and forces among the actuators to maintain the position of the floating drilling system while satisfying its mechanical constraints. The effectiveness matrix is key to the control allocation method, defining the mapping between actuators and virtual control signals. While previous methods for thrust allocation in vessels assume that the effectiveness matrix is known and fixed, this matrix can vary in DPS (Jafari et al., 2023). Therefore, the implementation of the control allocation method necessitates the online computation of the effectiveness matrix. In this paper, the effectiveness matrix is updated at each sampling time based on the angles of the thrusters. The control allocation is then formulated as an optimization problem to achieve the control targets and satisfy the constraints of the DPS (Tannuri et al., 2010). In Zhao and Roh (2015), the issue of thrust allocation in drilling vessels is tackled using a genetic algorithm to minimize thruster energy consumption. An alternative approach to thrust allocation, based on a quasi-inverse linear model, was presented in Ahani and Ketabdari (2019). Gopmandal et al. (2022) introduced an LQ optimal robust multivariable PID controller for the positioning system. In Gao et al. (2019), a control allocation strategy for DPS based on an improved non-dominant sorting genetic algorithm 2 was presented. Unlike previous methods, where all thrusters were used by the controller, the proposed approach in this paper allows a smaller number of thrusters to be used, accounting for system redundancy, with the remaining thrusters kept in reserve. Azimuth thrusters on drilling vessels may encounter issues such as propeller jamming or equipment failure (Lin et al., 2018). By using a reduced number of thrusters, the system can compensate for defective ones. Additionally, reducing the number of thrusters decreases the interactions between them. When two azimuth thrusters operate in close proximity, their slipstreams interact based on their relative angles. This interaction can substantially reduce the thrust produced, especially when one thruster operates within the slipstream of the other (Brown and Ekstrom, 2005).

The selection of the most effective thruster group is crucial in the control allocation method due to its direct impact on control performance and energy consumption. Various indexes have been used to identify suitable inputs. In Govin and Powers (1982), and Daoutidis and Kravaris (1992), graph-based methods and quantitative indexes such as steady state, time constant, and relative degrees are employed

for this purpose. Quantitative controllability criteria are also used in Hać and Liu (1993), Gawronski and Lim (1996), and Samar and Postlethwaite (1994) to determine the appropriate inputs. Additionally, an input selection index that avoids half-right zeros is discussed in Biss and Perkins (1993), Hovd and Skogestad (1993), Liu et al. (2003), Naderi and Khaki Sedigh (2020). The aforementioned methods are presented for square systems, where the number of DOFs and actuators are equal.

The actuator effectiveness index (AEI) was introduced in Naderi and Khaki Sedigh (2020) as a criterion for selecting the most suitable actuator to address actuator redundancy. This index depends on factors such as the location and direction of the thrusters, the type of control allocation, and the upper and lower limits of the thrusters (Naderi and Khaki Sedigh, 2020). In Jafari et al. (2023), the AEI is used to select the most effective actuator for implementing the control allocation algorithm in the level control system of four tanks, which involves a variable effectiveness matrix.

This paper presents a novel fault-tolerant control system for the DP of a floating oil platform, which is an overactuated system equipped with azimuth thrusters. Using the AEI, the thrusters are ranked and categorized accordingly into two distinct groups based on their effectiveness. The most effective thrusters, associated with the highest AEI values, are considered active actuators, and their commands are determined by solving a nonlinear optimal control problem to generate the total required control effort. The second group, comprising four thrusters with the lowest effectiveness, is kept on standby. In this approach, the control allocation problem is solved for a subset of thrusters rather than for all thrusters, as is done in conventional methods. With this enhancement, power consumption and maintenance costs are substantially reduced, while performance remains unaffected. Furthermore, solving the optimization problem for a smaller set of thrusters in the control allocation unit will noticeably reduce the computational burden.

The remainder of the paper is organized as follows: Section 2 presents the general modeling of drilling vessels, while Section 3 formulates the control allocation approach for drilling vessels. Subsequently, Section 4 explains the use of the AEI. Section 5 provides the block diagram of the proposed approach. Section 6 simulates a case study to validate the efficiency of the proposed approach. Finally, Section 7 presents the conclusion of the article.

2 Vessel dynamics and kinematics

In drilling vessels, the drilling system is located at the center of the vessel. Eight azimuth thrusters are responsible for maintaining the position of the vessel, as depicted in Figure 1. Marine vessels frequently experience distur-

bances such as waves, currents, and wind, which are typically characterized by low amplitude and high frequency. In drilling vessels, the primary task of the DSP is to maintain the position of the vessel for precise drilling and to protect the drilling equipment. As a result, the DPS operates with low-speed control, focusing solely on horizontal plane forces and moments. When solving the optimization problem of thrust allocation in the DPS, the prevalent method involves modeling forces and moments based on the principle of superposition. Consequently, vessel motion (X) can be conceptually decomposed as (Perez et al., 2004):

$$X = X_w + \bar{X} \quad (1)$$

$$\bar{X} = X_{sv-d} + X_c \quad (2)$$

where w is first-order wave-induced motion; $sv-d$ is the slowly varying disturbance motion produced by second-order wave effects, current, and wind (); c is the control-induced motion produced by the thrusters.

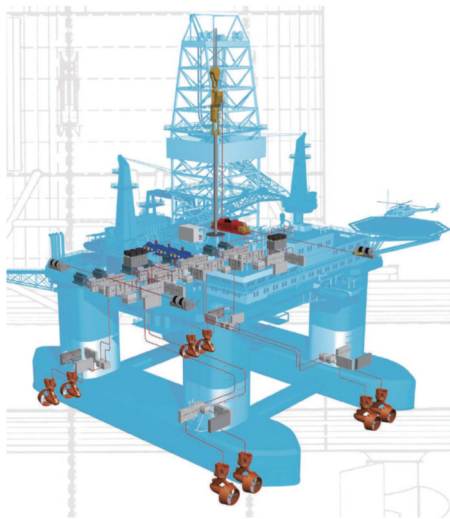


Figure 1 Dynamic positioning system configuration for eight thrusters of a semi-submersible drilling rig (Li et al., 2022)

However, DPS used in practice employs various filters to reject certain disturbances, applying only smooth disturbances to the position control unit for compensation. Therefore, the successful implementation of filters in the control system to eliminate oscillatory motion is assumed to accurately facilitate system modeling. In low-speed applications of DPS, a mathematical model that considers low-frequency waves, oscillations, and deflections is common. This model assumes small pitch and roll angles, as well as port–starboard symmetry of the ship. Coriolis parameters and center reluctance are negligible, and the linear part of the damping matrix makes a dominant contribution due to the damping from the laminar friction drift wave. Thus, the modeling of the drilling float with three degrees

of freedom, as described in Fossen and Johansen (2006), is as follows:

$$\dot{\eta} = J(\eta)v \quad (3)$$

$$M\dot{v} + D(v - v_c) = \tau + \tau_{env} \quad (4)$$

The output vector $\eta = [x, y, \psi]^T \in \mathbb{R}^{3 \times 1}$ includes the position of the ship (x, y), and the horizontal roll angle, $\psi \in [-\pi, \pi]$, and the vector $v = [u, v, r]^T \in \mathbb{R}^{3 \times 1}$ represent the forward speed, lateral speed, and angular speed of the float, respectively. $v_c = [u_c, v_c, r_c]^T \in \mathbb{R}^{3 \times 1}$ is the current velocity vector. The input vector $\tau = [\tau_x, \tau_y, \tau_n]^T \in \mathbb{R}^{3 \times 1}$ represents the forces and torques produced by thrusters, and the vector $\tau_{env} \in \mathbb{R}^{3 \times 1}$ denotes environmental disturbances. Additionally, $D \in \mathbb{R}^{3 \times 3}$ is the linear hydrodynamic damping forces and moments acting on the vessel, and $M \in \mathbb{R}^{3 \times 3}$ is the inertia matrix of the system. $J \in \mathbb{R}^{3 \times 3}$ is also the state-dependent transformation matrix from fixed vessel to ground surface frame.

$$J(\eta) = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (5)$$

$$M = \begin{bmatrix} m - X_{\dot{u}} & 0 & 0 \\ 0 & m - Y_{\dot{v}} & mx_G - Y_{\dot{y}} \\ 0 & mx_G - N_{\dot{v}} & I_z N_{\dot{y}} \end{bmatrix} \quad (6)$$

$$D = \begin{bmatrix} -X_u & 0 & 0 \\ 0 & -Y_v & -Y_y \\ 0 & -N_v & -N_y \end{bmatrix} \quad (7)$$

where $X_{\dot{u}}$, $Y_{\dot{v}}$, $Y_{\dot{y}}$, and $N_{\dot{y}}$ are the added mass forces for the DOFs of longitudinal and lateral motion and yaw, respectively. x_G is also the distance between the center of gravity of the vessel and the origin of the fixed frame of the float.

3 DPS and the thrust allocation model

Consider a drilling rig equipped with m azimuth thrusters with n DOFs. The generalized force vector τ produced by the thrusters is as follows (Johansen et al., 2004):

$$\tau = B(\alpha)u \quad (8)$$

where the vector $u \in \mathbb{R}^m$ contains the force and torque separately produced by each thruster. $\alpha \in \mathbb{R}^m$ is the angle vector related to the thrusters in the horizontal direction. $B \in \mathbb{R}^{n \times m}$ is the effectiveness matrix, which depends on the location and angle of actuators. Each column of B corresponds to a thruster that can be described as follows (Johansen et al., 2004):

$$\mathbf{B}_i(\alpha_i) = \begin{bmatrix} \cos\alpha_i \\ \sin\alpha_i \\ -l_{yi} \cos\alpha_i + l_{xi} \sin\alpha_i \end{bmatrix} \quad (9)$$

where (l_{x_i}, l_{y_i}) shows the i th thruster location in the horizontal plane. Given the angle of azimuth thruster i , $\mathbf{B}_i(\alpha_i)$ projects the force of actuator i into three degrees of freedom, namely, surge, sway, and yaw.

In marine vessels equipped with azimuth thrusters, the control allocation problem can be formulated as a nonconvex optimization problem, as shown in Equation (8), where α_i is the angle of i th azimuth thruster. In addition, the angle of the azimuth thrusters can only be changed in an allowable range, i.e., $\alpha_{i_{\min}} \leq \alpha_i \leq \alpha_{i_{\max}}$, based on their conditions (Johansen et al., 2004):

$$J(\alpha, \mathbf{u}, \mathbf{s}) = \sum_{i=1}^m W_i(u_i) + \mathbf{s}^T \mathbf{Q} \mathbf{s} + (\alpha_i - \alpha_{i_0})^T \mathbf{\Omega} (\alpha_i - \alpha_{i_0}) \quad (10)$$

Subject to

$$\mathbf{s} = \boldsymbol{\tau} - \mathbf{B}(\alpha) \mathbf{u} \quad (11)$$

$$u_{i_{\min}} \leq u_i \leq u_{i_{\max}} \quad (12)$$

$$\alpha_{i_{\min}} \leq \alpha_i \leq \alpha_{i_{\max}} \quad (13)$$

$$\Delta\alpha_{\min} \leq \Delta\alpha \leq \Delta\alpha_{\max} \quad (14)$$

In Equation (10), the first term reflects the total power consumption. The second term, namely $\mathbf{s}^T \mathbf{Q} \mathbf{s}$, penalizes the error between the command and generalized signals, and the positive definite diagonal weighting matrix $\mathbf{Q}$ is considered sufficiently large to obtain $\mathbf{s} \approx 0$ in constraint (11) whenever possible, adequately reducing the error between the command and generalized signals. This constraint is provided in the third part of the cost function of relation (10) and constraint (14). The weight matrix $\mathbf{\Omega}$ is used to set this target, and α_0 also contains the previous angles. The first constraint, namely (11), is included to ensure the generation of the required torque vector, which is determined by the main controller to reach the control goals, such as stability and elimination of steady-state errors, through appropriate actuator commands. Additionally, (12) and (13) reflect the force limits ($u_{\min}$ and $u_{\max}$) and angle limits ($\alpha_{\min}$ and $\alpha_{\max}$) of each actuator, respectively, which may be modified in case of fault or failure. Finally, the change rate of thruster angles is limited with (14), and a notable change is allowed only if necessary.

4 Actuator selection in a drilling platform

Figure 2 shows the placement of eight azimuth thrusters

on the drilling platform. The system has three DOFs, with the DP system exclusively focusing on horizontal movements. Four thrusters are used to establish the generalized force vector and sustain redundancy in the system. An actuator selection index, derived from the impact of actuators on the force vector, is introduced in Nederi and Khaki Sedigh (2020). This index depends on the parameters of the effectiveness matrix and the lower and upper limits of the thrusters. Computing a fresh effectiveness matrix for each iteration by integrating the latest azimuth thruster angles is necessary due to the variability of this matrix. Subsequently, the most effective thrusters should be selected based on this newly calculated matrix. This criterion is called the *AEI*, which is presented as follows:

$$AEI_i = \|\mathbf{B}_i(\bar{\mathbf{u}}_i - \underline{\mathbf{u}}_i)\| \quad (15)$$

where $\mathbf{B}_i$ is the i th column of the effectiveness matrix, and $\bar{\mathbf{u}}_i$ and $\underline{\mathbf{u}}_i$ are the upper and lower limits of the thruster i , respectively, which are related to the mechanical constraint of the thrusters. After *AEI* computation, the maximum and minimum index values correspond to the most effective and ineffective thrusters, respectively. After calculating the new effectiveness matrix, the *AEI* index is computed in each step. The four most effective thrusters with large values of *AEI* are selected to address the required force and torques, considering that the effectiveness matrix, which includes four columns corresponding to the selected actuators, is full rank. Otherwise, at least one of the actuators should be replaced by another to produce an effectiveness matrix with full rank.

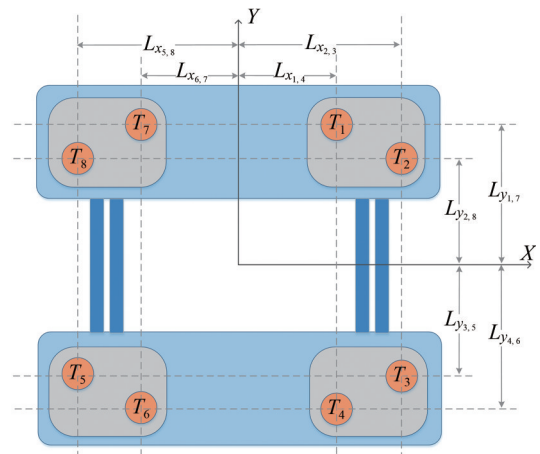


Figure 2 Placement of eight azimuth thrusters in the drilling rig

5 Power consumption analysis

The power consumption of azimuth thrusters relies on their thrust level and angle. The thrust (T) and torque (Q) produced by the propeller are functions of their rotation

speed, which can be calculated using Equations (16) and (17), respectively. The power consumption (P) of a propeller is given by (18).

$$T = \rho D^4 K_T n^2 \quad (16)$$

$$Q = \rho D^5 K_Q n^2 \quad (17)$$

$$P = 2\pi n Q \quad (18)$$

where ρ is the density of the water, D is the propeller diameter, and n is the propeller rotation speed. K_T and K_Q are the thrust and torque coefficients, respectively (Zhao and Roh, 2015).

6 Summary of the proposed method

The block diagram in Figure 3 illustrates the proposed approach, which employs a nonlinear PID as the main controller to generate virtual control signals and facilitate their delivery to the control allocation unit. Notably, the main controller is not the focus of this work; therefore, its details are not provided. The effectiveness matrix is updated in each step of the calculation section using the new thruster angles. In the actuator selection section, the effectiveness index of all thrusters is calculated and sorted from most to least effective. The top-performing thrusters are then chosen. The control allocation unit assigns control to these thrusters and transmits a corresponding command signal, including thrust and angle information. The steps of the method are summarized below:

Step 1 Calculate the effectiveness matrix assuming the initial angle for all thrusters.

Step 2 Calculate the AEI of all thrusters.

Step 3 Select the four thrusters with the largest AEI and update the effectiveness matrix by removing columns corresponding to the other thrusters.

Step 4 Ensure that the effectiveness matrix rank is full. If the rank matrix is incomplete, then go back to Step 3, remove the linearly dependent thrusters, and replace them with other thrusters.

Step 5 Solve the optimization problem related to the control allocation while considering the four selected thrusters to obtain the control signals.

Step 6 Update the overall effectiveness matrix with the new thruster angles.

Step 7 Repeat Steps 2 to 6.

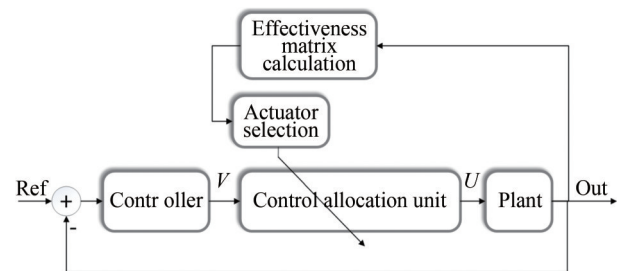


Figure 3 Closed-loop scheme of the proposed algorithm

7 Simulation results

This paper uses a semi-submersible drilling rig as a case study. The rig is a semi-submersible oil platform with four legs, each equipped with two azimuth thrusters as shown in Figure 2. Using eight azimuth thrusters, the effectiveness matrix is calculated by utilizing (9) as follows:

$$\mathbf{B} = \begin{bmatrix} \cos \alpha_1 & \cos \alpha_2 & \cdots & \cos \alpha_8 \\ \sin \alpha_1 & \sin \alpha_2 & \cdots & \sin \alpha_8 \\ l_{x_1} \sin \alpha_1 - l_{y_1} \cos \alpha_1 & l_{x_2} \sin \alpha_2 - l_{y_2} \cos \alpha_2 & \cdots & l_{x_8} \sin \alpha_8 - l_{y_8} \cos \alpha_8 \end{bmatrix} \quad (19)$$

Moreover, the center of the float is considered to be the origin of the coordinates. Thus, the vectors $\mathbf{l}_x$, $\mathbf{l}_y$, and $\mathbf{l}_z$, which are related to the displacement coordinates of the thrusters, are as follows:

$$\mathbf{l}_x = [40.81 \quad 44.749 \quad 44.749 \quad 40.81 \quad -44.741 \quad -40.735 \quad -40.735 \quad -44.741]$$

$$\mathbf{l}_y = [33.5 \quad 25.499 \quad -25.499 \quad -33.5 \quad -33.5 \quad -25.499 \quad 33.5 \quad 25.499]$$

$$\mathbf{l}_z = [-2 \quad -2 \quad -2 \quad -2 \quad -2 \quad -2 \quad -2 \quad -2]$$

The propeller diameter is 3.6 m, and K_T and K_Q coefficients are 0.501 6 and 0.081 4, respectively. The effectiveness of the proposed allocation was evaluated by comparing it with conventional methods. Therefore, a time-domain

simulation of the station-keeping operation was developed using the marine system simulator (MSS) toolbox in MATLAB (Fossen and Perez, 2004). The proposed method is tested in this section under two different scenarios. In the first scenario, the system operates under stable conditions without thruster fault. Meanwhile, in the second scenario, the system is subjected to thruster fault. The controller used in both methods is a nonlinear PID whose output is a virtual signal vector.

7.1 Scenario one: comparison of the conventional and the proposed methods without thruster fault

In this section, the proposed method is evaluated under a fault-free condition of thrusters. Disturbances, such as waves and wind, may affect the system by attempting to

disrupt the floating position. Therefore, perturbations occur in all three DOFs of the system, and their trajectories are depicted in Figure 4. Figure 5 also shows the virtual control vectors in this scenario, which are generated by the main controller (i.e., nonlinear PID). Notably, this paper specifically addresses the control allocation unit, thus excluding the details of the main controller, which is responsible for generating desired virtual control signals. Interested readers may find details in Fossen and Perez (2004).

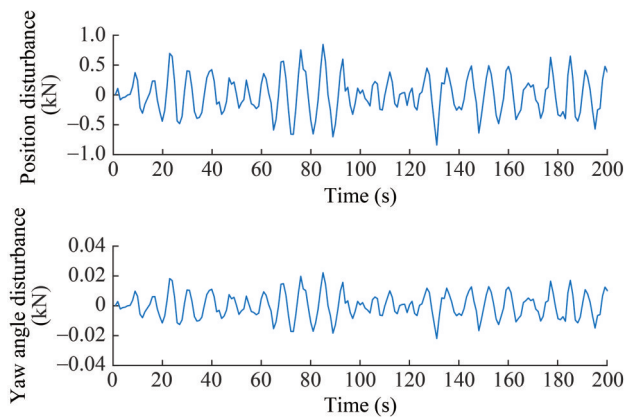


Figure 4 Applied disturbance to positions (x , y) and yaw angle of the rig

7.1.1 Conventional control allocation without actuator selection

In this section, a conventional control allocation unit,

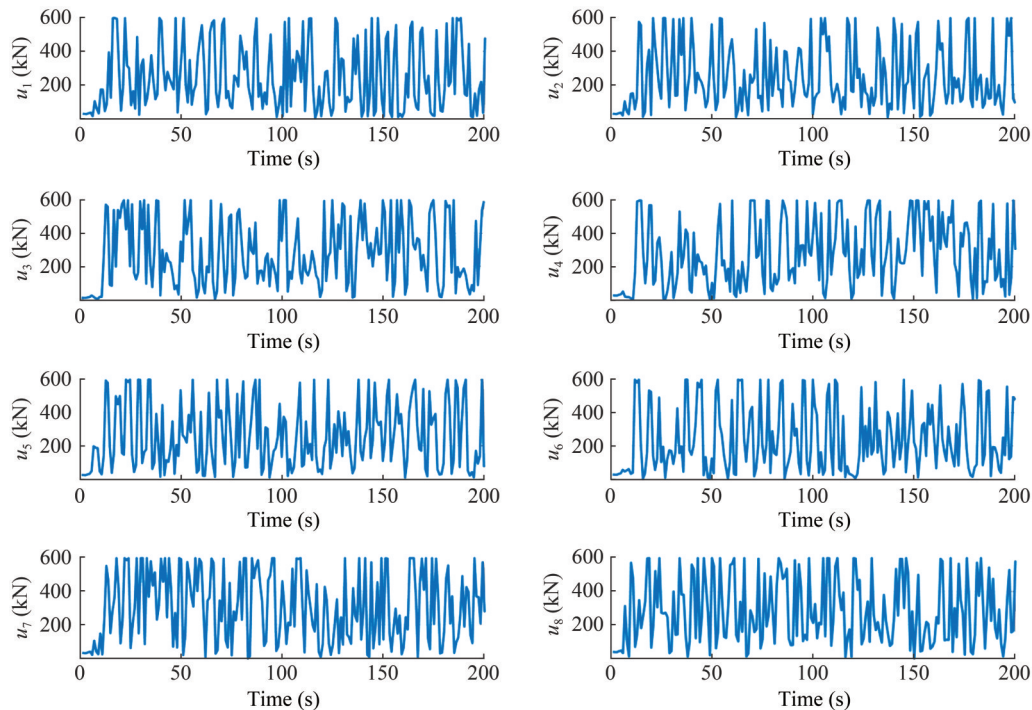


Figure 6 Thrust diagram of thrusters in control allocation with eight thrusters

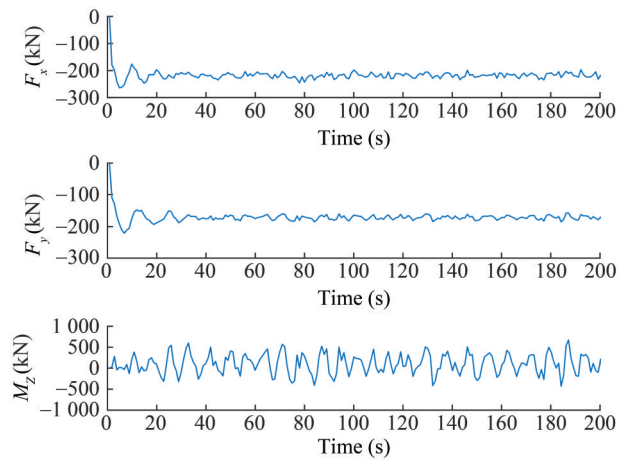


Figure 5 Virtual control vectors (Surge, Sway, and Yaw)

which continuously employs all eight azimuth thrusters and solves (10), is used to determine their commands for the given virtual control signals shown in Figure 5. The command signals of the generated actuator, including the thrust and angle, are illustrated in Figures 6 and 7, respectively. In these figures, all eight thrusters operate simultaneously to produce the necessary control effort. The continuous use of all thrusters may reduce their longevity and increase associated operation and maintenance costs. As depicted in Figure 7, the angle of all thrusters, i.e., α_i , $i = 1, 2, \dots, 8$, exhibits consistent variations, indicating their permanent rotation. These rotations are generated sequentially to neutralize the external disturbances.

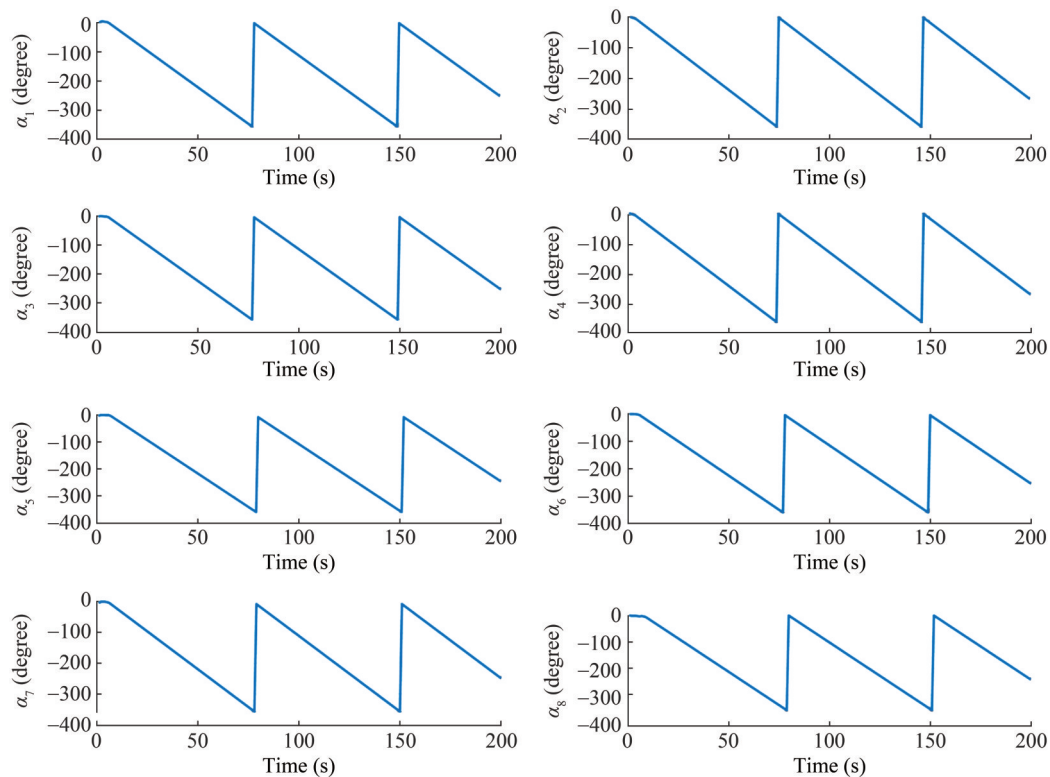


Figure 7 Angle diagram of thrusters in control allocation with eight thrusters

7.1.2 Proposed control allocation with actuator selection

In this part, the proposed method is tested in a fault-free scenario. To this end, at each sample, considering the instant status of thrusters, the *AEI* is calculated using (15), and the most effective thrusters are selected online. For example, in the first simulation, when the angles of all thrusters are zero, the effectiveness indexes are calculated as illustrated in Table 1. The four thrusters, with numbers 5, 8, 3, and 4, exhibit the highest index values and are used in the initial step. Thrusters 1, 2, 6, and 7 are designated as standby. Notably, the effectiveness matrix corresponding to the first group is full row rank. This process is conducted in each step, and the most effective thrusters are chosen. A thruster may exhibit a high effectiveness index in multiple steps and can be consistently chosen, or new thrusters may be replaced at each step. The selection of thrusters is determined based on their effectiveness during each step. However, once the system reaches a steady state, the chosen thrusters generally remain unchanged.

The results of the control allocation using the thruster division method are illustrated in Figures 8 and 9. In each step, only four thrusters are active, while the remaining thrusters remain in standby mode. Following the initial step, when the selected thrusters acquire new angles, the effectiveness index is recalculated, and a new set of four thrusters with high effectiveness indexes is utilized. Consequently, the use of the most effective thrusters is ensured at each step. Figures 8 and 9 show that the chosen thrust-

Table 1 Actuator effectiveness index for the drilling rig at the first simulation sample

Thruster	Type	AEI
1	Azimuth Thruster	19 525
2	Azimuth Thruster	20 674
3	Azimuth Thruster	36 115
4	Azimuth Thruster	35 577
5	Azimuth Thruster	61 564
6	Azimuth Thruster	10 780
7	Azimuth Thruster	32 922
8	Azimuth Thruster	42 819

ers undergo changes within the initial 20 s. However, after this period, thrusters 1, 2, 7, and 8 consistently exhibited the highest effectiveness indexes until the end of the simulation, with no further changes observed in their selection. Thrusters 3, 4, 5, and 6 remain on standby, ready to take over in case of malfunction. When a thruster is in standby mode, its thrust is equal to zero, and its angle remains fixed, leading to a reduction in power consumption. The consistent performance of the selected thrusters after passing the transient state addresses concerns regarding the expenses associated with their start and stop operations. The power consumption of the two aforementioned tests is calculated over the simulation by (18) and presented in Table 2, illustrating the reduction in the total power consumption of the

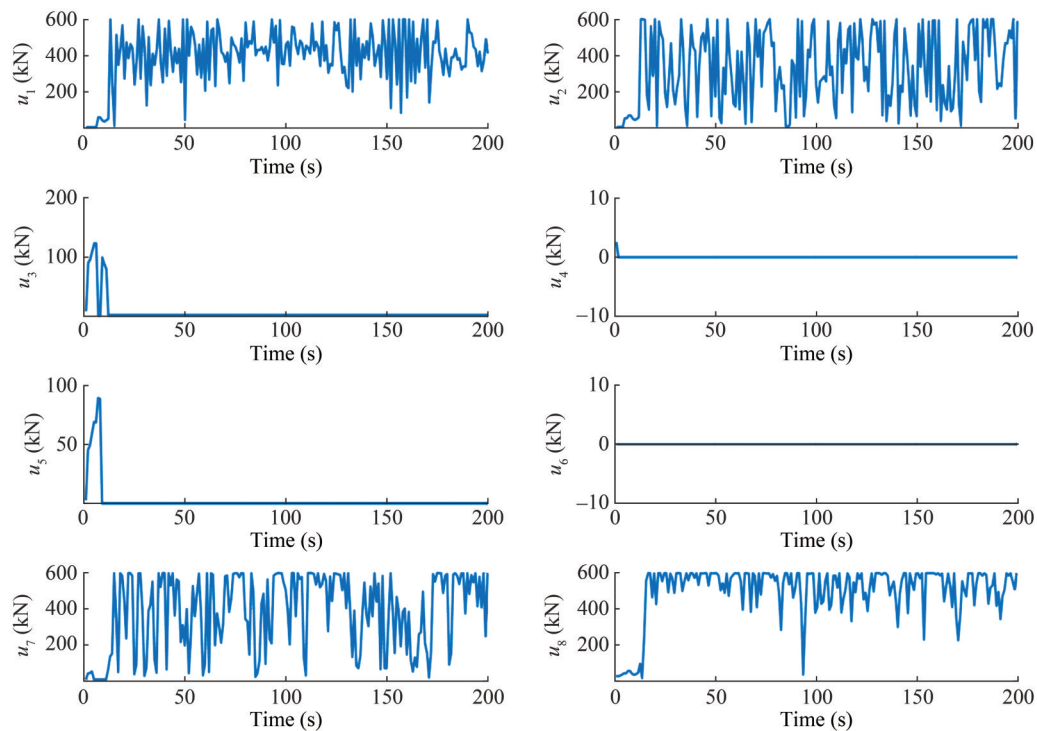


Figure 8 Thrust diagram of thrusters in control allocation with four thrusters

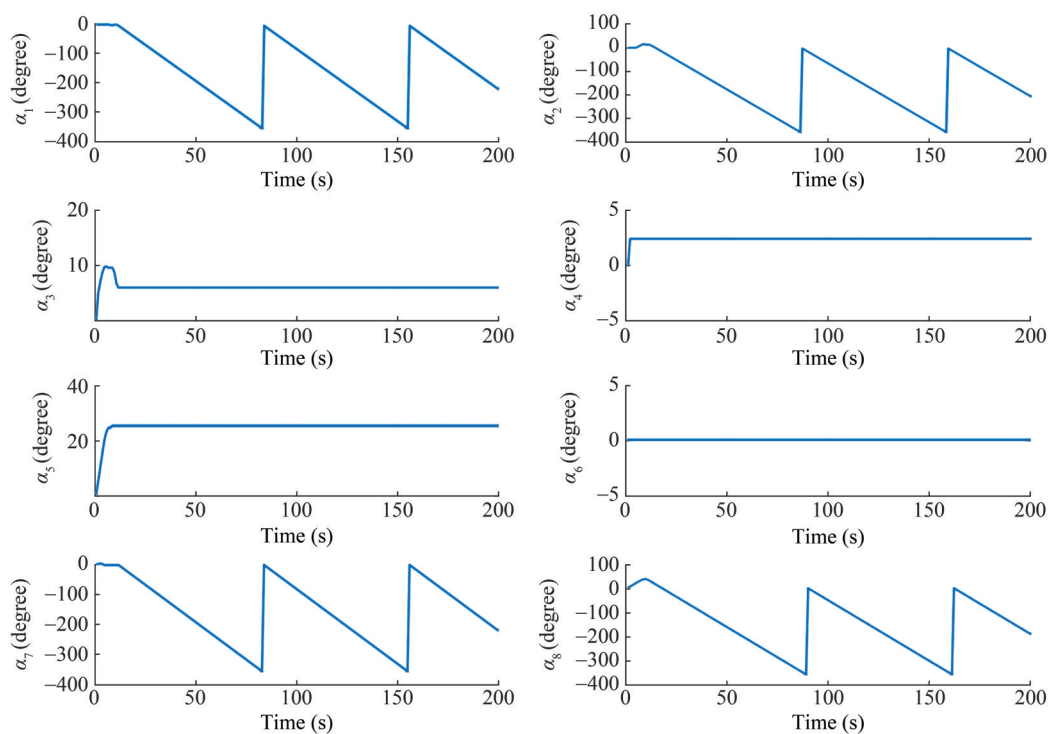


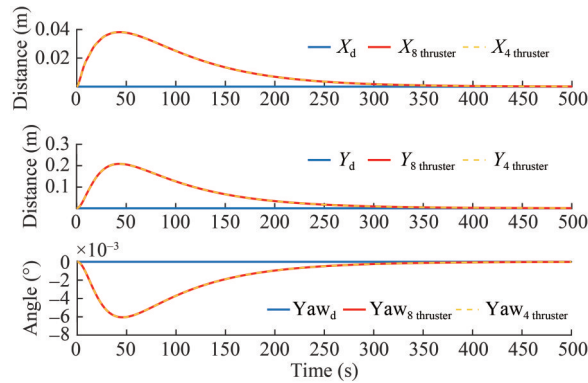
Figure 9 Angle diagram of thrusters in control allocation with four thrusters

thrusters by 36%. The output results of the system with the aforementioned controller are shown in Figure 10. Based on constraint (11) in the optimization of control allocation, the output of the control allocation block must be equal to the virtual control signal, and the virtual control signal must remain the same in both methods. Thus, the results of

the system output in the conventional and proposed methods are similar. The proposed approach introduces substantial energy saving without compromising performance. Furthermore, this approach profoundly reduces computational burden by solving the optimal problem, that is, (10)–(14), for four actuators rather than eight. More precisely,

Table 2 Consumed energy of thrusters

Method	Total power consumption (kW)	Thrust of thrusters (kN)
Normal control allocation (Eight thrusters)	1 317.90	427.65
Proposed control allocation (Four thrusters)	844.06	322.04

**Figure 10** Output results of the system

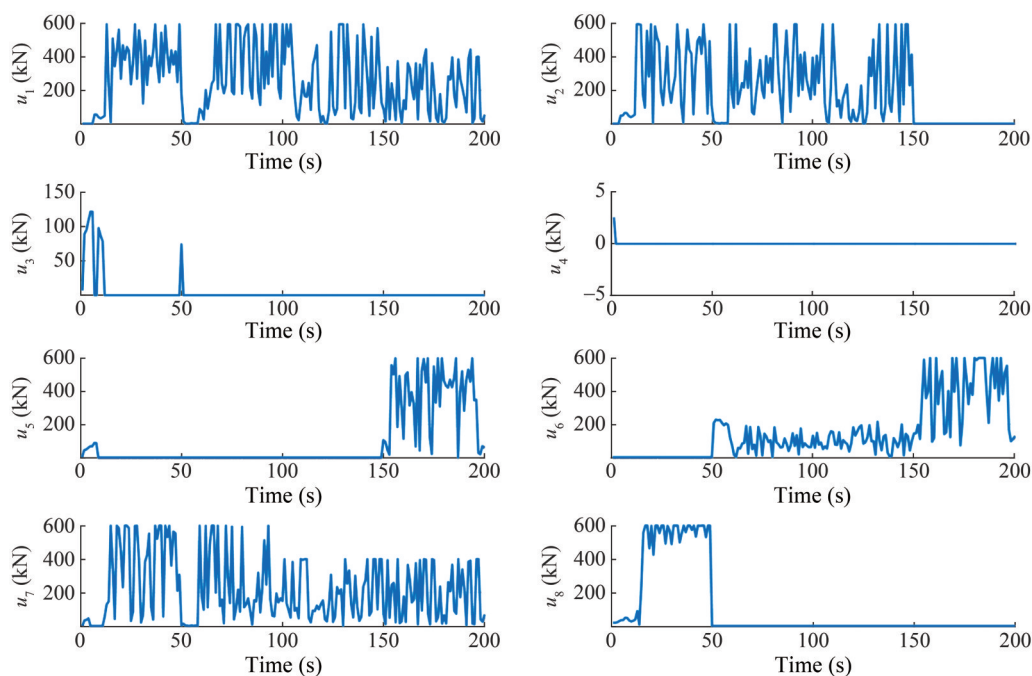
the computational time of the proposed approach is approximately 30% less than the conventional method.

7.2 Scenario two: proposed method with thruster failure

In the second scenario, the durability of the proposed method is assessed against thruster faults, presuming that

certain thrusters may experience failure or malfunction during operation, leading to their exclusion from the system or diminished functionality. Particularly, during the simulation, a malfunction occurs in thruster 8, rendering it inoperative at step 50. Furthermore, at step 100, thruster 7 encounters a functional limitation. Finally, at step 150, thruster 2 malfunctions and becomes inoperative. The virtual control signal vector remains consistent with the initial scenario, that is, the profiles displayed in Figure 6. The thrust and angle outcomes of thrusters are depicted in Figures 11 and 12, respectively.

Azimuth thrusters comprise two electric motors: one for thrust generation and the other for thruster rotation. Thus, issues such as shaft jamming, propeller damage, motor winding, and reduced functional range may arise. The following events occurred during the simulation: At step 50, thruster 8 failed and was consequently removed from the list of active actuators, and thruster 6 was activated as a replacement. At step 100, the functional range of thruster 7 was reduced from (0, 600) to (0, 400), prompting other thrusters to take on additional responsibilities. Figure 11 shows that this thruster does not violate the new limits. At step 150, thruster 5 replaced thruster 2 which failed, and the assigned thrust to thruster 6 was increased. Figure 12 illustrates that the angle of standby of failed azimuth thrusters remains constant, while that of other thrusters permanently changes to address the external disturbances. The output signals of the system remain unchanged, mirroring those of the initial scenario. This finding indicates that the proposed strategy effectively manages actuator faults and failures without impacting the system output.

**Figure 11** Thrust diagram of thrusters in control allocation with four thrusters

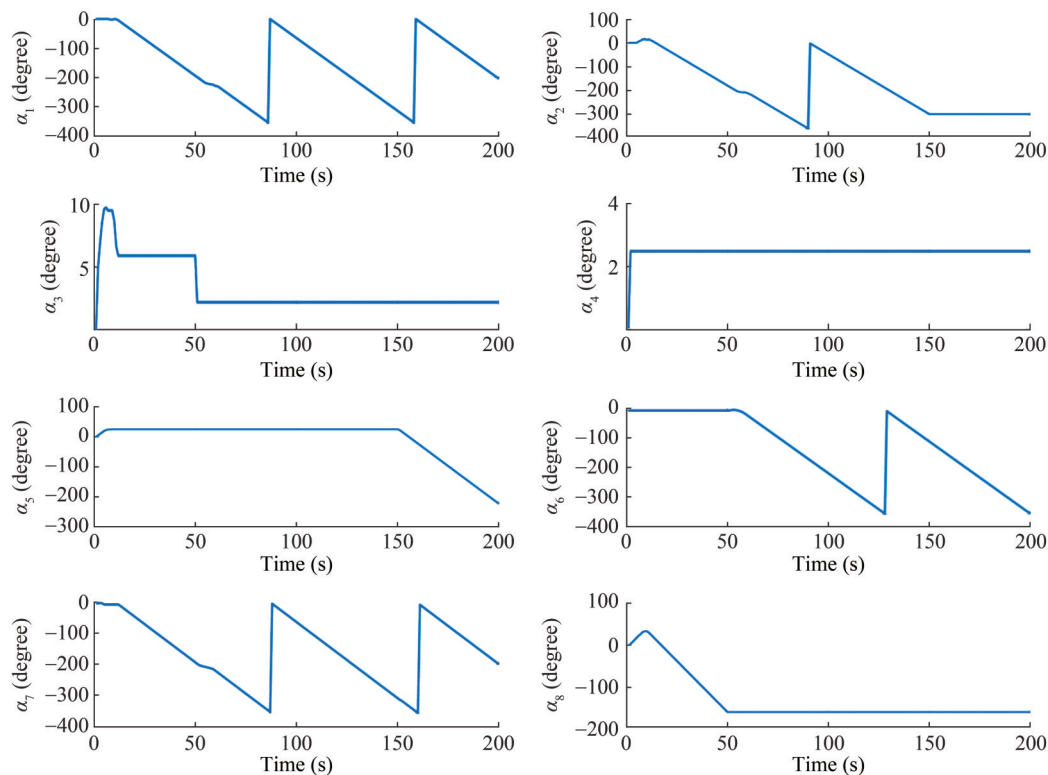


Figure 12 Angle diagram of thrusters in control allocation with four thrusters

The test results indicate that the system has successfully reduced energy consumption as well as computational burden by minimizing the number of active thrusters while maintaining thruster fault tolerance. In case of thruster failure, another thruster can seamlessly take over its responsibilities. The use of a unique thruster category not only reduces energy and maintenance expenses but also simplifies maintenance procedures. Furthermore, a reserve batch of thrusters can be maintained and replaced as needed, minimizing downtime and ensuring optimal system performance.

8 Conclusions

This paper investigates the application of a nonlinear control allocation algorithm in drilling vessels, aiming to enhance the resilience of the system to thruster failures and optimize thruster control. The control allocation unit optimizes and manages the control signal, leveraging the inherent redundancy of drilling rig thrusters to ensure efficient operation. The effectiveness matrix of practical systems is often dynamic, and engaging all thrusters is not always necessary. Thus, a nonlinear control allocation approach, which involves online calculation of the effectiveness matrix and categorization of thrusters into two groups using the AEI, is proposed. This approach can enhance fault tolerance while reducing energy consumption and operation and maintenance costs. Additionally,

the proposed approach enables faster operation during repairs due to periodic repairs of thrusters while in standby mode without fully shutting down the system. Simulation results show that this method reduces energy consumption compared to traditional control allocation methods that include all actuators, increasing its effectiveness against external disturbances, such as sea waves and winds, which attempt to disrupt the position of the float. Future work may focus on the implementation of a real-time fault detection unit to provide realistic information regarding actuator status and limits, including the actuator dynamics, as well as testing the proposed strategy on other floating systems with different kinds of thrusters.

Competing interest The authors have no competing interests to declare that are relevant to the content of this article.

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