

Scour Below Pipelines and Around Single Vertical Piles for Bichromatic and Bidirectional Waves

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Abstract

This article provides a method by which the scour depth and scour width below pipelines, and the scour depth around single vertical piles as well as the time scales of scour for both structures due to bichromatic and bidirectional waves are calculated. The scour and time scale formulae summarized by Sumer and Fredsøe (2002) as well as the bottom shear stress formulae under bichromatic and bidirectional waves by Myrhaug et al. (2023) are used. Results for unidirectional bichromatic waves and symmetrically bidirectional monochromatic waves are provided, showing qualitative agreement with what is expected physically. Qualitative comparisons are made with the data from Schendel et al.'s (2020) small scale laboratory tests on scour around a monopile induced by directionally spread waves. Applications to related cases for pipelines are also suggested. In order to conclude regarding the validity of the method for pipelines and vertical piles, it is required to compare with data in its validity range.

Keywords Scour depth and width; Time scales; Seabed pipeline; Vertical pile; Shear stress; Bichromatic and bidirectional waves; Qualitative comparison with data

1 Introduction

Both pipelines resting on the seabed and vertical piles mounted on the seafloor are affected by the near-bed boundary layer flow, which is due to combined waves and current in shallow and intermediate deep waters. A design condition for these structures in the vicinity of the seafloor is determined by local flow conditions, that is, depending on whether the flow is dominated by waves, tides or currents.

Article Highlights

- The scour depth and scour width below pipelines are calculated for bichromatic and bidirectional waves.
- The scour depth around single vertical piles is calculated for bichromatic and bidirectional waves.
- Time scales of scour for pipelines and vertical piles are calculated for bichromatic and bidirectional waves.
- Results for unidirectional bichromatic waves and symmetrically bidirectional monochromatic waves agree qualitatively with what is expected physically.

During the lifetime, a structure may experience a range of seabed conditions due to moveable bed effects and variation in metocean conditions. Consequently, the bed may be flat or rippled; the pipeline may be in free spans or partly/fully buried; the pile may be surrounded by scour holes. This is caused by the flow resulting from the interaction between the incoming flow, the structure, and the moveable seabed. The equilibrium result depends on the incoming flow velocity (e.g., the relative magnitude between waves and current), the bed geometry and the bed material, as well as the ratio between the near-bed oscillatory fluid particle excursion amplitude and the characteristic dimension of the structure. Moreover, real waves are irregular including directional spreading effects and nonlinear effects. The textbook by Sumer and Fredsøe (2002) gives more background and details of this topic. Fredsøe (2016) provided a review of the interaction between a pipeline and an erodible bed beneath waves and/or currents, covering scour, liquefaction, and lateral stability of pipelines.

From observations it appears that for pipelines and vertical piles a certain amount of time is required for an equilibrium scour to develop. This time T_s is referred to as the time scale of the scour process and is defined as (Sumer and Fredsøe, 2002):

$$S_t = S \left(1 - \exp \left(- \frac{t}{T_s} \right) \right) \quad (1)$$

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where S is the equilibrium scour depth, i.e. the scour depth corresponding to the equilibrium situation, and S_t is the instantaneous scour depth at time t .

The purpose of this article is to provide a hypothesized model to predict the equilibrium scour depths as well as time scales for scour below pipelines and around single vertical piles for bichromatic and bidirectional waves, which represent a first approximation for irregular and directionally spread waves. Results are obtained by using the Myrhaug et al. (2023) bottom shear stress formulae beneath bichromatic and bidirectional waves as well as the scour depths and time scales formulae for pipelines and monopiles as summarized in Sumer and Fredsøe (2002). Results are given for unidirectional bichromatic waves and symmetrically bidirectional monochromatic waves, yielding results which agree qualitatively with what are expected physically. Moreover, the results for a monopile are compared qualitatively with the experimental results by Schendel et al. (2020) for waves with directional spreading.

It should be noted that the seabed shear stress formulae in Myrhaug et al. (2023) were also applied by Myrhaug et al. (2024) providing a method to calculate the scour and self-burial of truncated cones, short cylinders and spherical bodies due to bichromatic and bidirectional waves.

2 Scour for regular waves

Here a summary of scour for regular waves below pipelines (Section 2.1), around single vertical piles (Section 2.2) and of the time scales of scour (Section 2.3).

2.1 Scour below pipelines

Sumer and Fredsøe (1990) provided the empirical formula for the two-dimensional equilibrium scour depth S below a fixed pipeline with diameter D (see Figure 1):

$$\frac{S}{D} = 0.1KC^{0.5}; \quad 2 < KC < 1000 \quad (2)$$

with the Keulegan-Carpenter number

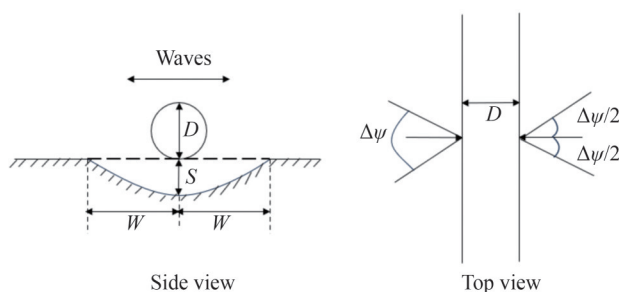


Figure 1 Definition sketch of scour below pipelines

$$KC = \frac{UT}{D} \quad (3)$$

Here U is undisturbed linear near-bottom wave-induced velocity amplitude, and T is wave period. Eq. (2) is valid for live-bed scour, i.e. $\theta > \theta_{cr}$, and θ is undisturbed Shields parameter:

$$\theta = \frac{\tau_w}{\rho g (s - 1) d_{50}} \quad (4)$$

Moreover, τ_w is maximum wave-induced bottom shear stress, ρ is density of fluid, g is acceleration of gravity, s = ratio of sediment density and fluid density, d_{50} is median grain size diameter, θ_{cr} is critical Shields parameter for initiation of motion at the bottom, i.e. $\theta_{cr} \approx 0.05$. Through a transition period the scour process attains its stage of equilibrium. The time scale of scour below pipelines is given in Section 2.3.

Sumer and Fredsøe (1990) also provided the empirical formula for the equilibrium scour width measured from the center of the pipeline to the end of the scour hole, W (see Figure 1):

$$\frac{W}{D} = 0.35KC^{0.65}; \quad 10 < KC < 1000 \quad (5)$$

Both Eqs. (2) and (5) are based on the same experimental data. More details are given in Sumer and Fredsøe (2002).

2.2 Scour around vertical piles

The scour depth in regular waves around single, slender vertical piles for a circular cross-section (Sumer et al., 1992), and a square cross-section at 90 and 45 degrees orientations to the flow (Sumer et al., 1993) was given as (see Figure 2):

$$\frac{S}{D} = C \{1 - \exp[-m(KC - n)]\} \text{ for } KC \geq n \quad (6)$$

where D is the cross-section of the pile, and C, m, n are coefficients with the values.

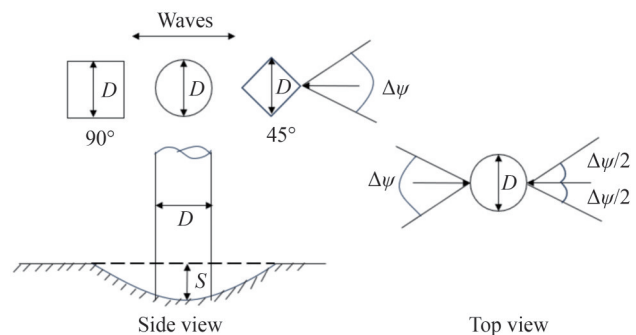


Figure 2 Definition sketch of scour around vertical piles

circular pile:

$$(C, m, n) = (1.3, 0.03, 6) \quad (7)$$

square pile, 90 degrees orientation:

$$(C, m, n) = (2, 0.015, 11) \quad (8)$$

square pile, 45 degrees orientation:

$$(C, m, n) = (2, 0.019, 3) \quad (9)$$

These formulae are based on extensive series of laboratory tests and are valid for live-bed scour as defined in Section 2.1. Equations (6) to (9) show that the onset of scour starts when $KC \geq n$, which coincides with the onset of vortex shedding around the pile. This vortex shedding is the main mechanism causing the scour (see Sumer and Fredsøe (2002) for more details). As for pipelines, the scour process for vertical piles attains its equilibrium stage through a transition period. The time scales for scour for pipelines and around circular vertical piles are given in Section 2.3.

2.3 Time scales for scour

Sumer and Fredsøe (2002) provided the following dimensionless time scale T_s^*

$$T_s^* = \frac{(g(s-1)d_{s0}^3)^{1/2}}{D^2} T_s \quad (10)$$

where T_s is defined in Eq. (1), and for the equilibrium scour depth below pipelines and around circular vertical piles T_s^* is given by

$$T_s^* = pKC^q\theta^{-r} \quad (11)$$

where p, q, r are coefficients with the values:

$$(p, q, r) = \left(\frac{1}{50}, 0, \frac{5}{3}\right) \text{ for pipelines} \quad (12)$$

$$(p, q, r) = (10^{-6}, 3, 3) \text{ for circular vertical piles} \quad (13)$$

Eq. (12) is valid for $0.05 \leq \theta \leq 0.19$, while Eq. (13) is valid for $0.07 \leq \theta \leq 0.19$ and $7 \leq KC \leq 34$.

Recently, Larsen and Fuhrman (2023) revised the equilibrium time scale formula in Eqs. (11) and (13) for a circular vertical pile by taking $(p, q, r) = (8 \times 10^{-5}, 2.5, \frac{3}{2})$ valid for $KC \geq 4$ and $0.33 \leq \theta \leq 0.74$, stating that this revised formula may be more representative of many field condi-

tions compared with using Eq. (13) based on smaller laboratory test conditions. However, the subsequent examples will apply the coefficients in Eq. (13), but can easily be changed using this revised formula.

3 Scour for bichromatic and bidirectional waves

Here results for bichromatic and bidirectional waves are presented by first providing the theoretical background (Section 3.1). Then, the results for scour below pipelines and around vertical piles for unidirectional bichromatic waves (Section 3.2) and symmetrically bidirectional monochromatic waves (Section 3.3) are provided by giving examples of results for realistic field conditions. Applications to related cases for pipelines are given in Appendix A, i.e. effects of pipeline position in scour and self-burial of pipelines at span shoulder.

3.1 Theoretical background

A summary of the Myrhaug et al. (2023) bottom shear stress formulae is provided.

Two monochromatic waves are considered with: wave frequencies ω_1, ω_2 , phase angles α_1, α_2 , propagation directions ψ_1, ψ_2 , horizontal free stream wave-induced velocity vectors:

$$U_n = \vec{u}_{0n} e^{i(\omega_n t + \alpha_n)}; n = 1, 2 \quad (14)$$

and bottom shear stress vectors:

$$\tau_n = \vec{\tau}_{0n} e^{i(\omega_n t + \hat{\phi}_n)}; \hat{\phi}_n = \phi_n + \alpha_n; n = 1, 2 \quad (15)$$

Here $\vec{u}_{0n}$ = horizontal free stream velocity amplitude vector, $\vec{\tau}_{0n}$ = bottom shear stress amplitude vector, $\hat{\phi}_n$ = phase of bottom shear stress, ϕ_n = phase lag between bottom shear stress and velocity, t = time, $i = \sqrt{-1}$ = complex unit. The combined bottom shear stress vector beneath bichromatic and bidirectional waves is:

$$\vec{\tau} = \vec{\tau}_{01} e^{i(\omega_1 t + \hat{\phi}_1)} + \vec{\tau}_{02} e^{i(\omega_2 t + \hat{\phi}_2)} \quad (16)$$

For $\omega_1 \neq \omega_2$ the maximum bed shear stress is (see Myrhaug et al. (2023) for more details):

$$\tau_{wm} = [\tau_{01}^2 + \tau_{02}^2 + 2\tau_{01}\tau_{02}\cos(\psi_1 - \psi_2)]^{1/2}; \omega_1 \neq \omega_2 \quad (17)$$

Here $\tau_{0n} = |\vec{\tau}_{0n}|$ = magnitude of $\vec{\tau}_{0n}$, and τ_{wm} is independent of the phase angles $\hat{\phi}_1$ and $\hat{\phi}_2$ for $\omega_1 \neq \omega_2$. Bichromatic waves (for $\omega_1 \neq \omega_2$) will be considered, except for a spe-

cial case given subsequently in an example.

The method is based on adopting Christoffersen and Jonsson's (1985) eddy viscosity valid for full rough turbulent boundary layer flow for large bottom roughness under harmonic waves, i. e. the amplitude of the bottom shear stress for a sinusoidal wave with wave frequency ω and free stream velocity amplitude u_0 is given by the well-known laminar solution $\tau_{wm}/\rho = \sqrt{\omega\nu} u_0$. The kinematic viscosity ν is then replaced by the time- and space- invariant eddy viscosity $\varepsilon_w = \beta k_s u_{*wm}$ where $\beta = 0.0747$, $k_s = 2.5d_{50} =$ Nikuradse's equivalent sand roughness, and $u_{*wm} = \sqrt{\tau_{wm}/\rho}$ = maximum bottom friction velocity. Thus, substitution of ε_w for ν gives $u_{*wm}^2 = \tau_{wm}/\rho = \sqrt{\omega\varepsilon_w} u_0 = \sqrt{\beta\omega k_s u_{*wm}} u_0$ for a sinusoidal wave, and thus $u_{*wm} = (\beta\omega k_s u_0^2)^{1/3}$, or

$$\frac{\tau_{wm}}{\rho} = u_{*wm}^2 = (\beta\omega k_s u_0^2)^{2/3} \quad (18)$$

Similarly, for each harmonic wave component the amplitude of the bottom shear stress is $\tau_{0n}/\rho = \sqrt{\beta\omega_n k_s u_{*wm} u_{0n}}$, which substituted in Eq. (17) and solved for $u_{*wm} = \sqrt{\tau_{wm}/\rho}$, yields:

$$\frac{\tau_{wm}}{\rho} = \left[u_{01}^2 \omega_1 + u_{02}^2 \omega_2 + 2u_{01}u_{02}\sqrt{\omega_1\omega_2} \cos(\psi_2 - \psi_1) \right]^{2/3} (\beta k_s)^{2/3}; \omega_1 \neq \omega_2 \quad (19)$$

This is valid for $1.3 < a_0/k_s < 50$ where $a_0 = u_0/\omega$ = free stream excursion amplitude.

Sections 3.2 and 3.3 include examples for unidirectional bichromatic waves and symmetrically bidirectional monochromatic waves by using Eq. (19).

First, unidirectional bichromatic waves are considered with $u_{01} = u_{02} = u_0$ and $\psi_1 = \psi_2$, Eq. (19), giving:

$$\frac{\tau_{wm}}{\rho} = \left(\sqrt{\omega_1} + \sqrt{\omega_2} \right)^{4/3} (\beta k_s u_0^2)^{2/3} \quad (20)$$

which corresponds to the maximum bottom shear stress under two waves with the near-bed wave-induced velocity:

$$u = u_0 \cos(\omega_1 t) + u_0 \cos(\omega_2 t) = 2u_0 \cos(\Delta\omega t) \cos(\dot{\omega} t) \quad (21)$$

where $\Delta\omega = (\omega_2 - \omega_1)/2$ and $\dot{\omega} = (\omega_1 + \omega_2)/2$. This corresponds to the equivalent monochromatic wave with the wave-induced velocity with the maximum amplitude $2u_0$ and mean wave frequency $\dot{\omega}$, when substituted in Eq. (18) gives:

$$\frac{\tau_{wm}}{\rho} = \left[\beta \dot{\omega} k_s (2u_0)^2 \right]^{2/3} \quad (22)$$

Second, for symmetrically bidirectional monochromatic waves with $u_{01} = u_{02} = u_0$, $\omega_1 = \omega_2 = \omega = 2\pi/T$ where T is the wave period, and $\psi_2 - \psi_1 = \Delta\psi$, Eq. (19) yields:

$$\frac{\tau_{wm}}{\rho} = \left[8\beta k_s u_0^2 \frac{\pi}{T} \cos^2\left(\frac{\Delta\psi}{2}\right) \right]^{2/3} \quad (23)$$

Eq. (23) represents the maximum bottom shear stress under two waves with frequency ω that propagate perpendicularly to the pipeline with $\Delta\psi$ as directional spreading, which corresponds to the equivalent monochromatic wave $u = 2u_0 \cos(\Delta\psi/2) \cos \omega t$, with equal and opposite directions from the perpendicular direction to the pipeline. Thus, Eq. (23) is obtained by replacing u_0 in Eq. (18) with $2u_0 \cos(\Delta\psi/2)$. One should note that Eqs. (17) and (19) are valid even for $\omega_1 = \omega_2$ if $\hat{\phi}_1 = \hat{\phi}_2$ (see Eqs. (14), (15) and Eq. (A4) in Myrhaug et al. (2023) for $\omega_1 = \omega_2$ and $\hat{\phi}_1 = \hat{\phi}_2$).

3.2 Scour for unidirectional bichromatic waves

An example with $d_{50} = 0.03$ m is considered (which corresponds to pebble; Figure 4 in Soulsby (1997)), $k_s = 2.5d_{50} = 0.075$ m. Further, the near-bed wave-induced velocity is given by Eq. (21) for $u_0 = 1.1$ m/s, $T_1 = 7.2$ s, $T_2 = 6.0$ s. This corresponds to $(a_{01}/k_s, a_{02}/k_s) = (16.8, 14) < 50$ (i.e. within the model's validity range). Then, the maximum wave-induced velocity amplitude is $2u_0 = 2.2$ m/s with the mean wave frequency $\dot{\omega} = (\omega_1 + \omega_2)/2 = 0.96$ rad/s. Then, for a pipeline/pile with $D = 1$ m, Eq. (3) gives $KC = \frac{UT}{D} = \frac{2\pi U}{\dot{\omega} D} = \frac{2\pi \cdot 2.2}{0.96 \cdot 1} = 14.4$ (see Table 1). Substitution in Eq. (22) yields $\tau_{wm}/\rho = (0.0747 \cdot 0.96 \cdot 0.075 \cdot 2.2^2)^{2/3} = 0.0878$ m²/s², and Eq. (4) gives the Shields parameter $\theta = 0.0878 / (9.81 \cdot 1.65 \cdot 0.03) = 0.18 > 0.05$ and < 0.19 for $g = 9.81$ m/s² and $s = 2.65$ (as for quartz sand) (see Table 1).

The results for pipelines and vertical piles are given in Table 1. First, for pipelines, substitution in Eqs. (10), (11), (2) and (5) yields T_s , S/D and W/D . Second, for vertical piles substitution in Eqs. (6) to (9) yields S/D for a circular pile, a square pile with 90 degrees orientation, and a square pile with 45 degrees orientation. Moreover, substitution in Eqs. (10) and (11) gives T_s for circular vertical piles.

Now, these results are compared with those for one of the wave components, for example, for $u_0 = 1.1$ m/s, $T_1 = 7.2$ s, as given in Table 1. Then, $KC = 7.9$, $\theta = 0.0673 > 0.05$ and < 0.19 . For pipelines it appears that S/D and W/D are smaller than for bichromatic waves, while T_s is larger than for bichromatic waves. Similarly, for vertical piles it appears that S/D for circular pile; square pile, 90 degrees orientation; square pile, 45 degrees orientation are smaller than for bichromatic waves, while T_s is larger than for bichromatic waves. All these results are as expected.

Table 1 Example of results for unidirectional bichromatic waves ($u_0 = 1.1$ m/s, $T_1 = 7.2$ s, $T_2 = 6.0$ s), and monochromatic waves ($u_0 = 1.1$ m/s, $T = 7.2$ s); $d_{50} = 0.03$ m, $k_s = 0.075$ m, $D = 1$ m

Wave conditions	KC	θ	Pipelines T_s (s)	Pipelines S/D	Pipelines W/D	Piles T_s (s) Circular	Piles S/D Circular	Piles S/D Square 90° orientation	Piles S/D Square 45° orientation
Unidir bichrom	14.4	0.180 0	16.7	0.38	1.98	24.5	0.290	0.099	0.39
Monochrom	7.9	0.067 3	85.9	0.28	0	77.4	0.072	0	0.18

3.3 Scour for symmetrically bidirectional monochromatic waves

Two monochromatic waves for $k_s = 0.075$ m with $u_{01} = u_{02} = u_0 = 1.1$ m/s, $T_1 = T_2 = T = 7.2$ s, directional spreading $\Delta\psi$ and perpendicular incidence to pipelines are considered. Then, based on the discussion of Eq. (23) in Section 3.1, this equation is identical to the use of the equivalent monochromatic wave replacing u_0 in Eq. (18) with $2u_0 \cos(\Delta\psi/2)$, which substituted in Eq. (23) gives:

$$\frac{\tau_{wm}}{\rho} = \left[8 \cdot 0.0747 \cdot 0.075 \cdot \frac{\pi}{7.2} \cdot 1.1^2 \cdot \cos^2\left(\frac{\Delta\psi}{2}\right) \right]^{2/3} = \left[0.02366 \cdot \cos^2\left(\frac{\Delta\psi}{2}\right) \right]^{2/3} \quad (24)$$

Moreover, substitution of this in Eqs. (3) and (4) yields, respectively,

$$KC = \frac{T}{D} U = \frac{7.2}{1} \cdot 2.2 \cdot \cos\left(\frac{\Delta\psi}{2}\right) = 15.8 \cdot \cos\left(\frac{\Delta\psi}{2}\right) \quad (25)$$

$$\theta = \frac{\tau_{wm}/\rho}{g(s-1)d_{50}} = \frac{\tau_{wm}/\rho}{9.81 \cdot 1.65 \cdot 0.03} = 0.170 \cdot \cos^{\frac{4}{3}}\left(\frac{\Delta\psi}{2}\right) \quad (26)$$

where τ_{wm}/ρ from Eq. (24) has been substituted in Eq. (26).

Table 2 gives the results for $\Delta\psi$ in the range 0–132 degrees; that is, for $\Delta\psi > 132$, $\theta < 0.05$, and thus no sediment motion. As $\Delta\psi$ increases, it appears that KC and θ decrease. Consequently, T_s increases and S/D and W/D

decrease as $\Delta\psi$ increases. Similarly, for circular vertical piles, T_s increases and S/D decreases; for square vertical piles with 90 and 45 degrees orientations, S/D decreases as $\Delta\psi$ increases. All these results are as expected.

One should note that the results for the pile exhibit the same qualitative behaviour for all the three different cross-sections. However, the directional spreading effect on the scour depth is largest for the square pile with 90 degrees orientation, followed by the circular pile, and the square pile with 45 degrees orientation. This is a result of the inherent physical features of Eqs. (6) to (9) as discussed in Section 2.2; for each value of KC in Table 1, S/D is largest for the cross-section where the critical value of KC (for the onset of scour) is smallest, i.e. for the square pile with 45 degrees orientation where $KC \geq 3$, followed by the circular pile where $KC \geq 6$, and the square pile with 90 degrees orientation where $KC \geq 11$. The following directional spreading effects are also observed from Table 2: The effect on T_s is larger for pipelines than for circular piles, and consistently, the effect on S/D is larger for circular piles than for pipelines.

For the pile with square cross-sections one should note that the 45 and 90 degrees cases will result in different mean flow around the cylinder. However, these different mean flow features are contained in the empirical formulae, which are based on the physical experiments for both cases. Thus, the present approach assumes that the directional spreading can be taken into account using the empirical formulae for the mean flow.

To the author's knowledge, no data are available in the open literature which can be used to make quantitative comparisons with the present results. However, Schendel et al. (2020) presented results from small scale laboratory tests on scour around a monopile induced by directionally

Table 2 Example of results for symmetrically bidirectional monochromatic waves; $d_{50} = 0.03$ m, $k_s = 0.075$ m, $u_0 = 1.1$ m/s, $T = 7.2$ s, $D = 1$ m

$\Delta\psi$ (°)	KC	θ	Pipelines T_s (s)	Pipelines S/D	Pipelines W/D	Piles T_s (s) Circular	Piles S/D Circular	Piles S/D Square 90° orientation	Piles S/D Square 45° orientation
0	15.8	0.170 0	18	0.40	2.10	38	0.330	0.140	0.43
30	15.3	0.162 0	20	0.39	2.06	40	0.320	0.120	0.42
60	13.7	0.140 0	25	0.37	1.92	45	0.270	0.079	0.37
90	11.2	0.107 0	40	0.33	1.68	55	0.190	0.006	0.29
120	7.9	0.067 5	85	0.28	0	77	0.072	0	0.18
132	6.4	0.051 2	136	0.25	0	93	0.016	0	0.13

spread irregular waves in combination with oblique currents, which enables qualitative comparison with their results for waves alone and the present results.

First, regarding the time scale of scour for waves alone for long-crested (2D) and short-crested (3D) waves (see Figure 10 in their paper), Schendel et al. (2020) found that the time scale for 2D waves was smaller than that for 3D waves for $KC \approx 8.5$, which agrees qualitatively with the present example results (see Table 2). However, for $KC = 3.6$ the 2D and 3D time scales were almost equal, and for $KC = 12.5$ the time scale for 2D waves was larger than that for 3D waves. The reason for this is unclear to the present authors. Second, regarding the scour depth for waves alone (see Figure 12 in their paper), they found a clear tendency for the scour depths to decrease as the directional spreading increases, i.e. consistent with the present example results (see Table 2).

However, in order to conclude regarding the validity of the method for pipelines and vertical piles, it is required to compare with data in its validity range.

Applications to related cases for pipelines are given in Appendix A, i.e. effects of pipeline position in scour and self-burial of pipelines at span shoulder.

4 Conclusions

The present proposed method appears to yield results that agree qualitatively with what are expected physically, and are summarized as follows:

1) For symmetrically bidirectional monochromatic waves it appears that:

i) The scour depths decrease as the directional spreading increases, while the time scales of scour increase as the directional spreading increases.

ii) The directional spreading effect on the time scale is larger for pipelines than for circular piles, and consistently, the effect on the scour depth is larger for circular piles than for pipelines.

iii) The directional spreading effect on the scour depth is largest for the square pile with 90 degrees orientation, followed by the circular pile, and the square pile with 45 degrees orientation.

2) The data from Schendel et al.'s (2020) small scale laboratory tests on scour around a monopile induced by directionally spread waves showed a clear tendency for the scour depths to decrease as the directional spreading increases, which is consistent with the present example results. This is also the case for the time scale of scour for $KC \approx 8.5$, which for 2D waves was smaller than for 3D waves. However, for $KC = 3.6$ the 2D and 3D time scales were almost equal, and for $KC = 12.5$ the time scale for 2D waves was larger than for 3D waves, which is not captured by the present model.

3) Applications to related cases for pipelines are also suggested, such as effects of pipeline position in scour and self-burial of a pipeline at span shoulder.

4) In order to make firm conclusions regarding the validity of the present method, it is required to compare with data in its validity range.

Appendix A. Application to related cases for pipelines

A1. Effect of pipeline position in scour

Some additional tests on the effect of pipeline position in scour were also made by Sumer and Fredsøe (1990) by measuring the equilibrium scour depth for regular waves over pipelines fixed at different levels e relative to the undisturbed bed (see Figure A1). Based on these results for regular waves Sumer and Fredsøe (2002) proposed the following empirical formula valid for live-bed scour:

$$\frac{S}{D} = 0.1KC^{0.5} \exp\left(-0.6\frac{e}{D}\right); 0 < \frac{e}{D} < 2 \quad (A1)$$

It is noticed that this formula is a generalization of Eq. (2) by re-arranging it to:

$$\frac{S}{D \exp\left(-0.6\frac{e}{D}\right)} = 0.1KC^{0.5} \quad (A2)$$

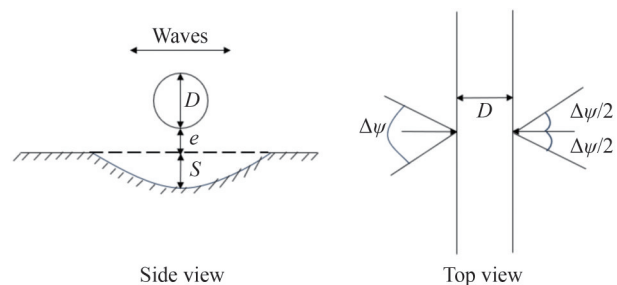


Figure A1 Definition sketch of scour below pipelines with position e above the undisturbed bed

Thus, the results given for the scour depth and the time scale of scour for a pipeline initially resting on an undisturbed bed provided in Sections 3.2 and 3.3 can be used in this case by multiplying by the factor $\exp(-0.6e/D)$. To the best knowledge of the authors, no data exist for scour in bichromatic and bidirectional waves in this case.

A2. Self-burial of pipelines at span shoulder

Sumer et al. (2001) performed an experimental study on the onset of scour below pipelines and self-burial of pipelines in regular waves and currents, finding that the equilibrium self-burial at span shoulders was the same as the

scour depth below a fixed pipeline with an initial zero gap. Consequently, the equilibrium self-burial depth at a span shoulder e for regular waves (see Figure A2) is given by Eq. (2) where e replaces S :

$$\frac{e}{D} = 0.1KC^{0.5}; KC < 100 \quad (\text{A3})$$

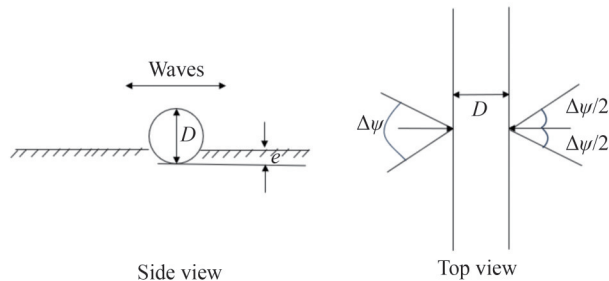


Figure A2 Definition sketch of self-burial depth e at span shoulder of a pipeline

Thus, the results given for the scour depth and the time scale of scour for pipelines initially resting on an undisturbed bed provided in Sections 3.2 and 3.3 can be used in this case. To the best knowledge of the authors, no data exist for self-burial of span shoulders in bichromatic and bidirectional waves in this case.

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