

Wave Characteristics in the Vicinity of the Critical Radiation Frequency of a Horizontal Elastic Cylinder

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Abstract

For a generalized radiation problem, an infinitely long submerged horizontal cylinder is forced to vibrate periodically in the transverse direction, with a described elastic harmonic motion along its longitudinal direction. A critical frequency corresponds to the described wave number of elastic vibration, and the generalized hydrodynamic coefficients abruptly change in the vicinity of critical frequency. In this work, a numerical examination is carried out to study the characteristics of wave profiles and wave propagation in the vicinity of the critical frequency. Results show that below the critical frequency, the real parts of complex wave profiles have large values in the vicinity of the cylinder and decay to zero with the increasing distance from the cylinder. Meanwhile, the imaginary parts of complex wave profiles are all zero, which explains why the generalized radiation damping is zero when the vibration is less than the critical frequency. At far distances, no radiation wave is observed. When the vibration exceeds the critical frequency, the real and imaginary parts of the wave profiles oscillate harmonically and keep steady amplitudes. In addition, the generated radiation wave propagates obliquely outward. The influence of the cylinder's submergence depth on the wave profile is also studied, and the results indicate that the amplitude of the wave profile decreases as the submergence depth of the cylinder increases. The 3D wave profiles are graphically presented to show the wave propagation characteristics in the vicinity of the critical frequency for this generalized radiation problem. This study provides a good reference for the interaction between fluid and slender elastic structures.

Keywords Submerged floating tunnel; Boundary element method; Eigenfunction expansion method; Critical frequency; Generalized radiation problem

1 Introduction

A submerged floating tunnel (SFT) is a novel type of tunnel suspended under water level and maintains its balance through buoyancy, its self-weight, and a support system (Engelbrechtsen et al., 2017; Zhang et al., 2021). SFTs

become competitive when the water depth is large, the wave and wind conditions are severe, and the ship traffic is heavy (Moan and Eidem, 2020). Some projects have considered SFTs for strait crossing, especially wide crossing such as the Messina straits (Martire et al., 2009) and Qiandao Lake in China (Mazzolani et al., 2008). Similar to other offshore structures, an SFT is subject to various environmental loadings from waves, currents, tsunamis, and seismic waves; the moving traffic loads of vehicles; and the possible impacting loads from falling objects.

In dealing with wave interaction with slender horizontal structures, such as SFTs, mega-floating bodies, or large ships, a common practice is to simplify them into 2D structures. This approach is based on the assumption that the incident wave potential and body response in the longitudinal direction are uniform or that the variations of incident wave and body response in the longitudinal direction can be separated from the total functions. These simplified 2D problems have significant advantages in computation over 3D problems.

The radiation and diffraction problems of 2D bodies floating in inviscid and incompressible fluid have been studied using different techniques, such as multipole expansion

Article Highlights

- Based on the potential flow theory, the generalized radiation problem of an infinitely long submerged horizontal cylinder being forced to vibrate periodically in the transverse direction and generate elastic harmonic motion along its longitudinal direction is studied.
- The characteristics of wave profiles and wave propagation in the vicinity of critical frequency are presented.
- This study provides a strong theoretical basis for the design and practical application of submerged floating tunnel.

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methods, finite element methods, and boundary element methods (BEMs). Ursell et al. (1949a, b) used a multipole expansion method to study the heave motion of a circular cylinder on a fluid surface and extended it to the roll motion of an arbitrary section on a fluid surface through isogonal transformation. Athanassoulis (1984) derived a general multipole expansion for an infinitely long cylinder of arbitrary section. He decomposed the wave potential into a regular wave, a wave source, a wave dipole, and a regular wave-free part and then used Texeira's series and the conformal mapping between the semicircular region and fluid domain to derive the solution. Martin and Dixon (1983) also studied the scattering problem of normal incident waves by a fixed half-immersed circular cylinder using Ursell's multipole method and presented accurate numerical results for wave reflection and transmission coefficients, the horizontal and vertical wave forces by tables. Bai (1975) analyzed the diffraction problem for 2D infinitely long fixed cylinders in the presence of oblique incident waves in finite water using a finite element method and examined the influence of wave direction on the wave reflection, transmission coefficients, wave forces, and moments on the horizontal rectangular cylinders on the free surface. Garrison (1984) and Politis et al. (2002) studied the diffraction problem of oblique incident waves by a 2D fixed cylinder in infinite water using a BEM with the wave Green's function. For this method, only the body surface must be discretized with the cost to develop an accurate and efficient program for computing the wave Green's function.

Predicting SFT response under various loadings is a typical fluid–structure interaction problem that is closely linked with the SFT hydrodynamic property and must be solved using a coupling method that considers hydrodynamics and structural mechanics. The mode decomposition method is convenient and widely used in the investigation of hydroelasticity and, thus, SFT response (Eatock Taylor and Ohkusu, 2000). In this approach, the structural vibration is divided into several modes that are determined in advance through a corresponding structural analysis.

Trigonometric functions play an important role in the hydroelasticity analysis of an SFT system. Lin et al. (2018) simplified an SFT subjected to moving vehicle loads in an ocean current environment as a beam on an elastic foundation with simple supports at the two ends. They found that the corresponding vibration modes of the SFT can be described by sine functions. However, the vibration modes of an SFT with mooring lines and end built-in conditions could not be represented by any trigonometric function along its longitudinal direction. Thus, the hydrodynamic problem of a long SFT cannot be directly simplified into a two-dimensional problem. When the vibration modes of STF are expanded into Fourier components that vary with the trigonometric functions along its longitudinal direc-

tion, the radiation problem corresponding to the Fourier expansion component can be simplified as a 2D problem and resolved as a generalized radiation problem.

For a generalized radiation problem, a certain normal velocity is prescribed on the SFT to generate water waves traveling obliquely outward. In the radiation problem, no wave incident from infinity exists. When the prescribed velocity has the same phase everywhere on the cylinder, the waves normally travel outward, and the generalized radiation problem is reduced to the ordinary radiation problem of a rigid cylinder. In addition, the hydrodynamic coefficients corresponding to the generalized radiation problem become the generalized hydrodynamic coefficients.

Research efforts have been devoted to the study of generalized radiation problems. Bolton and Ursell (1973) considered the generalized heaving problem of an infinitely long half-immersed circular cylinder under the harmonic elastic vibration $\exp[i(k_y y - \omega t)]$ in the heave direction, where ω is the vibration frequency and k_y is the elastic vibration wave number in the longitudinal direction of the cylinder. They derived the radiation potential using the multipole expansion method for the modified Bessel equation and computed the heave load using Haskind's relation. Teng et al. (2023) proposed a BEM with a simple Green function for calculating the generalized hydrodynamic coefficients of a horizontal vibrating cylinder and obtained accurate results. They found that a critical vibration frequency exists for every elastic vibration wave number. Below this limit, no radiation wave can be generated. The generalized hydrodynamic coefficients in the vicinity of the critical vibration frequency abruptly change, characterized by a "sharp peak" amplitude. To further understand this phenomenon, we use the same numerical model to examine the wave propagation characteristics in the vicinity of the critical vibration frequency. Considering that Teng et al. (2023) already carried out the validation and convergence examination, we will not provide any further analysis for the validation of this numerical model. The present work may be regarded as a sequel to Teng et al.'s (2023) earlier study.

2 Governing equation and boundary conditions

In this section, mathematical formulations for the generalized radiation problem are given. A right-handed Cartesian coordinate system, denoted as $Oxyz$, is chosen with $z = 0$ being the undisturbed free surface, the y -axis along the cylinder's longitudinal direction, and the positive z -axis vertically upward. We assume that the fluid is homogeneous, incompressible and inviscid, and the fluid motion is irrotational. We consider an infinitely long horizontal cir-

cular cylinder with a radius a submerged with a submergence depth T in a constant finite water depth d , as shown in Figure 1. The elastic cylinder experiences a harmonic transverse motion with a frequency of ω in the cross-sectional plane and an elastic wave of $\cos(k_y y - \omega t)$ along its longitudinal direction. Here, k_y is the wave number of the cylinder's elastic vibration along its longitudinal direction. For this generalized radiation problem, the translation displacements in the x and z directions and the rotation about the y axis can be written as

$$\begin{cases} \Xi_x = \text{Re}[\zeta_x e^{ik_y y} e^{-i\omega t}] \\ \Xi_z = \text{Re}[\zeta_z e^{ik_y y} e^{-i\omega t}] \\ \Theta_y = \text{Re}[\theta_y e^{ik_y y} e^{-i\omega t}] \end{cases} \quad (1)$$

where ζ_x and ζ_z are the amplitudes of transverse vibrations in the x and z directions, respectively, θ_y represents the amplitude of rotational vibration about the y axis, and Re means taking the corresponding real part. For simplification, Eq. (1) is presented in a generalized form as

$$\Xi_j = \text{Re}[\zeta_j e^{ik_y y} e^{-i\omega t}] \quad (j = 1, 2, 3) \quad (2)$$

where $(\Xi_1, \Xi_2, \Xi_3) = (\Xi_x, \Xi_z, \Theta_y)$ and $(\zeta_1, \zeta_2, \zeta_3) = (\zeta_x, \zeta_z, \theta_y)$. The vibration amplitudes of the cylinder are sufficiently smaller than its diameter.

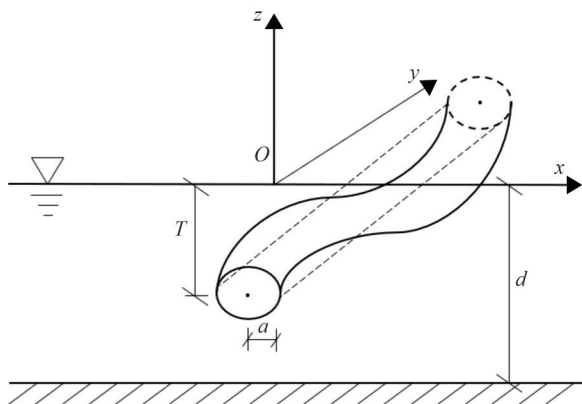


Figure 1 Sketch of a horizontal cylinder under elastic vibrations in a finite water depth

According to small amplitude wave theory with harmonic motion, we can express the fluid motion of an incompressible and inviscid fluid by introducing velocity potential. The radiation potential $\phi_j(x, y, z, t)$ caused by the elastic vibration of the cylinder in the j -th direction can be written as

$$\Phi_j(x, y, z, t) = \text{Re}[\phi_j(x, y, z) e^{-i\omega t}], \quad j = 1, 2, 3 \quad (3)$$

where $\phi_j(x, y, z)$ is a time-independent complex potential.

The body boundary condition for the complex radiation potential can be written as

$$\frac{\partial \phi_j(x, y, z)}{\partial n} = -i\omega \zeta_j e^{ik_y y} n_j, \quad j = 1, 2, 3 \quad (4)$$

where the generalized direction vector $(n_1, n_2, n_3) = (n_x, n_z, (z - z_0)n_x - (x - x_0)n_z)$ and (x_0, z_0) is the rotation center. Under the excitation of the body elastic vibration, the radiation potential must have the same form as the vibration displacement of the cylinder along the y direction. Thus, the radiation potential can be written as

$$\phi_j(x, y, z) = -i\omega \zeta_j \phi_j(x, z) e^{ik_y y}, \quad j = 1, 2, 3 \quad (5)$$

where $\phi_j(x, z)$ is a 2D velocity potential under unit amplitude motion. Taking $\phi_j(x, y, z)$ into the 3D Laplace equation, we can obtain the following governing equation for the 2D velocity potential.

$$\frac{\partial^2 \phi_j(x, z)}{\partial x^2} + \frac{\partial^2 \phi_j(x, z)}{\partial z^2} = k_y^2 \phi_j(x, z), \quad j = 1, 2, 3 \quad (6)$$

The mean body surface, still free surface, and seabed boundary conditions for the 2D potential are as follows:

$$\frac{\partial \phi_j(x, z)}{\partial n} = n_j, \quad j = 1, 2, 3 \text{ at } S_B \quad (7)$$

$$\frac{\partial \phi_j(x, z)}{\partial z} = \frac{\omega^2}{g} \phi_j(x, z), \quad j = 1, 2, 3 \text{ at } z = 0 \quad (8)$$

$$\frac{\partial \phi_j(x, z)}{\partial z} = 0, \quad j = 1, 2, 3 \text{ at } z = -d \quad (9)$$

For this generalized radiation problem, we need to solve the 2D radiation velocity potential $\phi_j(x, z)$. The 3D radiation velocity potential $\phi_j(x, y, z)$ can be obtained by substituting $\phi_j(x, z)$ into Eq. (5). In the next section, we will present how to solve the 2D radiation velocity potential $\phi_j(x, z)$.

3 BEM and eigenfunction expansion

In solving the boundary value problem, the whole computational domain is divided into the inner domain (D^0) and outer domains (D^-, D^+) as shown in Figure 2. The velocity potential in each subdomain is solved using different approaches.

In the inner domain surrounding the cylinder, a simple Green's function is applied to form a boundary integral function, which is then solved using BEM. The fundamen-

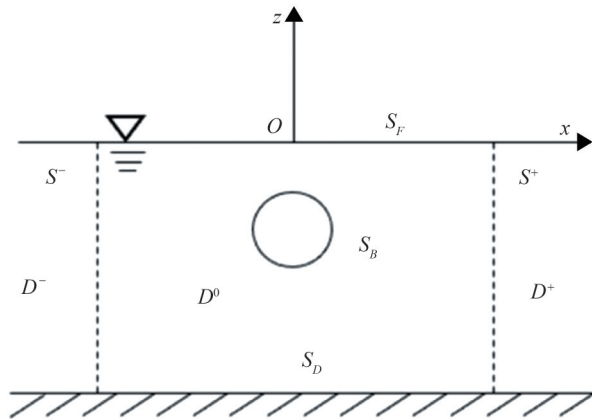


Figure 2 Division of the computational domain

tal solution of the governing equation Eq. (6) can be written as

$$G_0(\mathbf{x}; \mathbf{x}_0) = \begin{cases} \frac{1}{2\pi} \ln(r), & k_y = 0 \\ -\frac{1}{2\pi} K_0(k_y r), & |k_y| > 0 \end{cases} \quad (10)$$

where $\mathbf{x}_0 = (x_0, z_0)$ and $\mathbf{x} = (x, z)$ are the position vectors of a source point and a field point, respectively. $r = \sqrt{(x - x_0)^2 + (z - z_0)^2}$ is the distance between the field and source points. $K_0(\cdot)$ is the modified Bessel function of the second type of zero order.

In accordance with the principle of mirror image, we can obtain the simple Green's function satisfying the seabed boundary conditions as

$$G_1(\mathbf{x}; \mathbf{x}_0) = \begin{cases} \frac{1}{2\pi} [\ln(r) + \ln(r_1)], & k_y = 0 \\ -\frac{1}{2\pi} [K_0(k_y r) + K_0(k_y r_1)], & |k_y| > 0 \end{cases} \quad (11)$$

where $r_1 = \sqrt{(x - x_0)^2 + (z + z_0 + 2d)^2}$. Using this simple Green's function, we can exclude the bottom surface by employing the corresponding image source to reduce computational demand.

In the outer domains away from the cylinder, the velocity potential can be represented by eigenfunctions as

$$\varphi_j(x, z) = \begin{cases} \sum_{l=0}^{\infty} R_{j,l}^- \psi_l^-(x, z), & x < X^- \\ \sum_{l=0}^{\infty} R_{j,l}^+ \psi_l^+(x, z), & x > X^+ \end{cases} \quad (12)$$

where $X^\pm$ are the x coordinates of the interfaces between the outer and inner domains, $R_{j,l}^\pm (l = 0, 1, 2, \dots)$ are the expanding coefficients to be determined, and $\psi_l^\pm(x, z)$ are

the eigenfunctions denoted as

$$\psi_l^\mp(x, z) = \frac{\cos k_l(z + d)}{\cos k_l d} e^{\pm k_{lx} x}, \quad l = 0, 1, 2, \dots \quad (13)$$

By substituting the velocity potential expansion into the free-surface boundary conditions, we can obtain the dispersive equation as $\omega^2 = -gk_l \tan k_l d$, where $k_l (l = 1, 2, 3, \dots)$ and k_0 are the real and negative imaginary roots of the dispersion equation, respectively. The eigenvalue k_{lx} can be determined by $k_{lx} = \sqrt{k_l^2 + k_y^2} (l = 0, 1, 2, \dots)$ from the governing equation (Eq. 6). When $|k_0| > |k_y|$, $k_{0x} = -i \sqrt{|k_0|^2 - k_y^2}$ is imaginary, the corresponding mode is a propagation wave. When $|k_0| \leq |k_y|$, $k_{0x} = \sqrt{k_y^2 - |k_0|^2}$ is real, the corresponding mode is an evanescent wave. The wave number k_0 corresponding to $k_{0x} = 0$ is called the “critical wave number” hereinafter. The frequency corresponding to the “critical wave number” by satisfying the dispersion equation is called the “critical frequency”.

The following integral equation can be obtained by substituting the radiation potential $\varphi_j(x, z) (j = 1, 2, 3)$ and the Green's function into the Green's second identity:

$$\begin{aligned} \alpha \varphi_j(\mathbf{x}_0) - \int_{S_B + S_F} \frac{\partial G_1(\mathbf{x}; \mathbf{x}_0)}{\partial n} \varphi_j(\mathbf{x}) ds + \\ \frac{\omega^2}{g} \int_{S_F} G_1(\mathbf{x}; \mathbf{x}_0) \varphi_j(\mathbf{x}) ds - \\ \sum_{l=0}^{L^*} \int_{S^*} R_{j,l}^\mp \psi_l^\mp(\mathbf{x}) F_l(\mathbf{x}, \mathbf{x}_0) ds = \\ - \int_{S_B} G_1(\mathbf{x}; \mathbf{x}_0) n_j ds, \quad j = 1, 2, 3 \end{aligned} \quad (14)$$

where α is the free term, L^* are the numbers of terms applied for the eigenfunction expansions in the left and right outer domains, and $F_l(\mathbf{x}, \mathbf{x}_0)$ is determined as

$$F_l(\mathbf{x}, \mathbf{x}_0) = \partial G_1(\mathbf{x}, \mathbf{x}_0) / \partial n + k_{lx} G_1(\mathbf{x}, \mathbf{x}_0), \quad l = 0, 1, 2, \dots \quad (15)$$

With the discretization of the second-order elements, a higher-order boundary element method (HOBEM) is implemented on the above integral equation. The velocity potential $\varphi_j(x, z)$ and expanding coefficients $R_{j,l}^\mp$ can be determined by solving the algebraic equations.

The hydrodynamic coefficient f_{ij} , which is the wave force in the i -th direction due to the unit amplitude body vibration in the j -th direction, can be computed as

$$f_{ij} = \omega^2 a_{ij} + i \omega b_{ij} = \rho \omega^2 \int_{S_B} \varphi_j(x, z) n_i ds, \quad i, j = 1, 2, 3 \quad (16)$$

where ρ is the water density, a_j is the added mass, and b_j is the radiation damping.

4 Wave profile and wave propagation characteristics

Teng et al. (2023) found that the generalized hydrodynamic coefficients in the vicinity of the critical vibration frequency experience abrupt changes characterized by a “sharp peak” amplitude. The fluid in the vicinity of the cylinder oscillates strongly when the vibration frequency of the cylinder is close to the critical frequency. In this section, we will examine the wave characteristics in the vicinity of critical vibration frequency, including the variation of wave profile and wave propagation characteristics.

The 2D time-independent complex wave profiles at the first order of wave steepness due to cylinder vibration in the j -th direction can be computed using the radiation velocity potential at the still water surface as

$$\eta(x) = \frac{\omega^2}{g} \varphi_j(x, z), \text{ at } z = 0, \quad j = 1, 2, 3 \quad (17)$$

In this computation, the whole fluid domain is divided into three subdomains: left, middle, and right, as shown in Figure 2. In each domain, different methods are used to compute the wave potentials. In the left and right domains, the wave potentials are computed by the eigenfunction expansions. In the middle domain, the wave potential is computed by the interpolation of the node's potentials with the shape functions of boundary elements. In addition, the 3D wave profile at time t can be described as

$$\zeta(x, y, t) = \text{Re}[\eta(x) e^{ik_y y} e^{-i\omega t}] \quad (18)$$

Given that the fluid is considered inviscid, the rotation vibration of a circular cylinder about its axis will not generate any waves. Thus, we only need to consider the cases due to the elastic vibration of the cylinder in the x and z directions, i.e., the horizontal and vertical directions.

First, we consider a cylinder with a submergence depth of $T/a = 3.0$ in a water depth of $d/a = 6.0$. Interfaces $S^\mp$ are arranged at a distance 12 times the cylinder radius away from the cylinder center. A total of 350 discretized elements are involved in this case, including free surface, body surface, left and right interface elements, and 12 expansion terms in the outer domains. The cylinder experiences an elastic vibration with a wave number $k_y a = 0.200$ in the y direction. According to the analysis in section 3, the critical wave number $k_{0, \text{crit}} a = 0.200$ corresponds to $k_{0x} = 0$. We consider dimensionless wave numbers $k_0 a = 0.190, 0.193, 0.196, 0.199$ and $k_0 a = 0.201, 0.204, 0.207, 0.210$, which are below and beyond the critical wave num-

ber, respectively, as shown in Figures 3 and 4.

Figure 3 shows the distributions of the real and imaginary parts of the complex wave profiles formed due to the horizontal vibration of the submerged cylinder against the dimensionless horizontal coordinate x/d , where $A = |\xi_j|$ is the amplitude of body vibration in the j -th direction and x is measured from the center of the cylinder. When the vibration frequency is lower than the critical frequency $k_{0, \text{crit}} a$ (Figures 3(a) and (b)), the real parts of the complex wave profiles are antisymmetric about the centerline of the cylinder. However, the imaginary parts are all zero. In addition, the real part of each wave profile curve peaks near the cylinder and then decays to zero with the increasing distance from the cylinder, obeying the relationship of $e^{-|k_0 x|}$. When the vibration frequency $k_0 a$ is close to the critical frequency, the real part of the wave profiles has a relatively high peak. At far distances from the cylinder, no radiation wave is observed, which explains why the generalized radiation damping is zero when the vibration frequency is less than the critical frequency. When the vibration frequency is larger than the critical frequency (Figures 3(c) and (d)), all the real and imaginary parts of the wave profiles are antisymmetric about the origin of the x coordinate. At a large distance from the cylinder, the curves oscillate harmonically.

Figure 4 shows the distribution of the real and imaginary parts of the complex wave profiles formed due to the vertical vibration of the submerged cylinder against the horizontal coordinate x/d , where $A = |\xi_j|$ is the amplitude of body vibration in the j -th direction and x is measured from the center of the cylinder. All the curves are symmetric about the origin of the x -coordinate. When the vibration frequency is less than the critical frequency $k_{0, \text{crit}} a$ (Figures 4(a) and (b)), the imaginary parts are all zero. Meanwhile, the real parts have their maximum absolute values at $x = 0$ and then approach zero with the increasing distance from the cylinder. The real part of the wave profiles has a relatively high peak when the vibration frequency $k_0 a$ is close to the critical frequency. At far distances from the cylinder, no radiation wave can be generated, which explains why the generalized radiation damping is zero when the vibration frequency is less than the critical frequency. When the vibration frequency is larger than the critical frequency (Figures 4(c) and (d)), the real and imaginary parts of the wave profile curves oscillate harmonically with x/d .

Second, we investigate the influence of the cylinder's dimensionless submergence depth $T/a = 1.5, 2.0, 2.5, 3.0$ on wave profiles and wave propagation characteristics in the vicinity of the critical vibration frequency $k_{0, \text{crit}} a = 0.200$ (Figures 5 (a–d) and 6 (a–d)). From these figures, we can verify the findings in Figures 3 and 4 because the propagation properties of the wave profiles are the same. Below

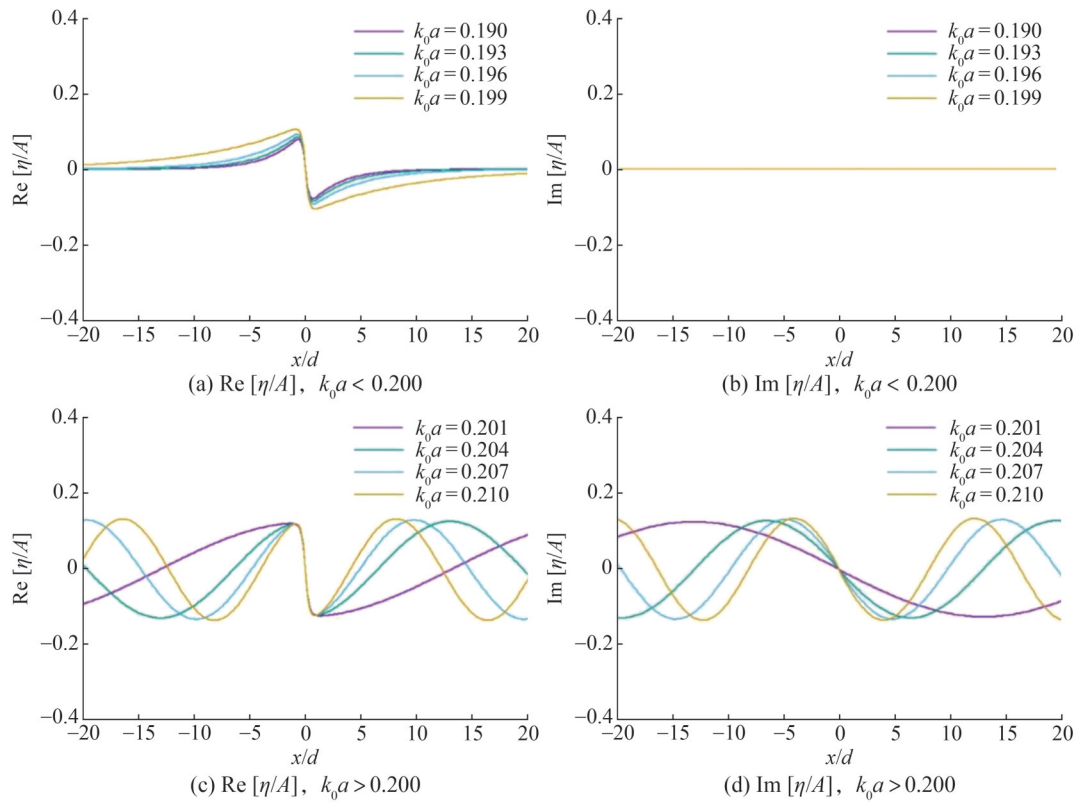


Figure 3 Real and imaginary parts of the complex wave profiles formed due to the horizontal vibration of the submerged cylinder with a submergence depth of $T/a = 3.0$ in the range below and beyond the critical frequency

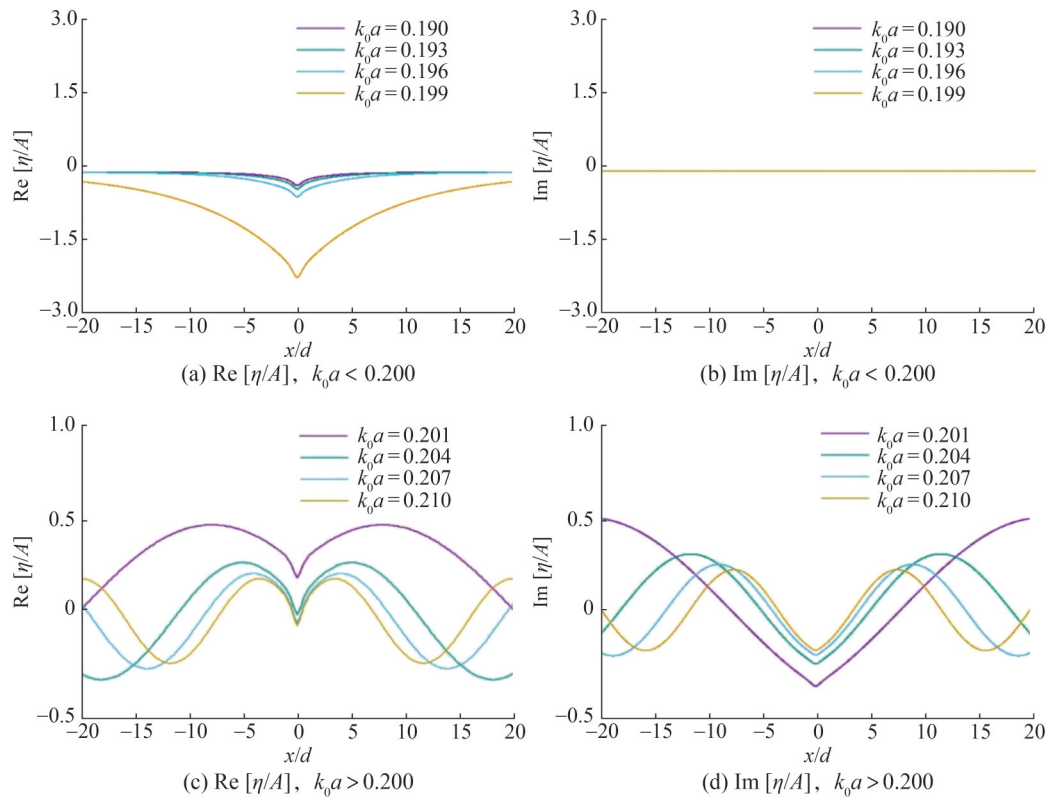


Figure 4 Real and imaginary parts of the complex wave profiles formed due to the vertical vibration of the submerged cylinder with a submergence depth of $T/a = 3.0$ in the range below and beyond the critical frequency

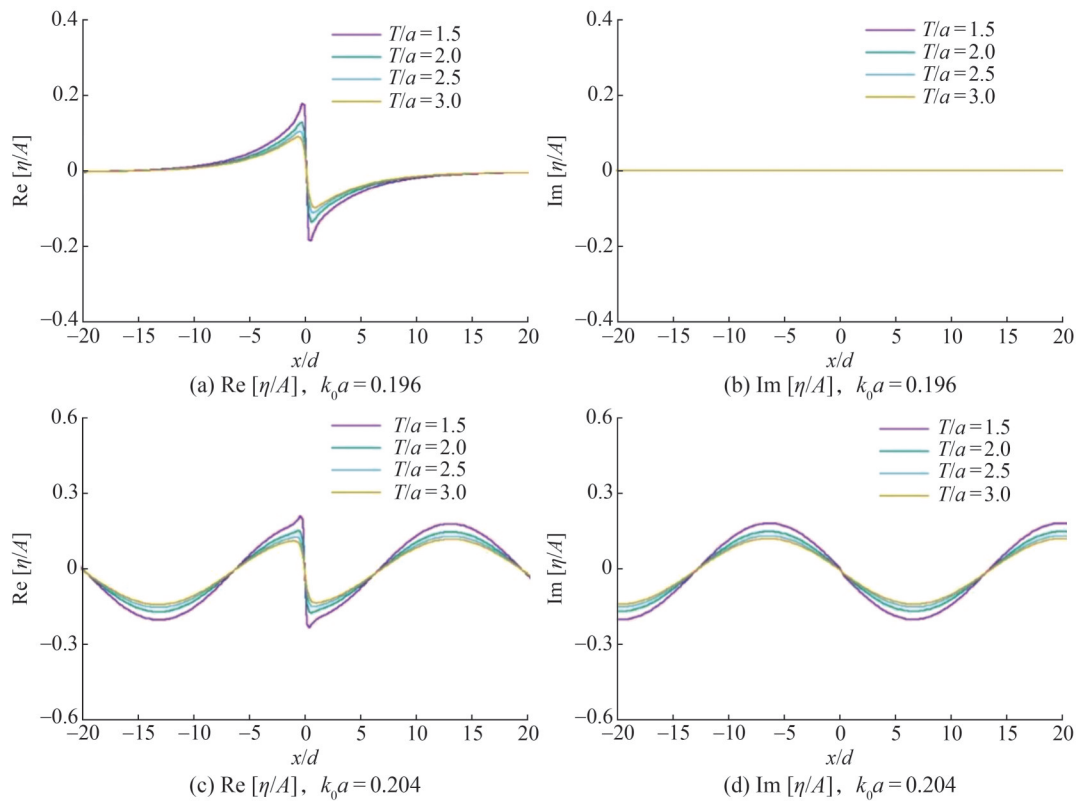


Figure 5 Variations with the submergence depths of the real and imaginary parts of the complex wave profiles formed due to the horizontal vibration of the submerged cylinder at the frequency of $k_0 a = 0.196$ and $k_0 a = 0.204$

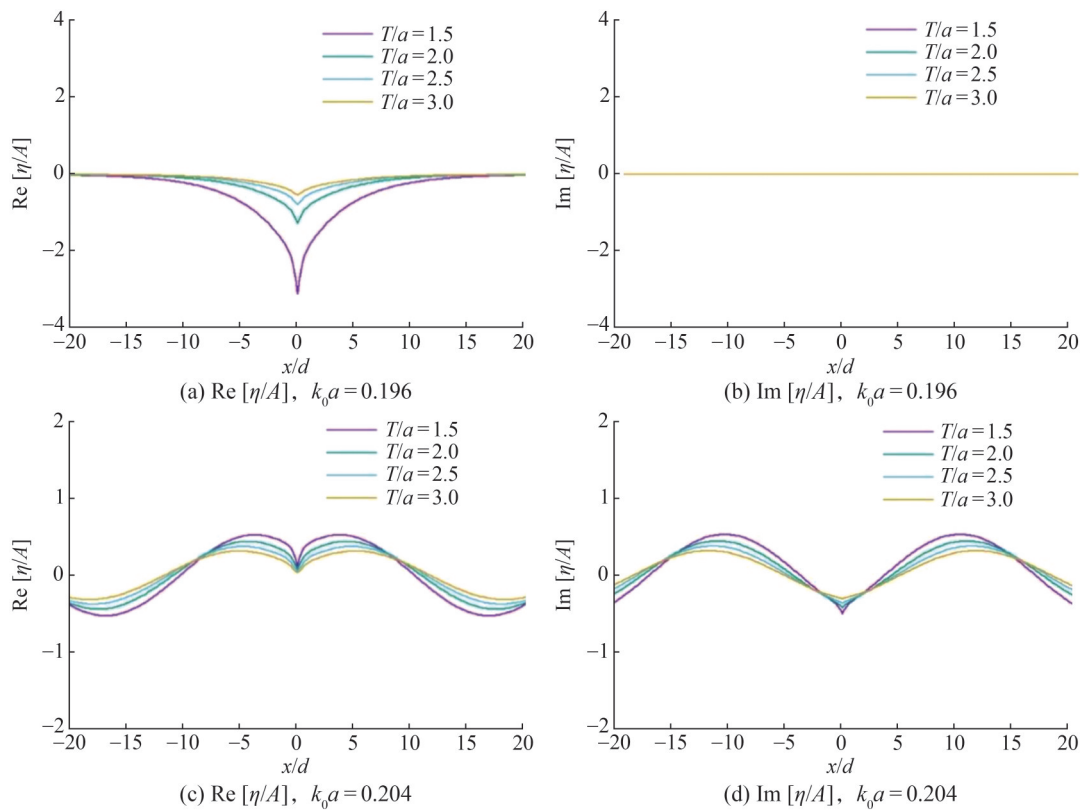


Figure 6 Variations with the submergence depths of the real and imaginary parts of the complex wave profiles formed due to the vertical vibration of the submerged cylinder at the frequency of $k_0 a = 0.196$ and $k_0 a = 0.204$

the critical frequency, the real parts of the complex wave profiles have large values in the vicinity of the cylinder and decay to zero with the increasing distance from the cylinder. However, the imaginary parts of the wave profiles are all zero, which explains why the generalized radiation damping is zero when the vibration frequency is less than the critical frequency. When the vibration exceeds the critical frequency, the real and imaginary parts of the wave profiles oscillate harmonically and keep steady amplitudes. As shown in the figures, the submergence depth of the cylinder has a significant impact on the wave profile amplitude, which decreases with the increasing submergence depth.

Finally, the 3D wave profiles formed due to the horizontal and vertical vibration of the submerged cylinder at time $t = 0$ are graphically presented in Figures 7 and 8 to provide a clear visualization of the wave profile and wave propagation characteristics. Figure 7 shows the 3D wave profiles formed due to the horizontal vibration of the submerged cylinder with a submergence depth of $T/a = 3$ at $k_0 a = 0.196, 0.204$ in the vicinity of the critical vibration frequency $k_{0, \text{crit}} a = 0.200$, where $A = |\xi_j|$ is the amplitude of body vibration in the j -th direction. The wave profiles

propagate in the y direction with an elastic vibration wave number of k_y , and those at the left- and the right-hand sides of the cylinder have a phase difference of π . Below and above the critical frequency, the wave profiles in the x direction completely differ. At the frequency $k_0 a = 0.196$ below the critical frequency, the wave profile amplitude decays with the increasing distance from the cylinder. At the frequency $k_0 a = 0.204$ above the critical frequency, the wave profile amplitude keeps steady with the increasing distance from the cylinder, and the generated radiation waves spread obliquely outward.

Figure 8 shows the 3D wave profiles formed due to the vertical vibration of the submerged cylinder with a submergence depth of $T/a = 3$ at $k_0 a = 0.196, 0.204$ in the vicinity of the critical vibration frequency $k_{0, \text{crit}} a = 0.200$, where $A = |\xi_j|$ is the amplitude of body vibration in the j -th direction. The wave profiles at the left- and right-hand sides of the cylinder propagate with the same phase. In the x direction, the wave profile amplitude decays with the increasing distance from the cylinder at $k_0 a = 0.196$ below the critical frequency $k_{0, \text{crit}} a$. By contrast, it keeps steady at $k_0 a = 0.204$ above the critical frequency, and the generated radiation waves spread obliquely outward.

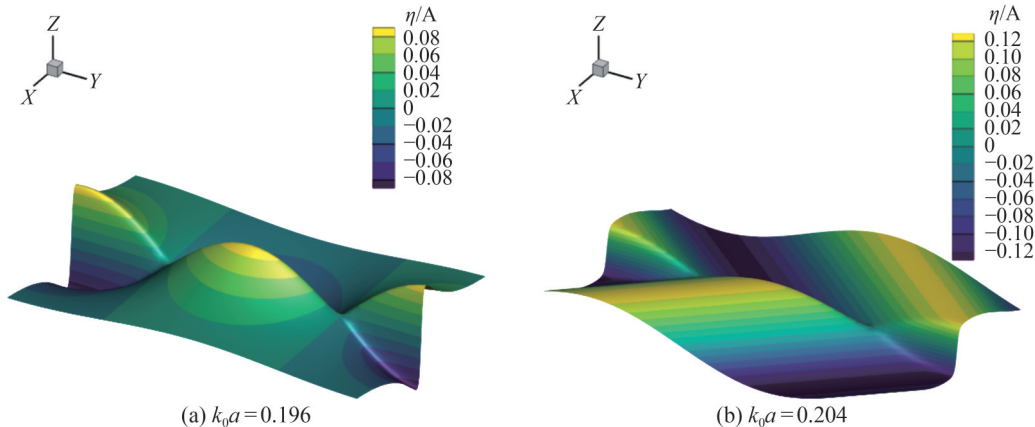


Figure 7 3D wave profiles at time $t = 0$ formed due to the horizontal vibration of a submerged cylinder with a submergence depth of $T/a = 3$ in the vicinity of the critical vibration frequency $k_{0, \text{crit}} a = 0.200$

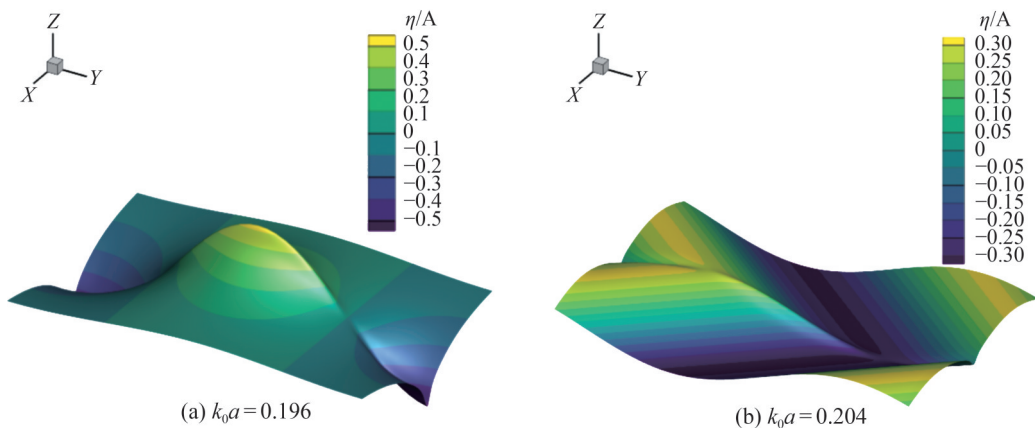


Figure 8 3D wave profiles at time $t = 0$ formed due to the vertical vibration of a submerged cylinder with a submergence depth of $T/a = 3$ in the vicinity of critical vibration frequency $k_{0, \text{crit}} a = 0.200$

5 Conclusions

On the basis of potential theory, an investigation is performed on the generalized radiation problem in which a cylinder is forced to vibrate periodically in the horizontal and vertical directions in the cross-sectional plane and experiences a number of elastic vibration waves in its longitudinal direction. The whole computation is divided into an inner domain and outer domains. The radiation velocity potential in the inner domain is calculated by a BEM with simple Green's function, and that in the outer domain away from the cylinder is calculated through eigenfunction expansion. We examine the wave profiles and propagation characteristics in the vicinity of the critical vibration frequency. The influence of varying submergence depths on the wave profiles and propagation characteristics is also analyzed. The 3D wave profiles are graphically presented to show the wave propagation characteristics in the vicinity of the critical frequency for this generalized radiation problem. This study provides a good reference for the interaction between fluid and slender elastic structures. The following conclusions are drawn:

1) When the cylinder vibration frequency is smaller than the critical frequency, the real parts of the complex wave profiles have large values in the vicinity of the cylinder and decay to zero with the increasing distance from the cylinder. When the vibration frequency $k_0 a$ is close to the critical frequency, the real parts of the wave profiles have a high peak and the imaginary parts are all zero. Thus, no propagating wave and radiation damping occur.

2) When the vibration exceeds the critical frequency, the real and imaginary parts of the wave profiles oscillate with the distance from the cylinder, and the generated radiation waves propagate obliquely outward. With the increasing distance from the cylinder, the amplitude of the oscillation reaches a steady value. When the vibration frequency $k_0 a$ is close to the critical frequency, the amplitude of the oscillation has a high peak.

3) The submergence depth of the cylinder significantly influences the amplitude of the wave profile. As the submergence depth of the cylinder increases, the amplitude of the wave profile decreases.

4) Wave profiles differ for the horizontal and vertical vibrations of the submerged cylinder and are antisymmetric and symmetric about the centerline of the cylinder, respectively. In addition, the wave profiles at the left- and right-hand sides of the cylinder are out of phase during horizontal vibration and are in phase during vertical vibration.

This work still has some limitations, opening up avenues for further exploration and enhancement. One key aspect for future investigation involves the nonlinearity of cylinder vibration, where the relative vibration amplitude of the cylinder is not smaller than 1. Rather than ideal fluid, viscous fluid should be considered, and the flow should be

rotational with viscous separation. Exploring these aspects could contribute to a comprehensive understanding of the generalized radiation problem and pave the way for the investigation of SFT response in hydroelasticity.

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