

Hydrodynamic Characteristics of an Underwater Manipulator in Pulsating Flow

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Abstract

Pulsating flow is a common condition for underwater manipulators in Bohai Bay. This study aimed to investigate the effects of pulsation frequency and amplitude on the hydrodynamic characteristics of an underwater manipulator with different postures using the user-defined function (UDF) method. The lift coefficient (C_L), drag coefficient (C_D), and vortex shedding of the underwater manipulator in single- and dual-arm forms were obtained. Results indicated that the maximum increase in the lift and drag coefficients subjected to the pulsation parameters was 24.45% and 28%, respectively, when the fluid flowed past a single arm. Compared with the single arm, the lift and drag coefficients of the arms were higher than those of the single arm when arm 2 was located upstream. Additionally, the pulsation frequency had no obvious effect on the manipulator, but the C_L and C_D of arm 2 showed an obvious increasing trend with an increase in pulsation amplitude. Meanwhile, when arm 2 was located downstream, the C_L and C_D of arm 2 were reduced by 16.38% and 1.15%, respectively, with an increase in the pulse frequency, and the maximum increase in the lift and drag coefficients was 33.33% and 16.78%, respectively, with increasing pulsation amplitude. Moreover, the downstream wake morphology changed significantly, and a combined vortex phenomenon appeared. Finally, a theoretical basis for examining the hydrodynamic characteristics of marine engineering equipment was established to aid future marine resource exploitation.

Keywords Underwater manipulator; Pulsating flow; Hydrodynamic performance; Vortex shedding; Flow interference

1 Introduction

The problem of flow around bluff bodies is of theoretical importance and has a wide range of practical applications, such as flow around underwater manipulators, offshore platform support legs, and marine risers (He et al., 2022; Zhang et al., 2013; Willden and Graham, 2004). When the flow is around the bluff body, boundary layer separation, vortex shedding, and other phenomena occur. The alternating shedding of fluid through the structure leads to lateral flow and large pressure pulsations in a straight direction. The action of these forces causes vibrations, noise, and res-

onance of the structure and even fatigue damage. Therefore, the hydrodynamic characteristics of related marine equipment must be analyzed to ensure their stability and accuracy. To solve related practical problems, for a long time, people have conducted extensive research on the theory, experiment, and numerical simulation of the flow around bluff bodies.

In practical applications, to facilitate simulation analysis, scholars have simplified cylindrical marine engineering equipment into the problem of flow around a cylinder. The hydrodynamic characteristics of a single column with different section shapes have been studied with good results. Molochnikov et al. (2019) found that the shape of the velocity profile near the cylinder was governed not by the distance from the cylinder but by the location of the station with respect to the streamwise size of the separation region. Jiang and Cheng (2021) discussed the wake recirculation length and the hydrodynamic forces on the cylinder. An inverse relationship between the root mean square lift coefficient and wake recirculation length was confirmed. Kološ et al. (2021) compared numerical solutions with the available experimental and standard data. The analysis of average flow velocity showed that the wake center lengthens with a decrease in Reynolds number (Re). In addition to the typical circular section, the working section of some marine engineering equipment was also mostly elliptical. Shi et al. (2020) investigated the flow around the elliptical cylinder

Article Highlights

- The effect of pulsating flow on the underwater manipulator is determined.
- The flow pattern varies as the underwater manipulator postures are modulated.
- As the pulsating flow parameter increases, the hydrodynamic coefficients increase.
- The pulsation amplitude has a greater influence on the dual arm.

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with long- and short-axis ratios (axial ratio, AR) varying from 0.25 to 1.0. The time-averaged drag and fluctuating lift coefficients were small, and two steady bubbles were formed behind the cylinder when $AR < 0.37$. Cui et al. (2023) conducted a study on an elliptical cylinder with different AR values at a Reynolds number of 3 900 and analyzed the variation in the flow field with respect to AR. The drag coefficient was considerably influenced within the AR range of 0.5–1.0. Notably, when the AR of the elliptical column was 0.7, the recirculation length in the wake region was the smallest. Liu et al. (2021) and Wu et al. (2022) investigated the winding characteristics of an elliptic cylinder under different ARs. As the AR increases, the strength of the vortex in the wake decreases considerably, and the vortex shedding position from the center of the cylinder increases considerably. When $AR \geq 0.7$, the lift-to-resistance ratio was negatively correlated with the Reynolds number. The flow around two cylinders arranged in tandem and side-by-side was a typical research object in a multicylinder group. The flow around two cylinders arranged in tandem and side-by-side was a typical research object in a multicolumn group. Hu et al. (2019) found that when $L/D < 3.5$, the shear layer of the upstream cylinder re-adhered to the surface of the downstream cylinder. Moreover, the vortex street only formed behind the downstream cylinder, while the shear layer of the upstream cylinder alternately rolled up. When $L/D > 3.5$, vortex shedding was observed in the upstream and downstream cylinders. In addition, Hosseini et al. (2020) noticed that when the spacing was between 3.6 and 4.4, a full vortex shedding similar to a single-cylinder wake was formed in the gap between the two cylinders. Mentese and Bayraktar (2021) further observed that when arranged in series and with a spacing ratio of 4.0, vortex shedding continued behind the upstream cylinder, but vortices started to shed simultaneously from opposite sides of the downstream cylinder, and two rows of vortex streets were observed. The flow around the tandem cylinders highly depends not only on the spacing ratio between the centers of the cylinders but also on the cross-sectional shape of the cylinder. Cheng et al. (2022) investigated the effects of the cross-sectional shape and spacing ratio on hydrodynamic coefficients and flow characteristics in different flow fields over the subcritical Reynolds number range. The results showed that the hydrodynamic coefficients of the downstream elliptical cross-section cylinder increased with increasing spacing.

Although there is a wealth of research on flow through a stationary cylinder, most previous studies have focused on uniform flow. However, because of the presence of ocean currents and waves, the fluid was generally an oscillating flow rather than a uniform incoming flow. Guilmineau and Queutey (2004) conducted detailed studies on the trailing vortex shedding of a vibrating cylinder in an oscillating flow. With an increase in the vibration frequency of the

cylinder, the position of vortex generation would be closer to the cylinder until it reached a certain limit. Mikheev et al. (2017) found that under the condition of a pulsating flow through the experiment, there were four forms of vortex shedding around the cylinder. Kim et al. (2006) and Mud-dada et al. (2021) analyzed the dynamic behavior and evolution stage of vortex shedding in oscillatory flow. The results showed that the amplitude continually changed, and the size of the vortex behind the column varied. For higher amplitude oscillations, the length of the wake was shorter. After vortex locking, the average recirculation area and vortex formation area decreased obviously.

In addition, Konstantinidis and Bouris (2016) considered the influence of harmonic and nonharmonic perturbation waveforms on inflow velocity. A phase diagram of the lift force was used to identify the dynamic state of the wake of the cylinder and determine the range of motion parameters at which synchronization occurred. That is, the vortex formation is locked with the subharmonic phase of the microdisturbance frequency. The bubble phenomenon in pulsating flow also severely affects the hydrodynamic characteristics of the underwater manipulator.

The underwater manipulator, a standard cylindrical piece of marine engineering equipment, features a multiarm arrangement in addition to a constantly changing cross-section. Exploring the hydrodynamic performance of an underwater manipulator ensures its safety and efficiency and promotes the accurate positioning of an underwater manipulator. In addition, it provides a reference for research on hydrodynamic performance in the marine industry. Furthermore, various environmental loads, such as waves and eddies in the environment, cause the incoming flow to have a certain pulsation intensity. These uncertainties have a nonnegligible effect on the hydrodynamic characteristics of an underwater manipulator. Given the hydrological environment of the Shandong Peninsula, China, the complex flow field of the marine environment was simplified into a pulsating flow field. In this study, a two-dimensional numerical model was established using the unsteady Reynolds-averaged Navier–Stokes (URANS) equation and the shear stress transfer (SST) $k-\omega$ turbulence model. The winding characteristics of the cylinder in pulsating flow, such as the flow force and vortex shedding characteristics, were investigated numerically using Fluent software and the user-defined function (UDF). In this way, a basis was provided for maintaining the best performance of the underwater manipulators and related marine engineering equipment under working conditions.

2 Numerical approach

2.1 Mathematical model

When the dual-arm underwater manipulator was in a

working state, the upper arm (arm 1) was always in the vertical direction, while the lower arm (arm 2) rotated at multiple angles. Among them, the rotation of the lower arm in the clockwise direction was considered a positive direction. As shown in Figure 1, to facilitate simulation analysis, the underwater manipulator was simplified into a cylinder. Thus, a cross-section was taken from the midpoint of arm 2 to analyze the influence of changes in the incoming flow parameters on the hydrodynamic performance of the cross-section. In addition, in its flow field, the upstream arm was Cir 1, and the downstream arm was Cir 2.

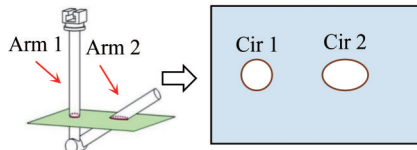


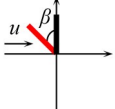
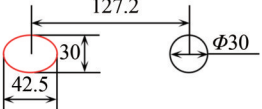
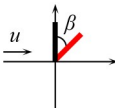
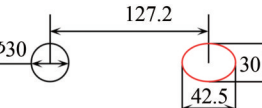
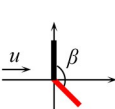
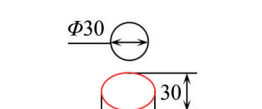
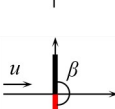
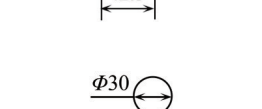
Figure 1 Flow field of the dual-arm underwater manipulator

To further analyze the impacts of pulsation parameter modifications on the hydrodynamic characteristics of the underwater manipulator, four representative poses were chosen, as given in Table 1. Among these poses, the working postures P1, P2, P3, and P4 correspond to rotation angles of arm 2 of -45° , 45° , 135° , and 180° , respectively. When the underwater manipulator was located at P1 and P2, there was an interaction between arms 1 and 2, and its flow field could be approximated as the flow field around a series of double columns. In P1, the cross-flow section of the upstream manipulator was elliptical, and its axial ratio AR was approximately 0.7. The cross-flow of the downstream manipulator was circular. However, the cross-flow section of the upstream and downstream manipulators in P2 was opposite to that in P1. When the underwater manipulator was located in P3 and P4, there was no interference between manipulators 1 and 2, and the flow field could be approximated as that around a cylinder.

2.2 Control equations

In this study, considering the effect of turbulent pulsation, Navier–Stokes equations were adopted to determine the transient pulsation quantities in the time-homogenized equations using some type of model. In addition, the two-dimensional unsteady Reynolds-averaged Navier–Stokes (URANS) method has the advantages of a low spatial resolution requirement, a small computational effort, and wide application (Shukla et al., 2021). Therefore, the two-dimensional URANS method was employed to solve the flow field, which was assumed to be incompressible, and the viscosity and density of motion were constant. The Reynolds mean equation for the conservation of mass and momentum is given as follows (Kim et al., 2006):

Table 1 Calculation model of the manipulator under different rotation angles

Postures	Mathematical model	Rotation angle β of arm 2	Computational model
P1		-45°	
P2		45°	
P3		135°	
P4		180°	

$$\frac{\partial u_i}{\partial x_i} = 0 \quad (1)$$

$$\rho \frac{\partial u_i}{\partial t} + \rho \frac{\partial (u_i u_j)}{\partial x_j} = - \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} (2\mu S_{ij} - \overline{\rho u'_i u'_j}) \quad (2)$$

where ρ , μ , u , p , and u' represent the fluid density, molecular viscosity, average velocity, average static pressure, and fluctuating velocity, respectively. u_i and u_j are the velocity components in the x and y directions, respectively. x_i and x_j are the displacement components in the x and y directions, respectively.

The S_{ij} term is the mean strain rate tensor, and $\overline{\rho u'_i u'_j}$ is the Reynolds stress, indicating that the pressure was generated by turbulence. The superscripts denote shorthand for mean values.

Using Boussinesq's hypothesis (Tennekes and Lumley, 1972), the Reynolds stress under an incompressible flow can be expressed as:

$$\overline{\rho u'_i u'_j} = 2\mu_t S_{ij} - \frac{2}{3} \delta_{ij} k \quad (3)$$

where k is turbulent kinetic energy, μ_t is turbulent viscosity, and δ_{ij} is the Leopold–Kronecker trigonometric function.

The SST $k-\omega$ turbulence model is developed on the basis of the standard $k-\omega$ turbulence model. The model combines the reliability of the standard $k-\omega$ model for viscous flow in the near-wall region with the accuracy of the $k-\omega$ model for the far-field region where turbulence is fully developed (Qu et al., 2021).

The SST $k-\omega$ turbulence model includes the transport equation of turbulent kinetic energy k and dissipation rate ω (Yagmur et al., 2020). The transport equations for the turbulent kinetic energy k and ω are as follows:

$$\frac{\partial(\rho k)}{\partial t} + \frac{\partial(\rho k u_i)}{\partial x_i} = \frac{\partial}{\partial x_j} \left[\Gamma_k \frac{\partial k}{\partial x_j} \right] + G_k - Y_k \quad (4)$$

$$\frac{\partial(\rho \omega)}{\partial t} + \frac{\partial(\rho \omega u_i)}{\partial x_i} = \frac{\partial}{\partial x_j} \left[\Gamma_\omega \frac{\partial \omega}{\partial x_j} \right] + G_\omega - Y_\omega + D_\omega \quad (5)$$

G'_k represents the item that produced k :

$$G'_k = \min(G_k, 10\rho\beta^*k\omega) \quad G_k = -\rho \overline{u'_i u'_j} \frac{\partial u_j}{\partial x_i} \quad (6)$$

G_ω is the generating term of ω :

$$G_\omega = G'_k \frac{\alpha}{\nu_t} \quad (7)$$

Y_k and Y_ω are the dissipation term of k and ω , respectively:

$$Y_k = \rho\beta^*k\omega \quad Y_\omega = \rho\beta\omega^2 \quad (8)$$

D_ω represents the cross-diffusion term:

$$D_\omega = 2(1 - F_1)\rho \frac{1}{\omega\sigma_{\omega,2}} \frac{\partial k}{\partial x_j} \frac{\partial \omega}{\partial x_j} \quad (9)$$

2.3 Boundary condition setting

A reasonable computational domain was beneficial to the accuracy and speed of the calculation. The physical model of the computational domain is shown in Figure 2. The size of the entire computational domain was $(34D+L) \times 20D$ (D is the diameter of the cylinder). The cylinder was located at the center of the vertical plane. The upper and lower boundaries were located $10D$ from the center of the cylinder to ensure that they did not affect the flow around the cylinder. The inlet was located $10D$ upstream of the center of Cir 1, and the outlet was located $24D$ downstream of the center of Cir 2 to eliminate the influence of the far-field upstream and downstream of the cylinder.

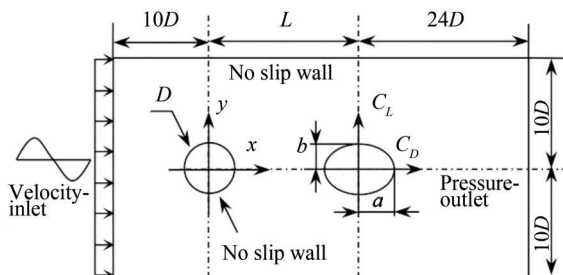


Figure 2 Fluid domain and boundary conditions

The inlet was the boundary of the velocity inlet, and the velocity of the sinusoidal pulsating flow was expressed as:

$$u(t) = u_m(1 + A \sin(2\pi ft)) \quad (10)$$

where $u(t)$ is the inlet pulsatile flow velocity, m/s; t is the flow time, s; u_m is the steady flow velocity, m/s; f is the pulsatile frequency, Hz; and A is the dimensionless pulsatile amplitude.

The outlet was the boundary of the pressure outlet, given the static pressure and appropriate reflux conditions. The inlet pulsating flow varied with changes in A and f . The values of A and f were determined based on the wave conditions in the Yellow and Bohai Seas of the Shandong Peninsula. Among them, the steady flow velocity was 0.2m/s, and the pulsating flow velocity was changed by changing A and f . Table 2 presents nine different pulsatile conditions.

Table 2 Basic parameters of pulsating flow

A	f (Hz)
0.2	0.2
0.3	0.3
0.4	0.4

2.4 Grid independence verification

As shown in Figure 3, a structured mesh was used to partition the computational domain. The mesh was locally encrypted to reduce the overall mesh size while ensuring an accurate solution in the cross-sectional region. To eliminate the influence of the number of grid cells on the calculation results, three meshes were used to verify the independence of the meshes. In addition, the verified meshes will be applied for subsequent calculations.

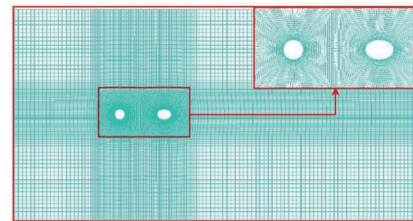


Figure 3 Meshing scheme

Table 3 provides the details of the grid independence tests performed at $Re=5900$. The results showed that there was little difference between the average drag coefficients $\overline{C_D}$ and root mean square of lift coefficient $C_{L,rms}$ under the three meshes. The maximum error in the root mean square of the lift coefficient was 0.67%, which was within acceptable limits. After comprehensive consideration, M2 was selected in the follow-up calculation.

2.5 Accuracy verification of numerical method

The validation of numerical methods plays an important

Table 3 Meshing details and convergence results

Mesh	Nodes	Elements	C_D	$C_{L,ms}$	St
M1	11 400	11 154	1.00±0.23	0.613	0.253
M2	18 564	18 880	1.00±0.26	0.617	0.255
M3	26 004	26 385	1.00±0.30	0.618	0.256

role in obtaining stable, reliable, and accurate data. To better verify the correctness of the numerical simulation method, the same computational domain and parameter settings as those of Harimi and Saghafian (2012) were adopted. That is, the overall computational domain was $30 \times 24D$, and the Reynolds number was 200. A uniform flow was applied at the inlet: $u = 1$, $v = 0$, and $\theta = 0$. The top and bottom boundaries were placed far from the cylinder surface; thus, the symmetry boundary condition was used: $u = 1$, $v = 0$, and $\theta = 0$. The manipulator in P4 was selected because the cross-section was circular. Table 4 compares the obtained lift coefficient and drag coefficient with those obtained by other researchers. The maximum error for the lift coefficient was 2.94%, and the maximum error for the drag coefficient was 1.47%. According to the results of the comparison of the lift and drag coefficients, the errors were within acceptable limits. Therefore, this numerical method can be adopted to simulate the hydrodynamic performance of the underwater manipulator.

Table 4 Verification result

Data source	Method	C_L	C_D
Present	Numerical	±0.68	1.36±0.05
Harimi and Saghafian (2012)	Numerical	±0.66	1.34±0.04
Ding et al. (2007)	Numerical	±0.66	1.35±0.05
Harichandan et al. (2010)	Numerical	±0.70	1.38±0.05
Wu et al. (2022)	Numerical	±0.69	1.30±0.04
Norberg (2003)	Experimental	±0.70	

3 Results and discussions

3.1 Effect of pulsating frequency on hydrodynamic performance

3.1.1 Flow past a single arm

The impacts of the manipulator at various pulsation frequencies on the hydrodynamic coefficients were examined for the pulsation amplitude $A = 0.2$, with the postures chosen to be A3 and A4 in Table 1. Figures 4 and 5 demonstrate, for the two postures, the time histories of the lift coefficient (C_L) and drag coefficient (C_D), respectively, evolving with pulsation frequency. Clearly, the C_L time history under the two postures exhibited a repetitive sinusoidal oscillatory trend, with the amplitude of the two adjacent oscillations jumping with time (Konstantinidis and Bouris, 2010; 2017).

$C_{L,max}$ was 0.49 under the conditions of $f = 0.2$ Hz and $\beta = 135^\circ$, where the C_L value steadily climbed in the first 10 s before exhibiting a stable periodic pattern. The time required for the C_L to stabilize was prolonged to 13.7 s when the rotation angle of arm 2 was increased to 180° . Additionally, $C_{L,max} = 1.78$ after stabilization, which was approximately 3.63 times higher than when $\beta = 135^\circ$. Similarly, the variation trend of the C_D time history curve was consistent with that of C_L , showing a periodic change law of first increasing and then decreasing. By comparison with Figures 4 and 5, it is clearly seen that the C_D of the elliptical section was considerably smaller than that of the cylindrical surface. In Figure 5(a), $C_{D,max}$ is approximately 1.88, which is almost twice that shown in Figure 4(a). This difference was caused by the variation in the pressure difference between the front and back of the cross-section. The elliptical cross-section had a more backward separation point, which led to a smaller differential pressure. Therefore, the drag force is less than that of a circular cross-section. This comparison indicated that the lift force and drag force on the underwater manipulator were stronger when the axial ratio AR was larger. Moreover, with an increase in pulsation frequency f in the range of 0.2–0.4 Hz, the maximum values of C_L and C_D also increased. When $\beta = 180^\circ$, the variations in C_L and C_D were not obvious. However, when β was reduced to 135° , the positive peak value of C_L increased from 0.49 to 0.55, and its value increased by approximately 12.25%. Meanwhile, the value of C_D increased by 6%. Notably, the variation trend of $C_{L,max}$ and $C_{D,max}$ was more pronounced when the axis ratio was smaller. This comparison indicated that the greater the pulsation frequency, the greater the pulsation of the fluid in the flow field. This trend agreed well with the research results of Cao et al. (2020).

The wavelet transform of C_L was used to further investigate the impact of pulsation frequency on the flow field. Figure 6 shows the results of the time-frequency distribution of the response frequency and signal vibration intensity over the time transient. The color of the frequency band on the right side indicates the magnitude of the wavelet coefficients that reflect the amount of energy contained in that frequency component. The darker the color, the higher the amplitude of that frequency, and the higher the energy contained. In contrast to uniform flow, multiple peaks were present in the C_L spectrum under a pulsating flow (Cheng et al., 2022). Figure 6 illustrates the band's sinusoidal variations, which correspond to frequencies distributed around 2 Hz. Each spectrum was dominated by more than one frequency component around the main wake frequency. By contrast, the single frequency in a uniform flow does not change over time (Hosseini et al., 2020). At the same pulsation frequency, the frequency band corresponding to $\beta = 135^\circ$ was considerably wider than that at $\beta = 180^\circ$. However, the maximum value of the wavelet coefficient at $\beta = 180^\circ$ was approximately six times that at $\beta = 135^\circ$ when $f =$

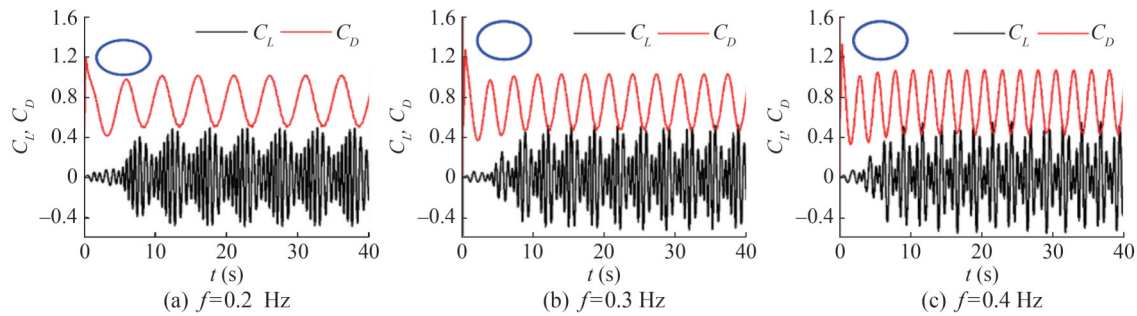


Figure 4 Time-history curve of the lift and drag coefficients when $\beta = 135^\circ$

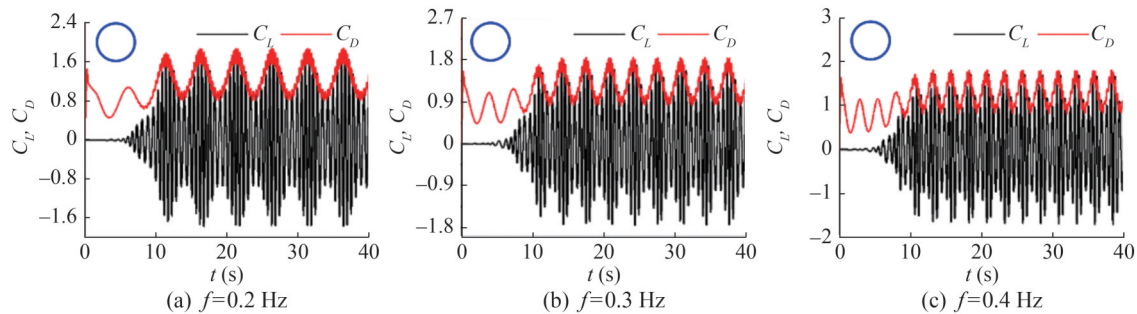


Figure 5 Time-history curve of the lift and drag coefficients when $\beta = 180^\circ$

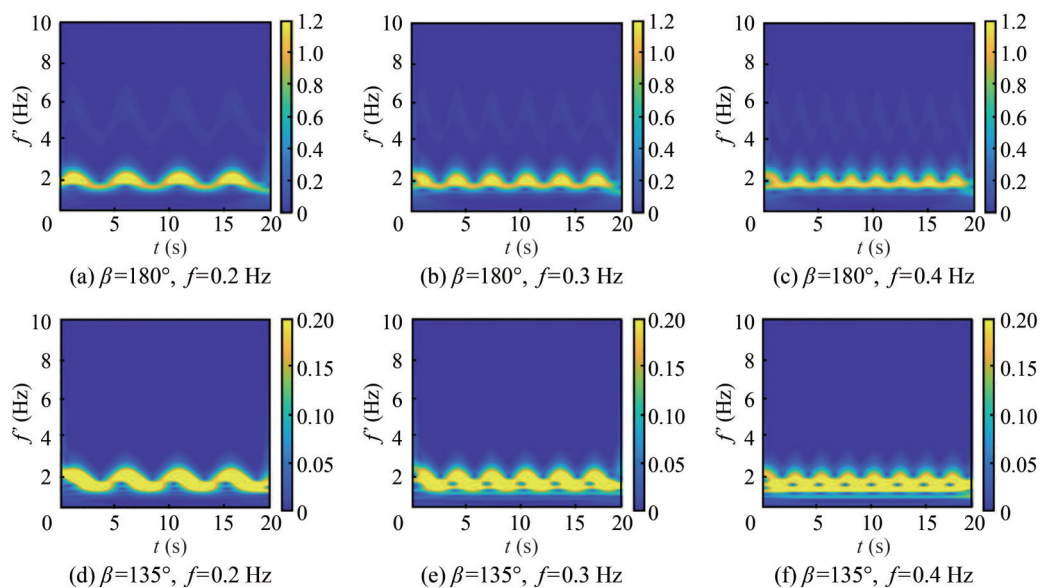


Figure 6 Lift coefficient time-frequency diagram with different pulsating frequencies

0.2 Hz. In addition, as the pulsation frequency increased from 0.2 Hz to 0.4 Hz, the frequency bands became tighter in the same period. The widths of both frequency bands were considerably shorter. This result could be attributed to the modulation of the wake frequency by the oncoming perturbations.

The vortex distribution of the single arm at different pulsation frequencies simultaneously is shown in Figure 7. Here, the red vortex is positive, the blue vortex is negative,

and h is the shedding length of the secondary vortex. When $\beta = 180^\circ$, the wake pattern was similar to that of uniform flow. The vortex street structure at the wake was regular, and the vortices were shed alternately up and down, exhibiting a typical 2S pattern. In addition, the cylinder near the vortex shedding mode, the scape wake vortex dynamics, and Jiang and Cheng (2021) around the escape of stationary flow vortex dynamics were basically identical. Nevertheless, the pulsation frequency had a considerable effect on the

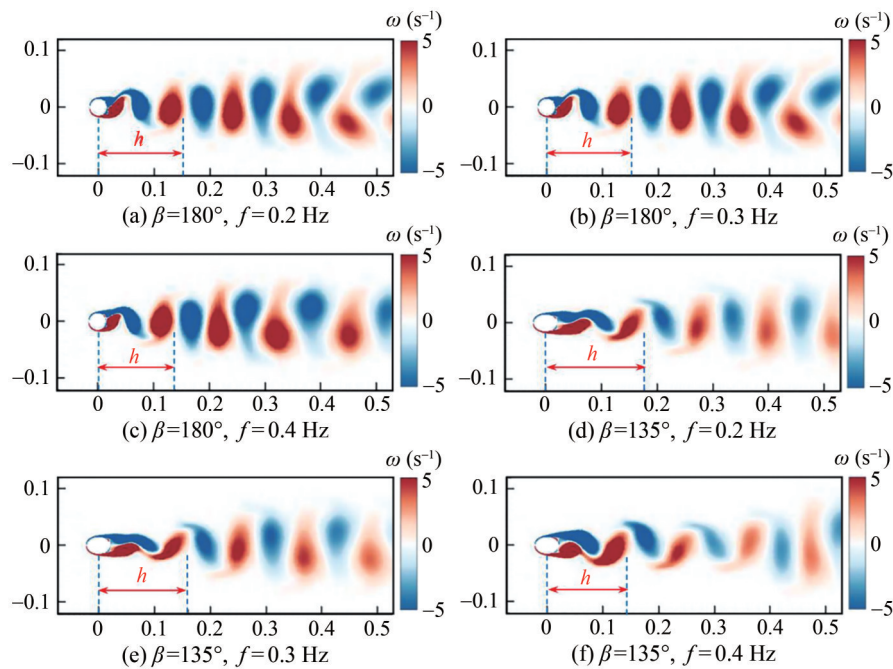


Figure 7 Vorticity cloud maps with different pulsating frequencies

arrangement of the vortex and a slight control effect on the strength of the vortex. As the pulsation frequency increased from 0.2 to 0.4 Hz, the vortex shedding length decreased from 0.148 to 0.132. Meanwhile, the shedding time accelerated. As the longitudinal distance between them decreased, the vortices became more crowded (Konstantinidis and Bouris, 2016). When $\beta = 135^\circ$, the vortex shedding was also in 2S mode. In contrast to the case with $\beta = 180^\circ$, the shedding vortex to the lamellar structure and the size and intensity of the vortices were markedly reduced. At the same pulsation frequency, the shedding length of the vortex was approximately 1.22 times that at $\beta = 180^\circ$. With an increase in pulsation frequency, the shedding velocity of the vortex increased by 25.87%. This phenomenon was consistent with the results reported by Mentese and Bayraktar (2021).

3.1.2 Flow past dual-arm underwater manipulator

When the manipulators are in operating condition, there may be interference between the arms, as in the P1 and P2 postures in Table 1. To investigate the effect of pulsation frequency on the arms, the hydrodynamic characteristics of the two postures are analyzed. Figures 8 and 9 depict the effect of pulsation frequency variation on the C_L and C_D of the manipulators in two postures, respectively. The subscripts 1 and 2 represent the upstream and downstream arms, respectively. Figure 8 clearly shows that the C_L of the downstream arm increased sharply to approximately 2.5 times that of the upstream arm when $f = 0.2$ Hz. This result was owing to the vortex generated by the upstream arm alternately hitting the surface of the downstream arm, resulting in the generation of additional pulse forces. Compared

with Figure 4, $C_{L, \max}$ of the upstream arm also increased considerably. This comparison also indicated interaction between the upper and lower arms. In contrast, the C_D of the downstream arm was much higher than that of the upstream arm and fluctuated. C_D of the upstream arm was smaller than that of the single arm. This result was obtained because the downstream arm prevented the vortex of the upstream arm from falling off and moving downstream, which decreased C_D . A comparison of Figures 8(a) and (b) shows that the time for the upstream arm to stabilize decreased continuously as the pulsation frequency increased. When $f = 0.2$ Hz, C_L tended to be stable after 24 s. However, when the pulsation frequency increased to 0.4 Hz, the time for C_L to gradually stabilize was shortened to 15 s. In addition, the period of C_L was continuously shortened from 10.1 to 5.1 s. During the same period, the number of fluctuations decreased. With an increase in pulsation frequency, the C_D of the upstream arm did not increase substantially, and the C_D of the downstream arm increased by 8.12%. This comparison implied that the increase in the inlet pulsation frequency also improved the pulsation of the fluid between the arms.

The data presented in Figure 9(a) show that $C_{L1} = 1.64$ and $C_{L2} = 4.58$ in the case of $\beta = 45^\circ$. The cross-flow section of the upstream arm was circular, and the cross-flow section of the downstream arm was elliptical. At a pulsating frequency of 0.2 Hz, the C_L of the downstream arm was approximately 5.6 times that of the single arm, and C_D was approximately two times. This result was due to the impact of vortex shedding generated by the upstream arm on the downstream arm and the vortex shedding of the downstream arm itself. In addition, the C_L of the upstream arm was always smaller

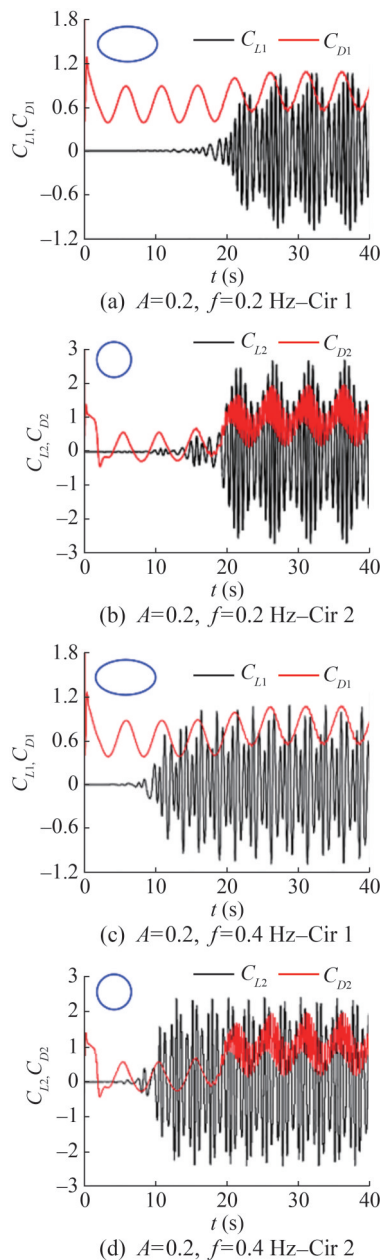


Figure 8 Time history curve of lift and drag coefficients when $\beta = -45^\circ$ and $A = 0.2$

than that of the downstream arm, and the C_L of the upstream arm increased with pulsation frequency, while the C_L of the downstream arm was the opposite. This trend was strikingly similar to that of Figure 8. In contrast to Figure 8, the C_D of the upstream arm was always higher than that of the downstream arm. This comparison indicates that the wake shielding effect was stronger when the upstream cross-section was circular (Assi, 2014). As the pulsation frequency continued to increase, the period of the upstream arms C_L and C_D decreased from 10.3 to 4.6 s. In addition, the maximum value of C_L increased from 1.64 to 1.69, while the maximum value of C_D decreased from 2.22 to 1.76, indi-

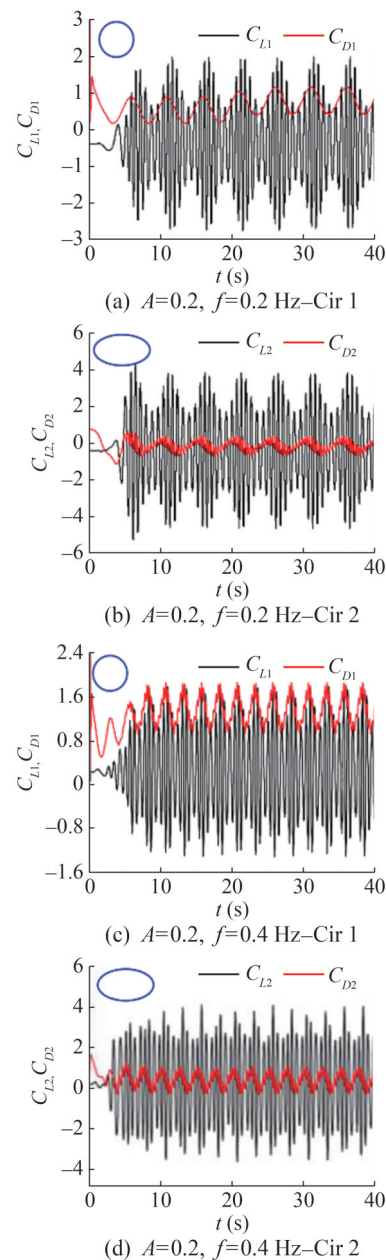


Figure 9 Time history curve of lift and drag coefficients when $\beta = 45^\circ$ and $A = 0.2$

cating that the influence of the downstream arm on the upstream arm was related to the pulsation frequency. The negative drag phenomenon appeared in the downstream arm as the pulsation frequency increased, indicating that the downstream arm was attracted to the upstream arm. This result was probably caused by the presence of a reflux zone in the gap between the two columns (Du et al., 2018).

As shown in Figures 10 and 11, the effect of the pulsation frequency on vortex shedding frequency in a series flow field was further observed. This change was consistent with the above results obtained by fast Fourier transform, but it was more intuitive and obvious. The width of the frequency

band was narrower than that of the single arm, as shown in Figure 6. However, the variation trend was consistent with Figure 6. As the pulsation frequency increased, the waveform tightened, and the peak continued to decrease. This result indicated that the pulsation frequency also affected the fluctuation of fluid between arms. At the same time, it was evident that the amplitude corresponding to the vortex shedding frequency of the downstream arm was much higher than that of the upstream arm. This result was due to the vortex shedding of the upstream arm and the change in wake velocity impacting the downstream arm.

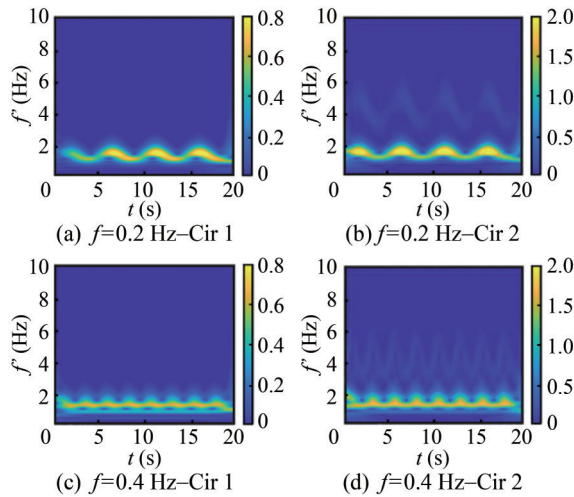


Figure 10 Lift coefficient time-frequency diagram of different frequencies when $\beta = -45^\circ$ and $A = 0.2$

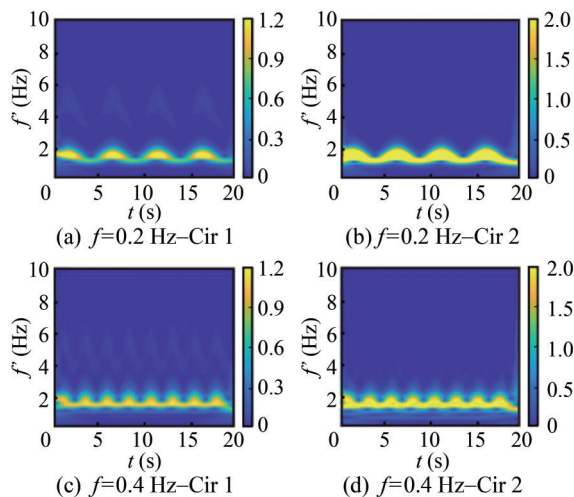


Figure 11 Lift coefficient time-frequency diagram of different frequencies when $\beta = 45^\circ$ and $A = 0.2$

Figure 12 shows a vortex cloud diagram at different pulsating frequencies. The left column in Figure 12 clearly shows that the free shear layer of the upstream arm developed to a certain extent. The wake in the upper part of the arm was obviously longer than that in the lower part, and the shedding vortex was obviously apparent in the lower

part. In the downstream arm, the vortex exhibited a regular vortex-shedding phenomenon. The vortex was arranged in a synchronous reverse pattern and continually moved away from the back of the downstream arm until the energy of the vortex diminished and disappeared. In contrast to the behavior shown in the left column in Figure 12, the wake morphology and vortex shedding mode of the flow field changed when $\beta = 45^\circ$. When $f = 0.2$ Hz, there was an obvious secondary vortex shedding behind the upstream arm. Affected by the impact of the shedding vortex and the elliptic cross-section of the downstream arm, the vortex shedding position at the lower part of the downstream arm was substantially forwarded, and an obviously combined vortex was generated at the upper part. With the increase in the pulsation frequency, the wake of the upstream arm gradually increased, and the secondary vortex shedding was substantially accelerated. The impact on the downstream arm was gradually strengthened, which led to an increase in the length of the combined vortex on the upper side of the downstream arm and an acceleration of the vortex shedding on the lower side. In addition, the asymmetry and instability of the vortex street were gradually enhanced, and the energy dissipation rate of the vortex accelerated.

3.2 Effect of pulsating amplitude on hydrodynamic performance

3.2.1 Flow past a single arm

The maximum velocity varies with the dimensionless pulsating amplitude, and its hydrodynamic performance differs from that caused by the pulsation frequency. Therefore, the flow structure caused by the variation in the pulsation amplitude deserves further study. Figure 13 illustrates the trend of C_L and C_D when pulsation frequency $f = 0.2$ Hz and dimensionless pulsation amplitude A are 0.2, 0.3, and 0.4. It is clearly seen that although the C_L and C_D of the cylindrical section were much higher than those of the elliptical section, the rate of increase with the pulsation amplitude of the cylindrical section was slightly lower than that of the elliptical section. When $\beta = 180^\circ$, the difference between C_L and C_D was small, and its growth trend was consistent.

The maximum values of the lift and drag coefficients were 1.78 and 1.88 for a pulsation amplitude of 0.2, respectively. The values increased by 41.5 and 32.3, respectively, compared with the study by Cheng et al. (2022). This result indicated that the pulsation amplitude increased the pulsatility of the flow field and force on the manipulator. As the pulsation amplitude increased from 0.2 to 0.4, C_D increased by 24.17%, and C_L increased by 21.36%. When $\beta = 135^\circ$, C_D was much higher than C_L , and $C_{D, \max}$ was approximately twice $C_{L, \max}$. C_L and C_D increased linearly with increasing pulsation amplitude; C_D increased from 1.00 to 1.40, and C_L increased from 0.48 to 0.59. This result was due to the

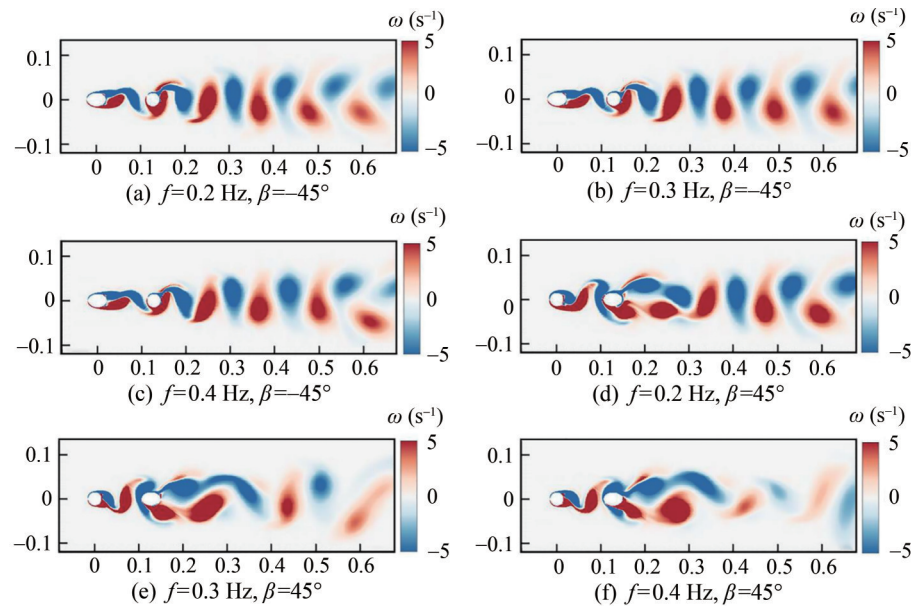


Figure 12 Vorticity cloud maps with different pulsating frequencies under $\beta = 45^\circ$ and $\beta = -45^\circ$

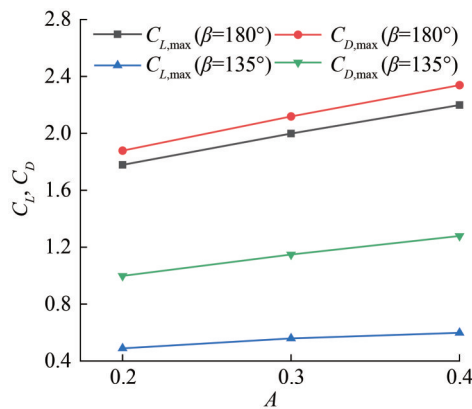


Figure 13 Variation trend of lift and drag coefficients at different pulsating amplitudes when $f = 0.2$ Hz

increase in the inlet flow velocity and Reynolds number with the increase in pulsation amplitude. This result was consistent with the study of Zhai et al. (2018), where the separation point continually moved forward with an increasing Reynolds number, and the drag gradually increased.

The lift coefficient history curve can be transformed into a frequency spectrum using the fast Fourier transform method, which transforms the time domain value into a frequency domain value and obtains the vortex shedding frequency f_v . Because the fluctuation of C_L in the time domain was disorderly, its time history curve was not sinusoidal with a stable amplitude. Therefore, several obvious peak frequencies were observed in the frequency domain.

From Figure 14(a), it is clear that under different pulsation amplitudes, the lift coefficients exhibit similar response characteristics. C_L was scattered in the frequency domain to 1–3 Hz; meanwhile, there was no obvious main frequency. When the pulsation amplitude was 0.4 Hz, the vortex shed-

ding frequency f_v was 2.05 Hz. Moreover, the remaining frequencies were 1.65, 1.85, 2.25, and 2.45 Hz. Interestingly, the frequency difference was 0.2 Hz, which was equal to the pulsation frequency.

The reason behind this phenomenon was that the other dominant frequencies resulted from the superposition of the pulsation frequency and vortex shedding frequency, potentially induced by vortex interactions (Cao et al., 2020; Konstantinidis and Bouris, 2009). When the pulsation amplitude was 0.2, the main peak frequency decreased to 1.82 Hz, and the number of main frequencies decreased. This phenomenon was also observed at other pulsation amplitudes, which will not be further discussed here. In Figure 14(b), the lift coefficient response became more complex relative to that of the circular cross-section because of the streamline property in the elliptical section. However, the difference between the frequency peaks was also equal to the pulsation frequency.

Figure 15 shows the vorticity clouds at the same time with different pulsation amplitudes when arm 2 was rotated 180° and 135° . When $\beta = 180^\circ$, alternating vortices were observed on both sides of arm 2 in the wake region, and the vortices dispersed downstream after shedding. With an increase in the pulsation amplitude, the vertical influence range of the wake region decreased, and the vortex formation area in the wake region shortened. Therefore, the vortex formation area was closer to the back of the column surface. The frequency of vortex shedding increased, and the time of cross vortex formation decreased. In addition, with an increase in pulsation amplitude, the distance between the vortices gradually increased, and the vorticity gradually decreased. Tail fluctuations and vortex strength were enhanced with increasing pulsation amplitude (Konstantinidis and Balabani, 2008; Konstantinidis and Liang, 2011).

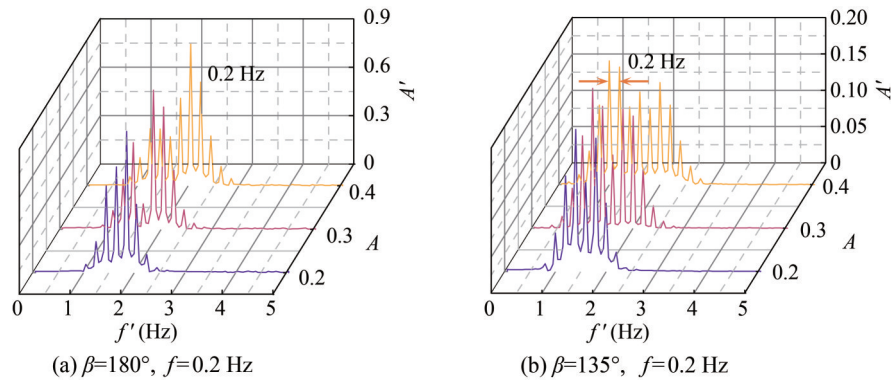


Figure 14 Lift coefficient spectrum of different postures

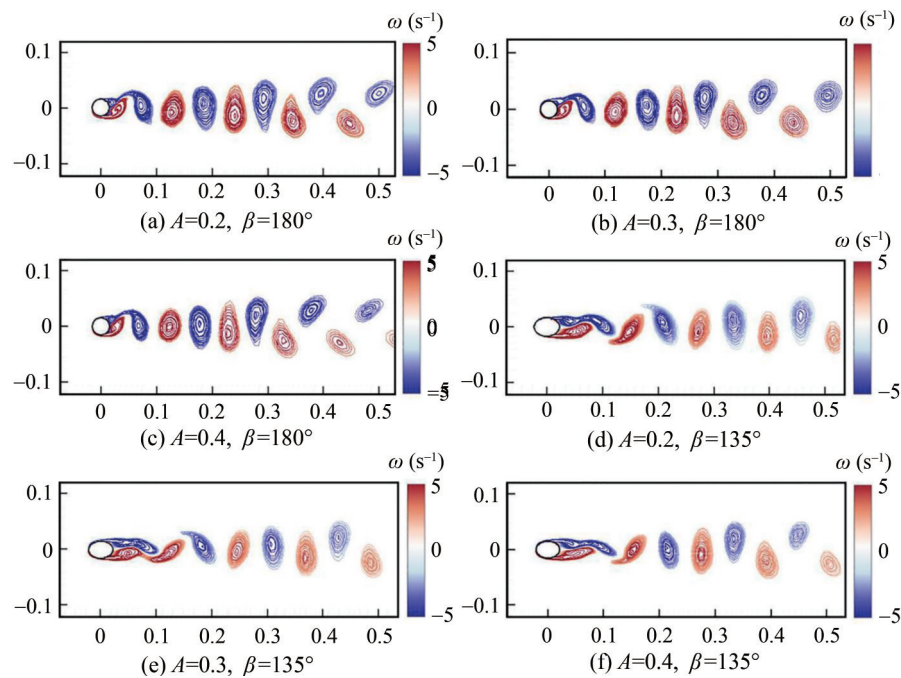


Figure 15 Vorticity cloud maps with different pulsating frequencies under $\beta = 180^\circ$ and $\beta = 135^\circ$

When $\beta = 135^\circ$, the cross-section of arm 2 was elliptical, and its vortex shedding situation was similar to that of the circular cross-section; however, the vortex shedding time and the formation of cross-vortex time were longer, and the reduction in vortex size was more obvious.

3.2.2 Flow past dual-arm underwater manipulator

The cross-flow section shape of the upstream arm was inconsistent, while the underwater manipulator was in P1 and P2. When the pulsation amplitude was changed, differences were observed in the hydrodynamic characteristics of the two postures. As shown in Table 5, $C_{L, \max}$ of the upstream and downstream arms increased by 65.71% and 38.06%, respectively, with an increase in the pulsation amplitude when $\beta = -45^\circ$. Compared with Figure 8, C_L increased more noticeably because of the pulsation amplitude. This result indicated that the pulsation amplitude had a greater influence on the pulsation of the fluid. However,

the trend of C_D was quite different from that of C_L . When the pulsation amplitude was 0.2, the C_D of the downstream arm was higher than that of the upstream arm. As the pulsation amplitude increased to 0.4, the C_D of the upstream arm increased by 114.02%, while that of the downstream arm decreased by 48.73%, and even negative resistance occurred. This result indicated that the downstream arm was pushed forward by the fluid.

Table 6 shows the variation in the lift and drag coefficients for a 45° clockwise rotation of arm 2. Table 6 shows that the C_L growth trend of the upstream and downstream arms was consistent with those in Table 5. It was clearly illustrated that the $C_{L, \max}$ of the upstream and downstream arms increased by 34.15% and 15.07%, respectively. The magnitude of the increase was approximately half that of $\beta = 45^\circ$. In addition, the increase in the C_D of upstream arm 1 was small. Regardless of whether the pulsation ampli-

Table 5 Value of lift and drag coefficients under $\beta=-45^\circ$ and $f=0.2$ Hz

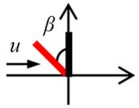
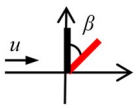
Postures	Parameters	$A=0.2$ -Cir 1	$A=0.2$ -Cir 2	$A=0.4$ -Cir 1	$A=0.4$ -Cir 2
	$C_{L,max}$	1.07	2.68	3.12	3.70
	$C_{L,min}$	-1.06	-2.68	-3.10	-3.73
	$C_{D,max}$	1.07	1.97	2.29	1.01
	$C_{D,min}$	0.54	0.17	0.46	-0.59

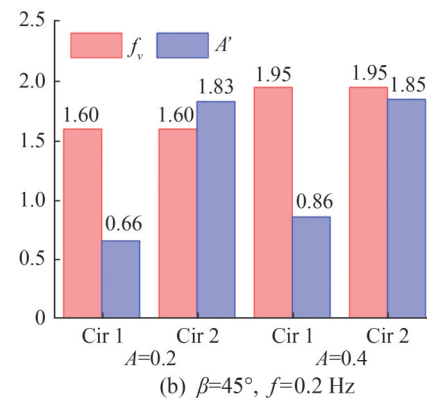
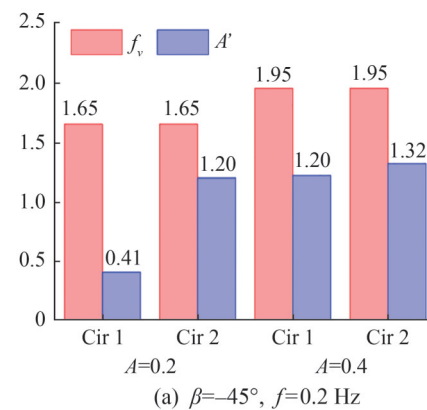
Table 6 Value of lift and drag coefficients under $\beta=45^\circ$ and $f=0.2$ Hz

Postures	Parameters	$A=0.2$ -Cir 1	$A=0.2$ -Cir 2	$A=0.4$ -Cir 1	$A=0.4$ -Cir 2
	$C_{L,max}$	1.64	4.58	2.20	5.27
	$C_{L,min}$	-1.64	-4.58	-2.18	-5.29
	$C_{D,max}$	2.22	0.87	2.29	1.02
	$C_{D,min}$	0.86	-0.31	0.46	-0.59

tude increased, the downstream column exhibited a negative drag phenomenon. This result may be attributed to the turbulence of the fluid between the arms and the shape of the cross-sectional flow in the upstream arm.

Figure 16(a) shows the values of the vortex shedding frequency and amplitude when the manipulator was rotated counterclockwise by 45° . The vortex shedding frequencies of the upper and lower arms were 1.65 Hz when the pulsation amplitude was 0.2. The vortex shedding frequency of the manipulator increased by 18.18% with an increase in the pulsation amplitude from 0.2 to 0.4. In other words, the pulsation amplitude further increased the pulsation of the flow field. When the manipulator was located in P2, the vortex shedding frequency increased by 39.29%, which was approximately twice as much as that in P1, indicating that the degree of flow field pulsatility caused by the pulsation amplitude varied with the arrangement. In addition, the increase in vortex shedding frequency was greater when compared with that of Figure 11, indicating that the pulsation amplitude affected the fluid pulsatility to a greater extent.

Figure 17 shows the velocity contours of the flow field for the underwater manipulator at angles of -45° and 45° . Under the two arrangement modes, it was coshedding, and vortex generation and separation phenomena were observed in the upstream and downstream columns. When $\beta = -45^\circ$, the flow field development process and vortex shedding state obtained are basically identical to the state under uniform flow, and the vortex distribution behind the downstream column has obvious regularity and periodicity. However, the intensity of the vortex shedding differs, and the turbulence of the water flow is more intense, which is manifested by the change in the uplift coefficient of resistance when acting on the cylinder. With an increase in the pulsation amplitude, the upstream arm vortex strength

**Figure 16** Lift coefficient spectrum with different pulsating amplitudes

increased, and the vortex shedding accelerated. Thus, the separation points of the downstream arm gradually moved forward. The time for the formation of the Karman vortex street appearing alternately on both sides became shorter. Compared with $\beta = -45^\circ$, the vortex distribution of the downstream arm was irregular under $\beta = 45^\circ$. When $A =$

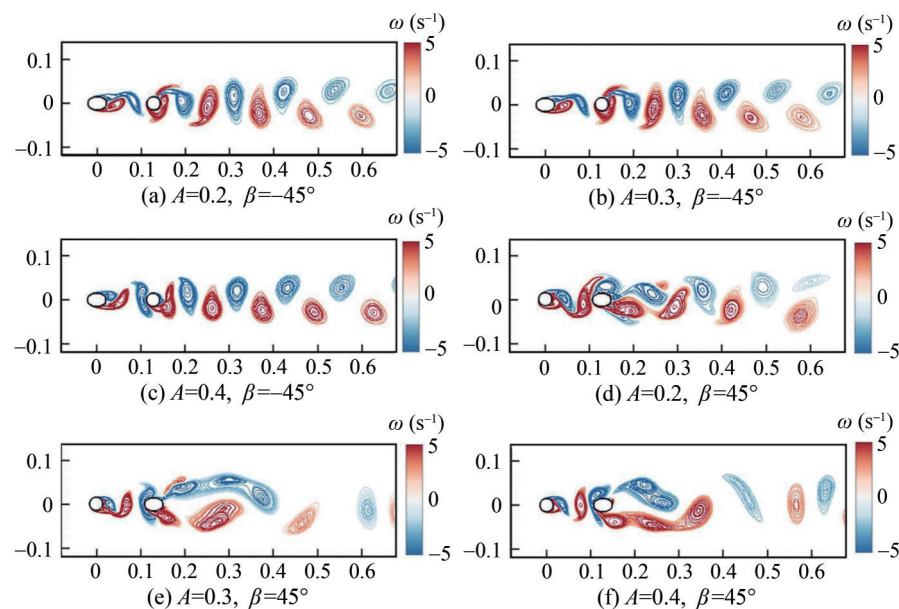


Figure 17 Vorticity cloud maps with different pulsating amplitudes

0.2, the vortex generated by the upstream arm impacted the downstream arm. Because of the influence on the cross-sectional shape of the downstream arm, a large separation bubble appeared on the upper part of the downstream arm. Separation bubbles were typically spatially and temporally unstable. Meanwhile, complex energy transfers occurred between them. The bubble phenomenon in pulsating flow also severely affects the hydrodynamic characteristics of the manipulator (Zhao et al., 2023; Rodríguez et al., 2015). When the pulsation amplitude increased to 0.3, the number of vortices shedding on the upstream arm increased, resulting in vortex shedding on the upper side of the downstream arm. The shedding vortices merged periodically. As the pulsation amplitude further increased to 0.4, the shedding speed and quantity of the trailing vortices of the upstream arm accelerated. The repulsive force generated by the wake disturbance further increased, and combined vortices appeared on both sides of the downstream arm.

4 Conclusions

This study examined the influence of pulsating settings on an underwater manipulator's hydrodynamic characteristics. To better observe this influence, the effects of the pulsating frequency and amplitude on the hydrodynamic properties of the underwater manipulator under four typical postures were compared and examined. The specific conclusions were as follows:

1) When arm 2 was located upstream, a stronger effect of the pulsation amplitude on the hydrodynamic coefficient was obtained because of the interaction between the two arms.

2) When arm 2 was located downstream, in contrast to the effect of pulsation frequency, the maximum C_L and C_D of the arms increased by 33.33% and 16.78%, respectively, with an increase in pulsation amplitude.

3) The maximum increase in C_L and C_D with pulsation frequency was 12% and 6%, respectively, when the underwater manipulator was in P3 and P4.

4) When the manipulator was in P3, the increasing amplitude of C_L and C_D caused by dimensionless pulsation amplitude was approximately two times that caused by pulsation frequency.

5) When the manipulator was located in P1, the combined vortex phenomenon became more pronounced with an increase in the pulsation amplitude. In other positions, the vortex street structure at the wake was regular.

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