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REVIEW

A Review of Ice Deformation and Breaking Under Flexural–Gravity Waves Induced by Moving Loads

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Abstract

Ice-breaking methods have become increasingly significant with the ongoing development of the polar regions. Among many ice-breaking methods, ice-breaking that utilizes a moving load is unique compared with the common collision or impact methods. A moving load can generate flexural–gravity waves (FGWs), under the influence of which the ice sheet undergoes deformation and may even experience structural damage. Moving loads can be divided into above-ice loads and underwater loads. For the above-ice loads, we discuss the characteristics of the FGWs generated by a moving load acting on a complete ice sheet, an ice sheet with a crack, and an ice sheet with a lead of open water. For underwater loads, we discuss the influence on the ice-breaking characteristics of FGWs of the mode of motion, the geometrical features, and the trajectory of motion of the load. In addition to discussing the status of current research and the technical challenges of ice-breaking by moving loads, this paper also looks ahead to future research prospects and presents some preliminary ideas for consideration.

Keywords Ice-breaking; Moving load; Flexural–gravity wave; Ice sheet; Above-ice load; Underwater load

1 Introduction

The exploitation of valuable resources in the polar regions and the consequent need to get ships through the Arctic Passage is attracting increasing attention (Dalaklis et al., 2023; Song et al., 2023). Ice-breaking is a major challenge for polar marine structures in Arctic regions. The traditional method for breaking Arctic ice utilizes collision by an ice-breaker. However, under some conditions—such as extremely thick ice—the vessel may become trapped in the ice. Consequently, researchers are constantly exploring new methods to improve ice-breaking ability. Many new ice-breaking methods have been proposed, including the use of explosions (Wang et al., 2022), high-pressure air-gun bubbles (Wu et al., 2022; Zhang et al., 2023), high-speed water jets (Yuan et al., 2022; Yuan et al., 2023), moving loads (Zemlyak et al., 2023; Zhou et al., 2023), etc. Among these new ice-breaking methods, breaking the ice by utilizing a moving load has the advantages of high efficiency and cleanliness. In this paper, we review the progress in research on ice-breaking by moving loads.

According to water wave theory, gravity waves are generated on a free surface under the combined influence of inertial and gravitational forces (Lamb, 1932). On the other hand, flexural−gravity waves (FGWs) are formed when a large elastic structure (such as an ice sheet) that covers the free surface of a fluid moves in conjunction with the fluid. FGWs are produced by the combined effects of the elastic force and the inertial force on the ice sheet, together with the hydrodynamic force on the fluid (Korobkin et al., 2011). The FGWs induced on an ice sheet by a moving load involve the complex interaction between fluid and structure, and it is also a hot topic in hydrodynamics (Shishmarev et al., 2023).

Depending upon its position relative to the ice sheet, a moving load can be categorized as one of two types: above-ice loads and underwater loads. An above-ice load can originate from a high-speed hovercraft on the ice sheet, an aircraft on an ice runway, a vehicle on an ice road, etc. In

Article Highlights

- An overview of research on ice-breaking by using moving loads from the perspectives of both above-ice and underwater loads is presented.
- Different ice conditions when an above-ice load moves to generate FGWs are covered, including a complete ice sheet, an ice sheet with a crack and an ice sheet with a lead of open water.
- The influence on the ice-breaking characteristics of the motion mode, the geometrical features, and the trajectory of motion of the underwater load are discussed.

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1972, Canadian investigators discovered that hovercraft platforms had a certain icebreaking capacity when they broke 0.76 meters of thick ice during an experiment to convert the hovercraft platform ACT-100 into a drilling platform in the Arctic region (Fowler, 1976; Zhu, 2008). They subsequently confirmed and applied the principle of using a high-speed moving hovercraft for ice-breaking. However, it is necessary to assess the carrying capacity of an ice sheet before utilizing it for various operational activities. For Instance, an ice sheet can serve as a highway for transporting goods, or it can be used as a runway for aircraft (Babaei et al., 2016; Sanden and Short, 2017). Underwater moving loads can be generated by underwater vehicles in ice-covered areas, which can also induce FGWs on the ice sheet (Cherkesov, 1980).

The deflection and breaking of ice by the FGWs generated by a moving load has both rich academic interest and promising engineering applications. Consequently, there has been rapid development in these areas recently. In this paper, we therefore provide a comprehensive overview of research on ice-breaking by using moving loads from the perspectives of both above-ice and underwater loads.

2 Research on ice-breaking with a load moving above the ice

The study of FGWs generated by a load moving on an ice surface was initially based on the assumption of an infinite ice sheet. The fluid beneath the ice sheet was considered to be an ideal fluid, and the flow was assumed to be irrotational. Research in this field began in the 1950s and 1960s (Wilson, 1958; Kheisin, 1963), but significant progress has been made over the past few decades. In addition to infinite ice sheets, investigators have also studied semi-infinite ice sheets, ice sheets with water channels, ice sheets with cracks, etc. In the following, we present the progress of research on ice-breaking by a load moving above the ice in different ice conditions.

2.1 Infinite ice sheet

2.1.1 An intact infinite ice sheet

A moving load induces a hydroelastic response—namely, FGWs—in an infinite ice sheet. Research in this area has focused mainly on the deformation of an ice sheet caused by the pressure distribution on its surface (Davys et al., 1985), the distribution of strain in the ice sheet (Squire et al., 1985), and the resistance experienced by the moving load (Kozin and Pogorelova, 2003). Whether the ice sheet will be damaged by the moving pressure can be determined by comparing the calculated values with the yield stress of the ice sheet. Most of the relevant research findings prior to 1996 have been reviewed by Squire et al. (1996); they included primarily theoretical derivations and experimental results, with limited content on numerical simulations. In the present section, we therefore present the more recent research progress in theoretical research and experimental studies, and we also include the results of numerical simulations.

In the propagation of an FGW in an infinite ice sheet, when the wavelength is relatively long, the gravitational force dominates, and the wave propagation speed (the phase speed c_p) is larger than the propagation speed of the wave energy (the group speed c_g). On the other hand, when the wavelength is short, the elastic force dominates, and c_p is smaller than c_g. Thus, there exists a critical wavelength for FGWs in an infinite ice sheet; at this critical wavelength, $c_p = c_g$, which is known as the first critical speed $c_{\mathrm{cr},1}$ (Nevel, 1970). When a load moves at this speed on an infinite ice sheet, the energy transmitted to the ice sheet cannot escape from the load. If there is no energy dissipation, the energy gained by the ice sheet under the load will continue to accumulate over time, leading to an infinitely large deformation of the ice sheet (Schulkes and Sney, 1988; Milinazzo et al., 1995). The steady-state deformation of the ice sheet when the load moves at $c_{\mathrm{cr},1}$ can be obtained by taking into account factors such as viscosity (Hosking et al., 1988; Kozin and Pogorelova, 2009) or nonlinearity (Parau and Dias, 2002; Bonnefoy et al., 2009).

Theoretical studies usually adopt linear potential flow theory for the fluid and infinite elastic thin plate equation for the ice sheet. Taking the moving load to be a point load, Davys et al. (1985) used the asymptotic Fourier method to solve the steady-state response of an infinite elastic ice sheet caused by its uniform motion and obtained the far-field waveform characteristics. In comparison, Schulkes and Sney (1988) investigated the response of an infinite elastic plate subjected to a sudden application of a line load moving at a constant velocity. They obtained an expression for the deformation of the ice by using the Fourier transform method, and they analyzed the ice deformation as time approaches infinity by using an asymptotic expansion. They found that when the load moved at $c_{\mathrm{cr},1}$, the deformation of the ice increased with time t in proportion to $\sqrt{t}$. They also found that a second critical speed $c_{\mathrm{cr},2} = \sqrt{gH}$ exists, where g is the acceleration due to gravity, and H is the water depth. When the load moves at the speed $c_{\mathrm{cr},2}$, the ice deformation increased with time in proportion to $t^{1/3}$. The second critical speed is independent of the parameters of the ice sheet, and it reflects only the gravity wave effects of the fluid. This also indicates that wave energy accumulates when the load moves at the speed $c_{\mathrm{cr},2}$, which leads to a continuous increase in the amplitude of the deformation of the ice sheet. This phenomenon is analogous to the formation of solitary waves ahead of a ship navigating in a shallow water channel (Zhang and Gu, 2006), so $c_{\mathrm{cr},2}$, is also called the critical speed in shallow water. Milinazzo et al. (1995) further investigated the steady-state response of an infinite elastic thin plate induced by a rectangular pressure surface moving at a constant velocity. They found that the ice sheet did not achieve a steady-state deformation at $c_{cr,1}$, while a steady-state deformation of the ice sheet was obtained at $c_{cr,2}$. They speculated that Schulkes and Sneyd (1988) may not have obtained an actual steady-state solution, possibly because the solution of the initial value problem increased with time. Nugroho et al. (1999) utilized Fourier transform and asymptotic analyses to study the response of an infinite, thin elastic plate induced by a suddenly initiated point load and a circular pressure surface. They found that when the load moved at $c_{cr,1}$, the deformation of the ice sheet increased logarithmically with time, whereas they obtained a steady-state deformation of the ice sheet at $c_{cr,2}$. Research has also been performed on the response of an ice sheet induced by varying velocity loads (Miles and Sneyd, 2003; Hosking and Milinazzo, 2022; Pogorelova et al., 2014; Li et al., 2017a) and on the influence of varying water depths (Pogorelova and Kozin, 2014a), varying ice thicknesses (Pogorelova et al., 2016), and more recently on the response of an ice sheet induced by a load moving at constant velocity and by a current with a vertical shear of velocity (Tkacheva, 2023).

Kozin and Pogorelova (2006) studied the wave-making resistance experienced by a rectangular pressure distribution over an infinite elastic ice sheet. They found that—to minimize the wave-making resistance—it was necessary for the pressure distribution either to move at a subcritical speed (i.e., a speed smaller than $c_{cr,1}$) or to reach a supercritical speed (a speed larger than $c_{cr,1}$) with a large acceleration and then continue to move at a supercritical speed. Lu and Zhang (2013) used Fourier transforms and the residue theorem to investigate the two-dimensional (2D) problem of the resistance to wave formation experienced by a moving line load on an infinite elastic ice sheet floating on an infinite depth of water. They found that the wave-making resistance jumped from zero to a certain value as the velocity of the line load increased from zero to the critical speed $c_{cr,1}$. With a further increase in velocity, the wave-making resistance increased before subsequently decreasing. This indicates that, for a line load, the wave-making resistance peaks at a speed larger than $c_{cr,1}$. However, different from the phenomenon observed by Lu and Zhang (2013), Zhang (2013) found that in the three-dimensional (3D) case of a point load moving on an infinite elastic ice sheet, the wave-making resistance peaked at the critical speed $c_{cr,1}$.

In the aforementioned studies, the ice sheet was commonly treated as a thin elastic plate. Some researchers have argued that the thickness of the ice cannot be ignored and that the assumption of a thick elastic plate needed to be adopted for such problems. However, the final results showed that it was unnecessary to use a thick elastic-plate model (Balmforth and Craster, 1999). Furthermore, when an ice sheet is assumed to be an infinite thin viscoelastic plate, a steady-state ice sheet deformation response can be obtained when the pressure load moves at $c_{cr,1}$. Takizawa (1988) considered the effect of the viscosity of the ice sheet by introducing an additional time derivative term for the ice deformation in the beam vibration equation for a thin elastic plate, while Hosking et al. (1988) introduced a time-memory function into the bending force term of the equations for a thin elastic plate to account for the effects of the viscosity of the ice sheet. Hosking et al. (1988) used the Fourier transform method to solve the ice response induced by a point load and by a line load moving at a constant velocity on a viscoelastic ice sheet. They observed that the local response of the ice sheet induced by a pressure load moving at a subcritical speed was asymmetric and that a steady-state ice deformation was achieved when the pressure load moved at $c_{cr,1}$. However, when the load moved at a supercritical speed, the bending wave in front of the load was suppressed. Additionally, Takizawa (1985) found that the position of the maximum depth of ice sheet deflection occurred behind the load, which is consistent with his experimental results. Wang et al. (2004) used the same viscoelastic thin plate equations as had been employed by Hosking et al. (1988) to analyze the response of a viscoelastic ice sheet to a suddenly moving line load and point load starting from rest on the ice sheet. They found that the ice sheet deformation induced by the point load decayed faster than that induced by the line load. Zhestkaya (1999) proposed another viscoelastic model that included a strain-relaxation time. Dinvay et al. (2022) obtained new exact solutions to the problem of a load moving on a floating ice plate based on the two viscoelastic models considered by Hosking et al. (1988) and by Zhestkaya (1999). Kozin and Pogorelova (2003) adopted the Kelvin –Voigt viscoelastic thin plate model for an ice sheet and applied the Fourier transform method to study the wave-making resistance acting on a distributed rectangular pressure surface moving at a constant speed (Figure 1). They analyzed the effects of the ice thickness, the aspect ratio of the rectangular pressure surface, the elastic modulus of the ice sheet, and depth of the water on the wave-making resistance, and they found that the maximum wave-making resistance occurred near the critical speed. Pogorelova (2008) used the Fourier transform method to consider the wave-making resistance of an unsteady rectilinear motion of a rectangular distributed pressure over a viscoelastic ice sheet at various speeds, and they proposed a series of strategies to reduce the wave-making resistance. Kozin and Pogorelova (2009) analyzed the ice sheet deformation and the strain induced by a moving load on three viscoelastic ice sheet models: the Kelvin–Voigt, Maxwell, and Maxwell–Kelvin models. They compared the displacement and strain results obtained from these models with experimental data (Takizawa, 1985; Squire et al., 1988) and found that the Kelvin – Voigt viscoelastic model provided better agreement with experimental results at subcritical velocities, while the Maxwell viscoelastic model showed better agreement for supercritical velocities. The Maxwell–Kelvin viscoelastic ice sheet model combines the advantages of both the Kelvin–Voigt and Maxwell models. To obtain more accurate results, Kozin and Pogorelova (2009) suggested the viscous term in the viscoelastic ice sheet model should be a function of the load velocity. Furthermore, when nonlinear effects are considered, it has been shown that an ice sheet can exhibit a stable deformation response even when the load is moving at $c_{cr,1}$ (Squire et al., 1985; Parau and Dias, 2002; Bonnefoy et al., 2009).

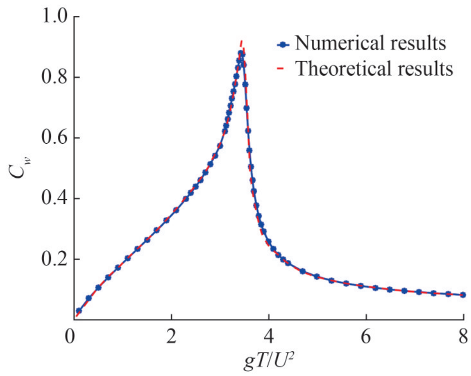


Figure 1 Wave-making resistance coefficient of a moving pressure on an infinite ice sheet as calculated theoretically by Kozin and Pogorelova (2003) and numerically by Xue et al. (2021a)

The earliest experimental research on loads moving on ice sheets dates back to the 1950s when Wilson (1958) observed that the maximum ice deformation occurred behind the load. Takizawa (1985; 1988) performed more comprehensive field experiments, measuring the deformation of the ice surface caused by loads moving at different speeds. These experiments further confirmed that the time at which the maximum ice deformation occurs lags behind the time at which the load passes through the position of maximum deformation. This time lag phenomenon also existed even when the pressure moved at a very low velocity. When the speed of the moving load exceeded $c_{cr,1}$, the time lag increased rapidly as the speed of the load increased. Additionally, Takizawa (1985; 1988) found that the depression depth of the ice sheet was largest when the load was moving at the first critical speed and that it was three times the depth produced by the gravitational effect of the load, as shown in Figure 2. In addition, Squire et al. (1985) performed field experiments on loads moving on freshwater ice and on Antarctic Sea ice. They measured the strain distribution caused by the movement of the load and found that the maximum strain of the ice surface occurred at $c_{cr,1}$. However, the strain amplification factor was 2.25 for freshwater ice and 1.45 for sea ice. The investigators attributed this difference mainly to the higher viscosity of sea ice compared to that of freshwater ice.

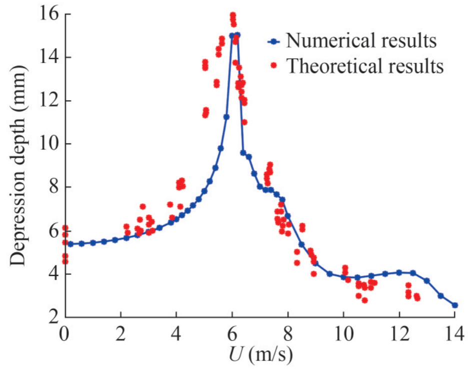


Figure 2 Depression depth of an ice sheet as a function of the speed of the moving load including experimental results of Takizawa (1985) and the numerical results of Zeng (2022)

Besides field tests, Zhang et al. (2014) performed model experiments in the laboratory, as shown in Figure 3, to establish a similarity relationship between prototype and model tests. However, in the model experiments, it was challenging to find an ice model with mechanical properties sufficiently similar to those of real ice, making it difficult to achieve completely similar model test conditions. Consequently, the investigators instead adopted polyurethane (PU) film to model the ice. They analyzed the effects of the load speed, the height of the pressure nozzle, the pressure amplitude, and the depth of water as parameters of the ice deformation response. They found that the load caused significant deformation of the ice sheet when it moved at $c_{cr,1}$. In addition, they found that the critical speed depended upon the parameters of both water and ice, as well as on the boundary conditions. Although the critical speed was independent of the pressure amplitude, the deformation amplitude of the ice sheet was positively correlated with the pressure amplitude.

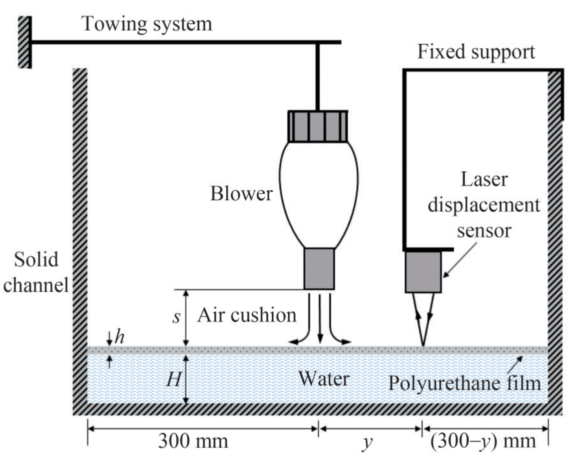


Figure 3 Schematic illustration of a model test of a load moving on a model ice sheet made of polyurethane (PU) film (Zhang et al., 2014)

Studies have also been performed using remote sensing satellites to measure the ice deformation caused by loads moving on polar ice (Babaei et al., 2016; Sanden and

Short, 2017). These studies observed the wave-like deformations of the ice sheet induced by the load, confirming some of the results predicted by the linear theoretical models. For instance, when a load was moving at a supercritical speed, a shadow zone of undisturbed ice appeared behind the load, as shown in Figure 4.

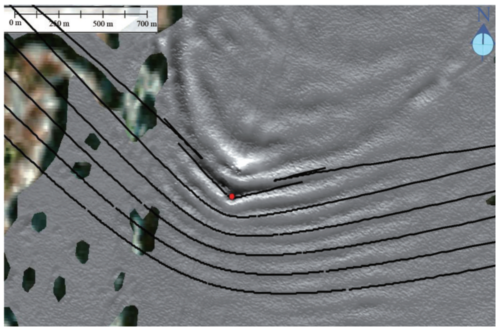


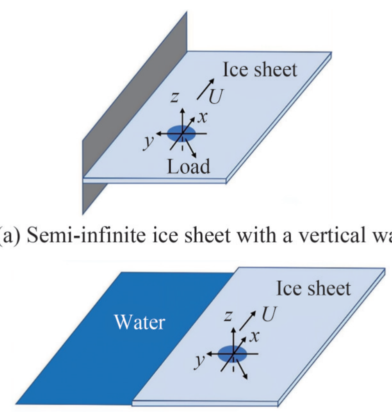
Figure 4 Comparison of the theoretical wave patterns (black lines) with satellite-observed wave patterns from a load moving across polar ice (Babaei et al., 2016)

There have been relatively few numerical studies of pressure-induced FGWs moving on ice sheets. However, FGWs in large floating structures have received more attention. Due to a focus on the strengths of these large floating structures, the finite element method (FEM) has been used extensively for analyzing this problem (Watanabe et al., 2004). In addition, Zhestkaya (1999; 2000) used FEM and the finite difference method (FDM) to study the deformations induced by loads moving on a viscoelastic ice sheet. Because FGWs on an infinite ice sheet are to some extent similar to capillary–gravity waves, based on the study by Parau et al. (2007) of capillary−gravity waves using the boundary integral method, Parau and Vanden-Broeck (2011) further developed a boundary integral algorithm applicable to FGWs on ice sheets caused by moving loads. They used an elastic thin plate model to simulate the ice sheet, considered the nonlinear effects of fluid kinematic and dynamic boundary conditions, and obtained the finite deformation of the ice sheet when the load moved at the critical speed. They found that the maximum deformation occurred near the critical speed. Other studies have simulated the motion of a load on an infinite viscoelastic ice sheet by combining FDM and the boundary element method (BEM). Liu et al. (2012; 2013) and Zeng (2022) used these methods to calculate the resistance and deformation response of the ice surface. Their results compared favorably with both the experimental results (Figure 2) and the theoretical results (Figure 1). Some studies have also simulated the stress distribution in an ice sheet induced by a moving load and have evaluated the ice-breaking capacity of a moving load based on different ice-breaking criteria. In particular, Li et al. (2017b; 2017c) found that the best ice-breaking effect occurs when the load is moving at a critical speed. Some other studies have used LS-DYNA dynamic analysis software to investigate the characteristics of the ice response when a load is moving on a finite ice sheet (Lu et al., 2014) and evaluate the ice-breaking process of the load (Lu et al., 2012). In the simulation of the ice-breaking process, these investigators found that the trough in the ice sheet behind the load was the first to be damaged before the damage extended rapidly forward and backward. As it extended forward, the damaged region encountered the wave crest in front of the load, causing lateral cracking that spread rapidly along the crest. The investigators also analyzed the effect of the water depth on the ice response, and they found that an increased water depth reduced the maximum principal stress induced by the load, thus reducing the ice-breaking capacity of the load (Lu et al., 2017).

The work cited above demonstrates that abundant research results exist regarding FGWs induced by a load moving on an intact, infinite ice sheet. However, there has been relatively little research on the time-domain response of an ice sheet to non-steady motions of the inducing load (Dinvay et al., 2019; Johnsen et al., 2022). Moreover, as research has progressed, studies of the ice fragmentation process caused by a moving load (Lu et al., 2012) have demonstrated the complex nature of the fluid−structure interactions in ice fragmentation. The FGWs induced by a moving load may lead to ice sheet damage that results in the creation of cracks or free water surfaces. Therefore, further development can be focused on the study of the ice-breaking process induced by moving loads.

2.1.2 Semi-infinite ice sheets

For a semi-infinite ice sheet, the boundary can be frozen onto a vertical wall, as shown in Figure 5(a), or it can be a free boundary next to a semi-infinite water surface, as shown in Figure 5(b). The existence of such a boundary significantly changes the response of the ice sheet to the moving load.



(a) Semi-infinite ice sheet with a vertical wall

(b) Semi-infinite ice sheet beside semi-infinite water surface

Figure 5 A load moving on a semi-infinite ice sheet (Xue et al., 2021a)

Brocklehurst (2012) studied the movement of a load on a semi-infinite ice sheet frozen onto the surface of a solid wall, using a viscoelastic thin plate model to simulate the ice sheet with a constant distance between the load and a solid wall. He employed the Fourier transform method to solve this linear problem, and he found that the existence of the solid wall makes the amplitude of the deformation response of the ice sheet smaller. When the load was moving at a supercritical speed, both the FGWs generated by the moving load and the solid-wall interaction were very strong, and the deformations of the ice between the load and the wall were obvious. Brocklehurst (2012) also used the boundary integral method to study the same problem, but he considered the nonlinear Bernoulli and kinematic conditions instead of their linear counterparts. He found that when the load was moving at a low velocity and was close to the wall, the deformation behind the load was greater than that produced in an infinite ice sheet—a phenomenon that had not been discovered under the linear assumption. However, Brocklehurst (2012) did not deal with the fixed boundary conditions of the ice sheet and the solid wall surface directly but instead used the mirror image method to take the fixed boundary conditions into account. This made it impossible to extend these calculations to other boundary conditions, such as simply supported or free boundary conditions.

For a semi-infinite ice sheet on a semi-infinite water surface, Zhestkaya and Kozin (1994) modeled the ice sheet as a thin viscoelastic plate and used FEM to investigate the distribution of stress and strain in the ice sheet when the load was moving either on the ice sheet or on the water surface. They found that the ice-breaking effect was best when the load was moving on the water surface at a given distance from the edge of the ice sheet. Under these conditions, a large area of the ice sheet was subjected to great stress. Li et al. (2017c) combined the BEM and FDM to analyze the stress distribution within the ice sheet when a load was moving on the ice sheet. Using different criteria for ice sheet failure, they analyzed the possible damage to the ice sheets and found that the best ice-breaking effect occurred when the load was moving at the critical speed of an infinite ice sheet. However, the mathematical model established by Li et al. (2017c) did not provide the boundary conditions for the ice sheet, nor did it explain how to deal with the boundary. Some other studies have investigated the motion of a load on a semi-infinite ice sheet with a semi-infinite water surface (Sturova and Tkacheva, 2018; Tkacheva, 2018; 2019a; Sturova, 2018). They used the Wiener–Hopf method, which can explicitly solve linear partial differential equations on a semi-infinite field. Sturova and Tkacheva (2018) and Tkacheva (2019a) studied the movement of a load on a water surface, assuming the ice sheet to be a thin elastic plate, and they found that the maximum deformation of the ice sheet and the water surface occurred when the load was moving at the critical speed of an infinite ice sheet and that the strain caused by the load may lead to ice sheet damage. In addition, they found that the wave-making resistance to the load oscillated near the critical speed of an infinite ice sheet. Tkacheva (2018) and Sturova (2018) further analyzed the movement of a load on a semi-infinite, elastic ice sheet and analyzed the ice deformation at different speeds of the load (Figure 6). Sturova (2018) also compared the ice deformations caused by the motion of a moving load for four ice boundary cases: an infinite ice sheet, a semi-infinite ice sheet with a semi-infinite water surface, a semi-infinite ice sheet with a fixed boundary at the surface of a solid wall, and a semi-infinite ice sheet with a free boundary at the solid wall surface. She found that the deformation of the ice was largest in the case where there was a free boundary between the semi-infinite ice sheet and the solid wall surface. The next largest was the case with a semi-infinite ice sheet and a semi-infinite water surface, then an infinite ice sheet, and finally, a semi-infinite ice sheet with a fixed boundary at a solid wall.

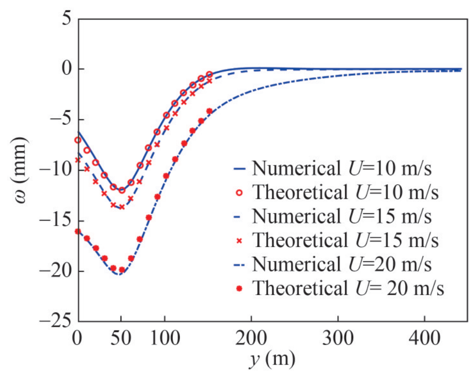


Figure 6 Deflections of a semi-infinite ice sheet calculated theoretically by Sturova (2018) and numerically by Xue et al. (2021a) for different speeds of a load moving along the edge of an ice sheet

Another important case involves the open water lead between two semi-infinite floating ice sheets, as shown in Figure 7, which is common in polar ship navigation (Ni et al., 2020). The Wiener–Hopf method mentioned above can also be used to solve the problem of a load moving in such a channel (Sturova and Tkacheva, 2019; Tkacheva, 2019b) since the two sides of the open water channel are semi-infinite ice sheets. Assuming the ice sheet to be a thin Kelvin–Voigt viscoelastic plate, Sturova and Tkacheva (2019) and Tkacheva (2019b) studied the steady-state response of the ice sheet caused by a load moving at a constant speed on the free surface of a water channel between two semi-infinite ice sheets, and they found that waves are generated in front of the load when it is moving at either a subcritical or a supercritical speed, due to the presence of edge modes. Furthermore, the ice sheet at the edge of the channel may be damaged under certain parameters.

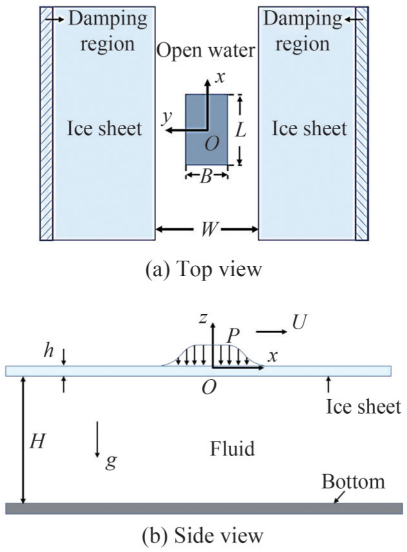


Figure 7 Sketch of a load moving in an ice-breaking channel (Xue et al., 2021a)

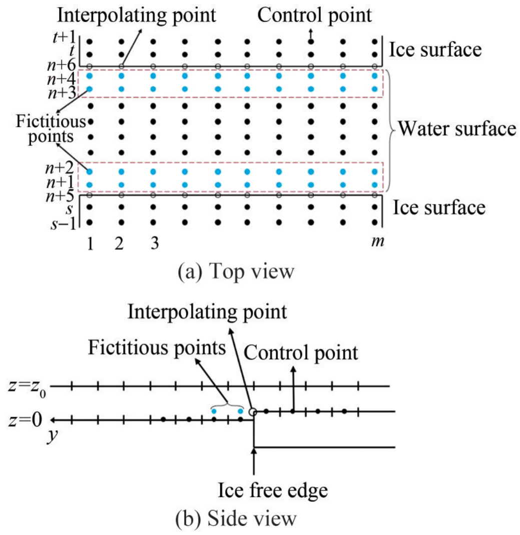


Figure 8 Relative positions of the control points, interpolating points, and fictitious points (Xue et al., 2021a)

The treatment of the ice – liquid boundary conditions within a semi-infinite ice sheet is a challenging problem compared to that of an infinite ice sheet. This is why the case of a semi-infinite ice sheet is more complex than that of an infinite ice sheet. Using a Rankine source and the FDM, Xue et al. (2021a) numerically solved the problem of FGWs induced by a load moving at a constant speed in an ice lead—or on an infinite ice sheet with a crack—by introducing external fictitious points at the ice sheet boundary (Figure 8) to enable the computation of the biharmonic operator within the boundary conditions. They employed the corresponding numerical algorithm to solve the problem for semi-infinite ice sheets with free edges, and they found that the deformations of the semi-infinite ice sheets were in good agreement with the theoretical results (Sturova, 2018), as shown in Figure 6. Xue et al. (2021a) focused on the FGWs caused by the steady motion of a load in the channel, and they found that the wave-making on the water surface was the superposition of wave-making by the load and of the waves reflected from the ice sheet. When the load was moving at the critical speed of an infinite elastic thin plate, wave-making on the water surface in the channel caused amplitude amplification. At the same time, the load moving on the water surface of the channel generated FGWs on the ice sheets on both sides.

As the width of a water channel tends to zero, it becomes a crack. Tkacheva (2019c) used the Wiener–Hopf method to study the problem of the generation of FGWs in an ice sheet with a straight crack under the action of a uniformly moving load. She assumed that the thicknesses of the ice sheets were different on the two sides of the crack and that the moving load remained at a given distance from the crack. She showed that waveguide edge modes traveling along the crack were excited when the load was moving at a supercritical speed. Xue et al. (2021a) calculated the wave-making resistance coefficient numerically for a load moving on an ice sheet with a crack right below the load, and their results were in good agreement with the theoretical results (Kozin and Pogorelova, 2003). They found further that the existence of the crack led to an increase in the critical speed of the ice sheet but that the amplitude of the stress in an ice sheet with a crack was smaller than that in an ice sheet without a crack at any speed. Based on the studies by Xue et al. (2021a), Zeng et al. (2021a) further considered a case in which a crack in an ice sheet was located outside the pressure surface. The effects of the cracks on the FGWs near the pressure surface were negligible when the distance between the crack position and the center of the pressure surface was greater than four times the width of the pressure surface, while the deformation and stress of the ice sheet near the pressure surface were significantly changed when the crack was closer than the width of the pressure surface to the center of the pressure surface.

2.2 Ice channel with two side walls

Model tests in an ice tank are important for investigations, such as the interactions between waves and ice. However, the effect of the wall of the ice tank is usually large because the width of the ice tank is usually small relative to the wavelength and cannot be ignored easily (Daly, 1993; Newman, 2016; Ren et al., 2020; Zeng et al., 2021b). Nevertheless, such tests are suitable for inland waterways since the load is usually moving near the wall of the ice channel. The problem of a load moving along an ice-covered channel such as a river or a canal has also received considerable attention. Therefore, it is also important to understand the FGWs induced by a load moving within an ice channel with two side walls.

Different from an infinite elastic ice sheet with just two critical speeds, a channel fully covered by an elastic ice sheet can support infinitely many hydroelastic waves, each with its own critical speeds (Korobkin et al., 2014; Batyaev and Khabakhpasheva, 2015; Ren et al., 2020). The steady-state response of an ice sheet to a load moving along the center of an ice channel at a constant speed has been studied by modeling the ice sheet as a Kelvin–Voigt viscoelastic sheet, using Fourier transforms along the channel direction and the orthogonal mode method along the direction of the channel width (Shishmarev et al., 2016). Korobkin et al. (2014) found the critical speed of the ice sheet by using the relationship between the maximum deformation of the ice sheet as a function of velocity, and they found that—due to the viscosity of the ice sheet—this critical velocity was slightly greater than the critical speed of the first-order FGW of an elastic sheet. Using these results, Khabakhpasheva et al. (2019) investigated the response of an ice sheet to a load moving along the center of the ice channel. They obtained a time-domain integral expression for the deformation of the ice when the load started from rest and then moved at a constant speed on a thin elastic plate by using the Fourier transform and orthogonal-mode methods, and they obtained the asymptotic steady state solution for the ice deformation as time tended to infinity by using the method of asymptotic analysis. They found that the large-time ice deformation consisted of a steady deflection, with standing waves in front of and behind the load in the coordinate system moving together with the load. However, they were not able to obtain an asymptotic steady-state solution for a load moving at the critical speed within a solid wall ice channel. Using the Kelvin–Voigt viscoelastic thin-plate model and linear potential-flow theory. Zeng (2022) further developed a mathematical model of a load with any functional form moving along the channel direction at a constant speed. When the distance y_0 between the center of the pressure surface and the ice channel (Figure 9) became zero, his result was reduced to that from the study by Shishmarev et al. (2016). The local deformation of the ice sheet and the maximum strain occurred in the vicinity of the load when it was moving at a subcritical speed, but the strain in the ice sheet near the wall gradually increased until it exceeded that near the load as the load gradually approached close to the wall. When the load was moving at the critical speed, it generated FGWs, as was demonstrated by the obvious periodic vibration of the ice sheet near the load. When the load was moving at a supercritical speed, however, higher-order FGWs contributed increasingly to the deformation of the ice sheet as the speed of the load increased. For a symmetrical load moving in the center of the channel, the ice sheet gradually formed multiple waves in the direction of the channel width as the speed of movement increased. In contrast, for an asymmetric load in the center of the channel, two rows of waves were formed behind the ice sheet at the center of the channel as the speed of the load increased, with their crests and troughs staggered, and the two rows of waves propagated farther and farther away. In addition, Zeng (2022) investigated the effects of the ice thickness on the maximum deflection of and strain in the ice. He found that the maximum deformation and strain of the ice plate decreased gradually as the thickness of the ice increased when the load was symmetrical about the center of the channel, while such a relation was not obtained when the load was asymmetrical about the center of the channel. Shishmarev et al. (2023) further studied the response of ice to a load moving with a constant speed along the center line in a channel. They assumed the thickness of the ice to vary linearly and symmetrically across the channel, being lowest at the center of the channel and highest at the channel walls. They showed that even a small variation in the thickness of the ice significantly changed the characteristics of the hydroelastic waves in the channel.

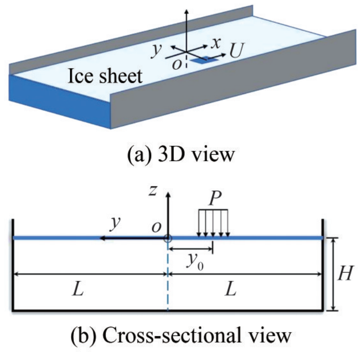


Figure 9 Sketch of a pressure surface moving on a channel covered by an ice sheet (Zeng, 2022)

Another condition of the ice in a channel involves the presence of cracks in the ice sheet. Different from the second critical speed $c_{cr,2} = \sqrt{gH}$ in an infinite ice sheet, which is independent of the ice parameters, the second critical speed in an ice channel is affected by the parameters of the ice plate, the channel parameters, and cracks in the ice sheet (Zeng, 2022). Zeng et al. (2022) studied the problem of FGWs produced by a uniform and rectangular pressure surface moving along an ice-covered channel with a crack in the center plane of the channel, using the Kelvin–Voigt model for the viscoelastic ice. The ice sheets on both sides of the crack were clamped to the walls of the channel, and the pressure surface was symmetric about the center line of the ice channel. The deflection of and strain in the ice sheet were stationary in the coordinate system moving together with the load. The investigators showed that the responses of ice sheets with cracks depended strongly on the delay time. The deflection of the ice and the strain along the crack reached a maximum at the first critical speed when the delay time was relatively small. For relatively large delay times, however, the maximum dimensionless deflection of the ice along the crack peaked exactly at the second critical speed (Figure 10(a)), while the strains along the crack achieved their maxima when the speed of the load was close to, but not equal to, the first critical speed. The minimum dimensionless ice deflection peaked near the second critical speed, and the 3D ice deflections differed significantly from those at the second critical speed (Figure 11), even though the speeds differed only by 0.01 m/s. Non-obvious extreme values of the maximum and minimum dimensionless ice deflection occurred near the first critical speed, and the speed corresponding to the extreme values was greater than the first critical speed (Figure 10) due to the viscosity parameters of the ice sheet. The results with no crack in the ice sheet were similar to those with a crack, but the critical speeds corresponding to the extreme deflections differed (Figure 10(b)). However, Shishmarev et al. (2016) found that the maximum deformation of the ice sheet occurred at the first critical speed for a load moving along a channel covered by a viscoelastic ice sheet without a crack. The reason for this was that their study considered a small water depth, which caused the first and second critical speeds to be close to each other.

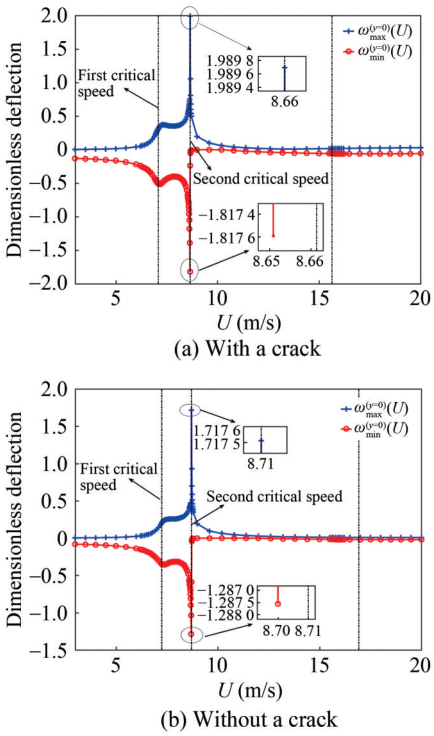


Figure 10 The maximum and minimum of the dimensionless ice deflection along $y = 0$ (Zeng et al., 2022)

To improve the ice-breaking ability of a load moving on an ice sheet with a crack, it is essential to find the parameters that affect the ice strains. The maximum strains in an ice sheet at the crack and at the channel wall do not decrease

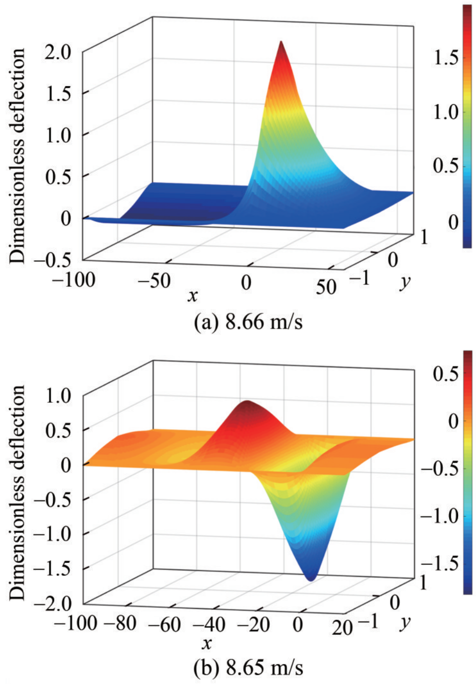


Figure 11 Dimensionless 3D ice deflections for the case in Figure 10(a) (Zeng et al., 2022)

monotonically as the width of the channel increases. As the ice thickness increases, the maximum strains decrease both along the crack and at the wall of the channel, which is the same as the trend when the water depth in a channel increases. Zeng et al. (2022) found that both the length and width of the pressure surface seriously affect its maximum ice-breaking ability. For a given total load and width of the pressure surface, the maximum ice-breaking ability gradually weakens when the load is elongated along the channel. On the other hand, for a given total load and length of the pressure surface, the strain in the ice sheet increases as the load is narrowed. However, the maximum ice-breaking ability of the pressure surface does not change significantly when its width is less than 1/10 of its length (Zeng et al., 2022).

Further damage of an ice crack can form a free surface of water in the ice channel, as shown in Figure 12. The problem of FGWs caused by a load moving along an ice channel on open water has not been adequately studied. Different from the propagation of waves along an ice channel completely covered by ice or in ice containing cracks, with one critical speed for each wave mode in each case, there may be multiple critical speeds for some modes in an ice channel with a lead of open water (Zeng et al., 2021b). It is hard to obtain a steady-state solution for the FGWs in an ice channel using common theoretical methods due to the infinite conditions of the free water region. Based on the treatment of the boundary conditions in Xue et al. (2021a), Zeng (2022) numerically studied the problem of a load moving at a constant speed along an ice channel with open water.

He used a Kelvin–Voigt model for the ice and clamped the ice sheet to the channel walls. He found that for the same physical parameters, the critical speed for FGWs under semi-ice−water conditions were less than that for FGWs under ice–water–ice conditions. The characteristics of the FGWs produced by a load moving at the same speed with two free surface conditions are thus much different than for a case with a single free surface. The maximum strain at the free boundary occurred near the first critical speed of the FGWs, and the maximum deformation at the free boundary occurred near the second critical speed of the FGWs, as did the maximum strain at the solid boundary.

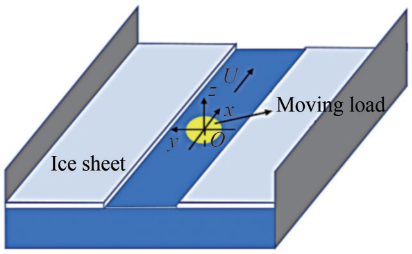


Figure 12 Sketch of a load moving on an open-water channel covered by an ice sheet (Zeng, 2022)

Present research thus yields the following results for the following three conditions: a fully covered ice channel, ice with a crack, and ice with open water. First, when a load is moving along an ice channel at a speed smaller than the first critical speed of the FGWs, the maximum strain in the ice sheet occurs either at the free boundary or right under the load. In contrast, when the speed is larger than the first critical speed, the maximum strain in the ice sheet occurs at the wall of the channel. However, the problem of the response of the ice to a load moving on an ice channel has not yet been studied sufficiently. The conditions of ice channels in real environments are even more complicated, and issues that remain to be explored—whether by theoretical, numerical, or experimental means—include considerations of different numbers and shapes of cracks, different edge conditions of the ice within the channel, etc.

3 Research on ice-breaking by loads moving under the ice

We next discuss ice breaking by a load moving under the ice, which is based on the theory of wave-making by the motion of an underwater object. The movement of the underwater object disturbs the surrounding flow field, and energy transfer from the flow field forms the moving underwater load that is applied under the ice sheet and stimulates the hydroelastic response of the ice sheet. The destruction of an ice sheet occurs when the stress inside the ice exceeds its limiting stress. Therefore, different from ice-breaking with a load moving above the ice, ice-breaking with a load moving under the ice is a non-contact ice-breaking method (Xue et al., 2021b). There exists a minimum energy for breaking the ice using this kind of method (Kozin et al., 2005).

At present, research on ice-breaking by FGWs induced by a load moving under an ice sheet is limited. The earliest theoretical research may have been performed by Kheisin (1967), who considered the 2D problem of the motion of a point vortex under a layer of broken ice. He found that the broken ice produced only minor changes in the gravity waves generated on the free surface by the motion of an underwater object. Cherkesov (1980) pointed out that the moving underwater objects also showed the formation characteristics of waves. Since then, theoretical studies of a load moving under the ice have been developing steadily. An ice sheet is considered to fracture when the strain exceeds its yield strain, which is the same as the condition employed in theoretical studies of a load moving above an ice sheet. There have been few numerical simulations of ice-breaking by FGWs produced by underwater moving loads (Li, 2023). The possibility that a load moving under an ice sheet may lead to its destruction was first confirmed by a model experiment carried out by Kozin and Onishchuk (1994). Recently, additional model experiments have been utilized to explore the ice-breaking phenomenon and the ability of FGWs to break ice caused by underwater moving loads. In this section, we provide a general review of research on the problem of ice deformation and breaking by FGWs generated by underwater objects moving under an ice sheet, especially from the perspective of the modes of motion of the underwater object, its geometrical features, and its trajectory of motion.

3.1 Modes of motion of underwater objects

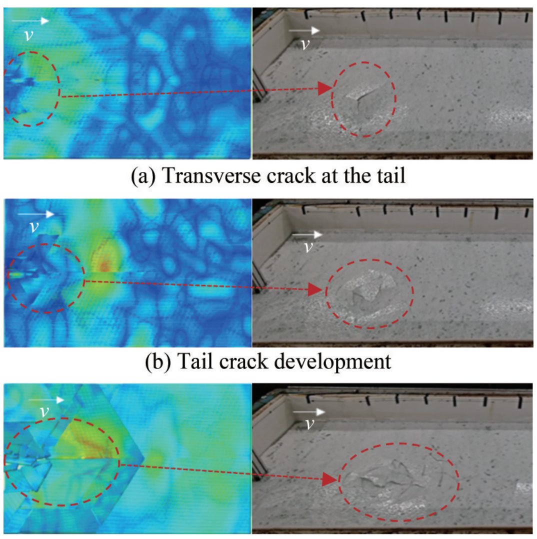
In theoretical studies, point sources have often been used as analytical objects to simulate the motion of an underwater load, and the effects on the response of the ice sheet have been explored for parameters such as the depth of immersion, speed of motion, etc., of an underwater object in low-speed steady motion (Bukatov and Zharkov, 1995; Kozin and Pogorelova, 2008; Pogorelova and Kozin, 2014b) or in variable speed motion (Pogorelova and Kozin, 2010; Pogorelova, 2011). Bukatov and Zharkov (1995) studied the perturbation problem of a point source of constant intensity moving with a small and constant speed in a heterogeneous liquid of finite depth under an elastic plate. Kozin and Pogorelova (2008) considered the hydrodynamic problem of a point source moving in an infinite depth of water under an elastic plate. Using the theory of complex variables and the method of integral transformations, they found that the inertial force of the ice sheet was negligible compared with its elastic force when the load was moving at high speed, while the inertial force cannot be ignored compared to the elastic force when the load is moving at low speed. Pogorelova and Kozin (2010) further investigated the effects of some parameters on the response of ice to a point source moving unsteadily in a liquid of finite depth under a floating elastic plate. They found that the deflection decreased when the ice thickness, Young's modulus, and the depth of immersion in the water increased (Pogorelova et al., 2019a). Based on an analytical model proposed by Kozin and Pogorelova (2008), Pogorelova (2011) further considered the influence of the acceleration, deceleration, and uniform motion of a point source on the ice deflection. She showed that a point source that remained in constant motion after accelerating did not produce an obvious maximum deflection near the critical speed, which was the minimum phase speed of FGWs propagating in deep water (Squire et al., 1996). The FGWs were not generated when the load moved at a speed smaller than the critical speed, and either acceleration or deceleration of the point source caused a temporary slight increase in the deflection of the ice sheet. Pogorelova and Kozin (2014b) studied the problem of the deflection of a floating elastic ice plate due to an underwater point source in a variable-depth basin. In a certain rather small region, the slope of the bottom surface affected the deflection of the ice. They showed that an increase in the plate thickness, or a decrease in basin depth and depth of submergence of the point source, led to an increase in the effect of the slope on the deflection of the ice plate. The basin depth was a more significant factor for the amplitude of the ice deflection than was the submergence depth.

In addition to these studies of point sources beneath an ice sheet, there have also been studies of an underwater cylinder that moves from stationarity to a state of constant acceleration (Kostikov et al., 2018) and of a source–sink system that moves in an unsteady motion (Pogorelova et al., 2012). It is common to simulate a moving cylinder or sphere by a moving dipole. The generation of FGWs by a 2D dipole moving under an ice sheet in a fluid of infinite depth was investigated by Savin and Savin (2012) and Il'ichev et al. (2012). They also considered the impulsive start of horizontal uniform motion by a dipole. Sturova (2013) investigated the disturbed velocity potential in the fluid caused by transient sources of arbitrary intensity in arbitrary 3D motion as well as the radiation of FGWs by an underwater sphere moving at a constant velocity. Her results showed that the hydrodynamic load of the underwater sphere depended mainly on its translational velocity and angular frequency as well as on parameters such as the thickness of the ice. Savin and Savin (2015) studied the 3D disturbance produced by a dipole moving at a constant speed under an ice sheet. They obtained the steady-state perturbation of the ice in the moving coordinate system as well as an analytical expression for the ice deflection after a long time of movement. They showed that the dipole can cause different ice sheet disturbances depending on the speed of the steady motion. If the speed of the dipole was smaller than a critical value, the largest perturbation occurred above the dipole, and it decayed rapidly with distance from the dipole without forming a distinct wave.

Considering the effects of viscosity and the boundary conditions of the ice sheet, Shishmarev et al. (2017; 2019) studied the strain distributed along an ice channel caused by the uniform motion of a dipole under a thin viscoelastic plate. They solved this problem by using a decoupled, two-step, linear approach. By ignoring the deflection of the ice sheet, they first determined the potential flow caused by an underwater dipole moving in a channel using the image method. This provided the hydrodynamic pressure on the rigid ice sheet in the channel. They then used the resulting hydrodynamic pressure as a forced load (Shishmarev et al., 2019) and solved for the deflection and strain by using Fourier transforms along the solid-wall ice channel and employing the normal mode method across the channel. They showed that the strain in the ice depended strongly on the critical speed at which the dipole was propagating along the ice channel for the hydroelastic wave. As the dipole moved along the channel to the wall, the strain in the wall closer to the dipole increased, while the strain in the other wall decreased (Shishmarev et al., 2017). The ice sheet can be broken either in front of the moving dipole or on the wall if the dipole moves close to the wall. The width of the channel also mattered in this result, even if it was much larger than the characteristic length of the ice sheet (Shishmarev et al., 2019). Pogorelova et al. (2023) investigated the influence of the viscous properties of ice—which she modeled using the Kelvin–Voigt model—on the wave resistance and lift force experienced by an underwater object moving in a liquid near an ice–water interface. The use of a viscoelastic model for the ice sheet resulted in a significant decrease in the maximum values of the wave resistance and lift coefficients compared to the use of an elastic plate model.

Relevant problems of the radiation and scattering of waves by an underwater cylinder stationary beneath an ice sheet with a uniform current have also been studied. (Das and Mandal, 2006; Chakraborty and Mandal, 2015; Maiti et al., 2015a; Li et al., 2019; Das and Sahu, 2019, 2020; Yang et al., 2021). Das and Mandal (2008) and Sturova (2012) modeled the underwater object as a 3D sphere. Semenov (2021) solved the nonlinear problem of a stationary 2D body in a uniform current. Ni et al. (2023) solved the nonlinear 2D problem of steady flow in a channel covered by broken ice with an arbitrary bottom topography that included a semi-circular obstruction, and Boral et al. (2023) solved the time-dependent problem of FGW scattering by an uneven bottom.

In numerical simulations—with the BEM used to treat the hydrodynamic part and the FDM to treat the elastic sheet equations—Kozin et al. (2010) studied the linear 3D problem of the deformation of ice caused by an underwater object moving at a uniform speed in infinitely deep water under an infinite floating ice sheet. Zemlyak et al. (2019a) investigated the influence of irregular bottom conditions in deep water of finite depth on the deformation of ice caused by the uniform movement of a rotating object. The bottom protrusions caused wave reflection and hydroshock, which aggravated the ice sheet deflection. An increase in ice deflection was also evident in the case of a smooth transition from deep water to shallow water (e.g., the presence of a sloping bottom), which is consistent with the results of Pogorelova and Kozin (2014a). In studies with numerical software, Hu (2019) established deformable ice sheets using the discrete element method and the FEM but without considering the fragmentation of the ice sheet. Bai et al. (2021), as well as Li et al. (2022), studied the hydrodynamic characteristics of an underwater object under a rigid ice sheet. Li (2023) studied crack propagation in an ice sheet caused by the uniform motion of an underwater object, modeling a breakable ice sheet by using the discrete element method. He found that transverse cracks first appeared before several longitudinal cracks. As the cracks extended, lateral and longitudinal cracks both developed fully throughout the ice sheet. The simulations agreed well with the results of model experiments carried out by Zhou et al. (2023), as shown in Figure 13.



(a) Transverse crack at the tail

(b) Tail crack development

(c) Fully extended cracks

Figure 13 Comparison of ice crack development caused by a body of revolution (numerical results on the left (Li, 2023) and experimental results on the right (Zhou et al., 2023))

3.2 Geometrical features and trajectories of motion

Similar to the way in which the length and width of a pressure surface seriously affect the maximum ice-breaking ability in research on ice-breaking by FGWs induced by a load moving above an ice sheet (Zeng et al., 2022), the geometric features of an underwater object also affect its ice-breaking ability. Theoretically, it has been common to consider the effect of the geometric profile by modeling the moving object with a circular profile (Sturova, 2015; Li et al., 2017d; Kostikov et al., 2018; Tkacheva, 2020; Stepanyants and Sturova, 2021a) or an elliptical profile (Korobkin et al., 2012; Tkacheva, 2015; Semenov, 2021), both of which limit the particularity of the mathematical model. The effects of its geometric features on the ice-breaking efficiency of an underwater object under an ice sheet have been explored through model experiments performed by Zemlyak et al. (2013, 2014, 2018, 2019b, 2022). The critical tangent of the ice sheet tilt angle α (the ratio of wave height to wavelength), above which the modeled ice sheet was considered to be broken, was chosen to be 0.04 in the work of Kozin et al. (2005). The deformation and destruction of the ice depend on the cross-sectional area, together with the form and relative elongation ratio, of the underwater object. An increase in the transverse cross-sectional area of the underwater model led to an increase in ice deflection, which increased the ice-breaking ability of the FGWs (Zemlyak et al., 2013; 2019b). Under the same working conditions, the form of an underwater object of rotation with a teardrop shape and moving at a critical speed caused strong fragmentation of an ice sheet (Zemlyak et al., 2014). An increased length of an underwater object can also be understood as an increased relative elongation ratio (Zemlyak et al., 2022), which causes an increase in the wavelength. This led to the formation of transverse main cracks and large-scale floating ice blocks after the destruction of the ice sheet, which reduced the ice-breaking efficiency (Zemlyak et al., 2019b). In addition, the changes in the cross-sectional area, the medium longitudinal section form, and the relative elongation ratio did not affect the value of the critical speed (Zemlyak et al., 2013; 2014; 2018).

A curved path of a load moving on the ice has a significant effect on the strain induced in the ice by the waves excited by the moving load (Johnsen et al., 2022). Different from the movement of a load above the ice, which is limited by space, the vast watershed space under the ice provides the possibility of more extensive and varied trajectories of motion of an underwater object beneath an ice sheet. However, the main body of research on ice cover deflections caused by a body moving under the ice has been carried out for straight horizontal motion. Lu and Dai (2008) analyzed the instantaneous vibrations of line, point, and ring sources that generated FGWs. Small horizontal, vertical, and rotational oscillations of underwater objects that cause ice deformations have also been studied by Maiti et al. (2015b), Tkacheva (2015), Sturova (2015), and Kostikov et al. (2018), who considered the object as a thin vertical plate, an elliptical cylinder, and a horizontal cylinder, respectively. Among these studies, Sturova (2015) modeled the ice sheet as a thin elastic plate with an infinite straight crack parallel to the axis of the cylinder. She found that the wave motion depended greatly on the position of the cylinder relative to the crack. Kostikov et al. (2018) proposed a special iterative algorithm to construct the small-time asymptotic solution for a cylinder. Stepanyants and Sturova (2021b) studied small periodic perturbations of the cylinder speed. They showed that the hydrodynamic load applied by an underwater object depends somewhat on its translation speed (Stepanyants and Sturova, 2021b) and oscillation frequency (Tkacheva, 2015; Stepanyants and Sturova, 2021b). Li et al. (2017d) investigated the small deflections produced by large oscillations of a submerged circular cylinder under an ice cover, including purely horizontal, vertical, and clockwise circular movements of the cylinder. Tkacheva (2020) studied the cylinder vibration problem in a finite-depth fluid under an elastic plate near a vertical wall. Shishmarev and Papin (2018) studied the periodic oscillations of a load under an ice channel, and Zemlyak et al. (2023) presented the prescribed lateral and angular acceleration maneuvers (pure sway and pure yaw) of an underwater object in an ice tank.

In the actual operation of underwater objects under ice, less research has been carried out for objects moving with variable speeds or variable trajectories. More complex modes of movement—such as a sinusoidal wave motion of an underwater body—can be expected to generate more complex FGWs, which may enhance the icebreaking ability of the body.

4 Summary and prospects

As a complex fluid – structure interaction problem, the FGWs of an ice sheet produced by a moving load are always hot topics in the hydrodynamics area. Such FGWs can be used to break the ice, which is of interest in the structural field. As a result, the deformation and breaking of ice by FGWs generated by a moving load has received greatly increasing attention in recent years. In this paper, we have reviewed the development of this problem and have identified some potentially helpful references.

1) The problem of the deformation and breaking of ice by FGWs produced by a load moving above a completely intact and infinite ice sheet has been well studied. This has laid the foundation for subsequent studies, e. g., by identifying the first and second critical speeds. Studies of the process of ice breaking have also appeared (Lu et al., 2012) that involve the complex phenomenon of ice fragmentation, which lies more in the structural area. We, therefore, recommend that fluid–structure interaction modeling should be developed further, especially for the deformation and fragmentation of ice.

2) A completely intact and infinite ice sheet can also involve more complex ice boundary conditions such as rigid walls, free surfaces, cracks, etc. These represent more realistic environmental conditions for navigation in ice-covered regions. In theoretical analyses, however, not all conditions can be solved due to the restrictions of the analytical methods. For example, a steady solution cannot be obtained for a load moving on a water surface confined between two ice sheets clamped by rigid walls because the condition of zero disturbance of the free surface at infinity cannot be satisfied concurrently. On the other hand, numerical simulations that can provide effective treatments for dealing with complex ice sheet boundary conditions should be considered further (Xue et al., 2021a; Zeng, 2022). The numerical method developed by Xue et al. (2021a) provides some techniques for solving such complex boundary conditions, and it may also be applicable to problems with loads moving uniformly under the ice. In experimental studies, it is currently still very hard to model very complex conditions.

3) Most theoretical and numerical studies have adopted uniform elastic or viscoelastic thin plate equations to simulate ice sheets. However, ice properties that are closer to reality should be considered in more studies of the deformation and breaking of ice by FGWs generated by moving loads, such as ice sheets with varying thicknesses (Shishmarev et al., 2023). Additionally, ice sheet models can be enhanced by adopting fifth-order, porous, sea-ice models (Meylan et al., 2017; Chen et al., 2019; Zheng et al., 2020; Wu et al., 2023a, 2023b) or by considering the addition of a viscous layer to the ice cover (Pogorelova et al., 2019b). Such models represent the real state of sea ice better (Ni et al., 2021), and they can be expected to provide more accurate and realistic simulations in icebreaking scenarios.

4) In comparison to the relatively abundant research on ice-breaking caused by loads moving above ice surfaces, the study of non-contact ice-breaking by loads moving underwater remains relatively limited. Most studies have focused on the response of the ice to the steady motion of an underwater object, using linear potential flow theory. More complex conditions need to be considered, for example, the nonlinear effects of boundary conditions and the unsteady motion of objects, including variable speeds and variable trajectories.

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