

Effect of Porosity on Wave Scattering by a Vertical Porous Barrier over a Rectangular Trench

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Received: 15 March 2023 / Accepted: 25 June 2023

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Abstract

The effect of porosity on surface wave scattering by a vertical porous barrier over a rectangular trench is studied here under the assumption of linearized theory of water waves. The fluid region is divided into four subregions depending on the position of the barrier and the trench. Using the Havelock's expansion of water wave potential in different regions along with suitable matching conditions at the interface of different regions, the problem is formulated in terms of three integral equations. Considering the edge conditions at the submerged end of the barrier and at the edges of the trench, these integral equations are solved using multi-term Galerkin approximation technique taking orthogonal Chebyshev's polynomials and ultra-spherical Gegenbauer polynomial as its basis function and also simple polynomial as basis function. Using the solutions of the integral equations, the reflection coefficient, transmission coefficient, energy dissipation coefficient and horizontal wave force are determined and depicted graphically. It was observed that the rate of convergence of the Galerkin method in computing the reflection coefficient, considering special functions as basis function is more than the simple polynomial as basis function. The change of porous parameter of the barrier and variation of trench width and height significantly contribute to the change in the scattering coefficients and the hydrodynamic force. The present results are likely to play a crucial role in the analysis of surface wave propagation in oceans involving porous barrier over submarine trench.

Keywords Water wave scattering; Rectangular trench; Vertical porous barriers; Havelock's inversion formula; Multi-term galerkin approximation; Reflection and transmission coefficients; Energy dissipation; Hydrodynamic force

1 Introduction

During the last few decades, the study of propagation of surface water waves over a rectangular trench was being considered by many researchers because of their possible applications in coastal and ocean engineering. It has a sig-

nificant role in understanding the characteristics of wave in harbour and offshore regions. The scattering of normally incident monochromatic plane progressive wave by rectangular submarine trench of constant depth containing two fluids of constant but different densities was studied by Lassiter (1972). Lee and Ayer (1981) investigated the effect of a symmetric rectangular trench by dividing the fluid domain into subregions. Miles (1982) used a conformal mapping to solve the trench problem for normal incidence of wave train. The diffraction of obliquely incident surface waves by an asymmetric trench was investigated using linearized potential theory by Kirby and Dalrymple (1983). They solved a set of integral equations derived by matching eigen function expansions of the velocity potentials. Later, Kirby and Dalrymple (1987) developed the theory of wave diffraction over a rectangular trench where currents flowing parallel to the trench boundary. The normal and oblique incidence by the two-dimensional wave scattering by a rectangular submarine trench was considered by Chakraborty and Mandal (2014; 2015) reducing the problem to solving appropriate integral equation which is solved by multi-term Galerkin approximation involving ultraspherical Gegenbauer polynomials. Later, Roy et al. (2017) considered the problem of water wave scattering by an asymmetric rectangular trench using an approach similar to Chakraborty and Mandal (2014; 2015).

Article Highlights

- Considering linear theory, problem of water wave scattering by porous thin submerged plate over rectangular trench is studied.
- Two different positions of the barrier are considered.
- A multi term Galerkin approximation technique together with taking orthogonal polynomials as well as simple polynomial as basis function. To test the rate of convergence compares the both results numerically.
- The reflection coefficient, transmission coefficient, energy dissipation coefficient and horizontal wave force are determined and depicted graphically.
- Porous parameter of the barrier, variation of trench width and height affect the energy coefficients significantly.

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Interaction of water waves with thin plate assuming the linear theory has been a subject of considerable interest as this phenomenon serves as a model for a wide range of physical situations which include wave interaction with breakwaters, very large floating structures. Breakwaters are coastal structures which are widely constructed to reduce the wave action in inshore water and thereby reduce the coastal erosion and protect a port or harbour from the effect of rough sea. Usually the breakwaters are mathematically modeled as rigid impermeable thin vertical plate either partially immersed or submerged in ocean. A number of researchers were engaged in the study to handle the boundary value problem associated with the study of water wave scattering by a thin rigid vertical plate present in ocean with free surface and consequently many sophisticated mathematical concepts have evolved. Dean (1945), Ursell (1947) and Evans (1970) worked with such types of problems. It may be noted here that exact solution of the afore-said boundary value problem exists when the barrier is in the form of a rigid vertical plate present in the deep ocean and for normal incidence of the incoming wave train. In all other cases only approximate analytical or numerical methods are used to obtain approximate solution. The scattering problem involving a partially immersed and submerged rigid plate using first kind of hypersingular integral equation was studied by Parson and Martin (1992; 1994). Based on application of Green's integral theorem they have reduced the boundary value problem to a solution of first kind hypersingular integral equation where the unknown function is the difference of velocity potential across the plate. The hypersingular integral equation was then solved by using collocation method based on approximating the unknown function satisfying the integral equation Chebyshev's polynomial. The problem of wave diffraction by rigid vertical barrier in finite depth water was considered by Goswami (1983). Goswami (1983) employed Green's function technique to reduce the scattering problem involving a fixed vertical rigid plate submerged in uniform finite depth water to an integral equation. The integral equation was then solved approximately by using a perturbation method and reflection and transmission coefficients are thereby obtained. Losada et al. (1992) obtained the reflection and transmission coefficients of the above mentioned problem by using an eigenfunction expansion method. Later on, Porter and Evans (1995) considered oblique wave scattering by a thin vertical rigid barrier in uniform finite depth water having four basic configurations namely, a surface piercing barrier, a bottom standing barrier, a barrier with a gap and a totally submerged barrier. For each case, they have used an approximate method based on the Galerkin approximation. Also, Mandal and Dolai (1994) considered oblique water wave scattering by thin vertical barrier in uniform finite depth water. They employed one-term Galerkin approximation to evaluate the upper and lower bounds

for the transmission and reflection coefficients.

During the later half of twentieth century, study of wave interaction with porous coastal structures like rubble mound breakwaters became important in coastal engineering as the structural voids in the porous breakwaters can dissipate wave energy efficiently. Also in coastal engineering, porous breakwaters drawn special attention to the scientist and researchers because of the rigid breakwaters were collapsed due to huge load, which were mainly constructed to protect harbours and coastal villages. Porous structures in the form of thin permeable barriers have been used to dissipate and reflect incoming wave energy from sea. Also, a porous breakwaters are moved eco-friendly as water can pass through the holes which helps to protect marine environment. For low engineering cost, porous breakwaters are more useful with respect rigid one. Mathematical modeling of porous structure as thin porous vertical wave maker was pioneered by Chwang (1983). Based on the model of Solitt and Cross (1972), Yu (1995) examined the wave diffraction by a semi-infinite porous barriers. The rapid convergence of the solution to the problem of water wave scattering by thin vertical porous barriers utilizing multi-term Galerkin approximation was examined by Li et al. (2015). Roy et al. (2016a; 2016b) analyzed water wave scattering by two unequal vertical barriers and two submerged plates by using Galerkin's approximation technique. More recently, Ray et al. (2021) considered the wave scattering problem by thin rigid plate above the rectangular trench and Sarkar et al. (2022) analyzed oblique wave scattering by two thin rigid plate over an asymmetrical trench. The above mentioned problems were solved by considering multiterm Galerkin approximation technique using simple polynomial as basis functions. But as per authors knowledge, no research work is done when porosity is present in vertical plate. Present of porosity is actually more effective in dissipating wave energy and it helps to reduce wave force on the barrier [cf. Manam and Sivanesan (2016)].

In the present paper, water wave scattering by a rectangular submarine trench in presence of a thin vertical porous barrier are investigated. Here we consider that the porous barrier positioned in the right side, at a distance ' l ' from the middle of the trench. We divide the whole fluid region into four subregions and using Havelock's inversion formula in four subregions and matching conditions at the interfaces of these subregions we construct three Fredholm type integral equations. To solve these integral equations we use multi-term Galerkin approximation technique taking orthogonal Chebyshev's polynomials and ultra-spherical Gegenbauer polynomial as its basis function due to the edge conditions at the submerged end of the barrier and at the edge of the trench respectively. We have also consider simple polynomial as basis function to solve the integral equation as done in Ray et al. (2021). Then the reflection coefficient and the transmission coefficient are determined

in terms of the solutions of the integral equations. Due to porosity, how the proposed system affects the reflection coefficient, transmission coefficient, energy dissipation, wave force acting on the structure are discussed in numerical section. In the absence of porosity, known result evaluated by Ray et al. (2021) is recovered. We may mention here that the use of special function as basis function in multi term Galerkin approximation gives better rate of convergence of the method than using simple polynomial as basis function as in the work of Ray et al. (2021).

2 Problem construction

We consider two dimensional, time harmonic, irrotational motion in water due to interaction of a wave train incident normally on a thin vertical porous barrier partially immersed in water region with a submarine trench at the bottom. We choose rectangular cartesian coordinate system where x -axis is along the mean free surface and y -axis is taken vertically downwards into the fluid region. A rectangular submarine trench symmetrical about y -axis, of width ' $2b$ ' and depth ' c ' below the mean free surface is situated at the bottom of water region so that water occupies the region $(-\infty < x < b, 0 < y < h) + (-b < x < b, 0 < y < c) + (b < x < \infty, 0 < y < h)$. A porous thin vertical barrier is situated along a vertical line $x = l$, which is partially immersed upto a depth ' a ' below the mean free surface so that configuration of the barrier is given by $x = l, 0 < y < a; l < b$. A train of time harmonic waves from negative infinity with angular frequency σ represented by the velocity potential function $\text{Re}[\varphi^{\text{inc}}(x, y)e^{-i\sigma t}]$, is incident normally on the barrier and is partially reflected and partially transmitted below the barrier so that R and T are the amplitudes of the reflected and transmitted waves respectively. A schematic diagram of the problem is shown in Figure 1.

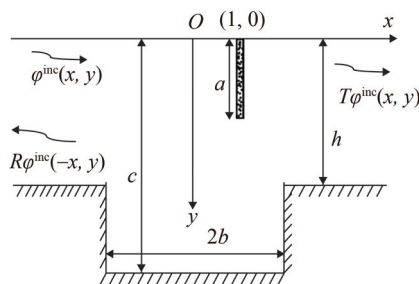


Figure 1 Schematic diagram

Here

$$\varphi^{\text{inc}}(x, y) = \frac{2 \cosh \alpha_0(h-y) e^{-i\alpha_0(x-b)}}{\cosh \alpha_0 h} \quad (1)$$

where α_0 is the unique real positive root of dispersion relation

$$\alpha \tanh \alpha h = K \quad (2)$$

where $K = \sigma^2/g$, g is the acceleration due to gravity.

To study the problem under consideration we divide the fluid region into four subregions, viz, R_1, R_2, R_3, R_4 (see Figure 2), where

$$R_1 \equiv (-\infty < x < -b, 0 < y < h)$$

$$R_2 \equiv (-b < x < l, 0 < y < c)$$

$$R_3 \equiv (l < x < b, 0 < y < c)$$

$$R_4 \equiv (b < x < \infty, 0 < y < h)$$

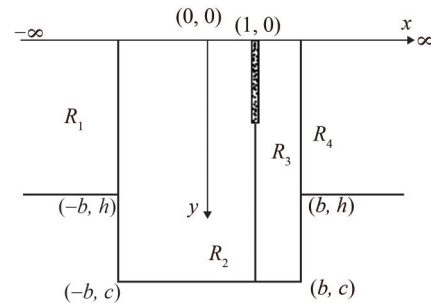


Figure 2 Subregions of the fluid domain

Let the resulting motion in the fluid be described by the velocity potential $\text{Re}[\varphi_j(x, y)e^{-i\sigma t}]$, then $\varphi_j(x, y)$ satisfies

$$\nabla^2 \varphi_j = 0, \text{ in the fluid domain} \quad (3)$$

the linearized free surface condition

$$\left(\frac{\partial}{\partial y} + K\right)\varphi_j = 0 \text{ on } y = 0, -\infty < x < \infty \quad (4)$$

the bottom boundary condition

$$\varphi_{jy} = 0 \text{ on } \begin{cases} y = c, j = 2, 3 \\ y = h, j = 1, 4 \end{cases} \quad (5)$$

the conditions on the two sides of the trench

$$\varphi_{jx} = 0 \text{ on } x = \pm b, j = 2, 3, h < y < c \quad (6)$$

the edge conditions

$$r_1^{\frac{1}{3}} \nabla \varphi_1, r_2^{\frac{1}{2}} \nabla \varphi_2, r_3^{\frac{1}{3}} \nabla \varphi_3 \text{ are bounded as } r_1, r_2, r_3 \rightarrow 0 \quad (7)$$

where r_1, r_2, r_3 are the distances from the submerged left edge of the trench, the submerged edge of the barrier and the submerged right edge of the trench respectively so that

$$r_1^2 = (x+b)^2 + (y-h)^2$$

$$r_2^2 = (x-l)^2 + (y-a)^2$$

$$r_3^2 = (x-b)^2 + (y-h)^2$$

The condition on the porous barrier surface is given by

$$\frac{\partial \varphi_2}{\partial x} = \frac{\partial \varphi_3}{\partial x} = -i\alpha_0 G(\varphi_3 - \varphi_2) \text{ on } x = l, 0 < y < a \quad (8)$$

Here $G = G' + iG''$ is the dimensionless porous parameter given by $G = \frac{\delta(f + iS)}{K\tau(f^2 + S^2)}$, where δ is the porosity, f is

the resistance force coefficient, S is the inertial force coefficient and τ is the thickness of the porous medium. The real part G^r represents the resistance force coefficient and the imaginary part G^i represents the inertial force coefficient of the porous material. The quantity G^r resists passage of fluid through the pores while G^i allows the fluid through the pores of the porous material. When $G^i \ll G^r$, i.e., when the inertial force coefficient is much less than the resistance force coefficient then G is considered as real.

The continuity of normal velocity and pressure across the free-flowing interfaces yields

$$\begin{aligned} \frac{\partial \varphi_1}{\partial x} &= \frac{\partial \varphi_2}{\partial x} \text{ and } \varphi_1 = \varphi_2, \text{ for } x = -b, y \in (0, h) \\ \frac{\partial \varphi_2}{\partial x} &= \frac{\partial \varphi_3}{\partial x}, \text{ for } x = l, y \in (0, c) \\ \varphi_2 &= \varphi_3, \text{ for } x = l, y \in (a, c) \\ \frac{\partial \varphi_3}{\partial x} &= \frac{\partial \varphi_4}{\partial x} \text{ and } \varphi_3 = \varphi_4, \text{ for } x = b, y \in (0, h) \end{aligned} \quad (9)$$

The far field behavior of the potential functions are described by

$$\begin{aligned} \varphi_1(x, y) &\sim \varphi^{\text{inc}}(x, y) + R\varphi^{\text{inc}}(-x, y) \text{ as } x \rightarrow -\infty \\ \varphi_4(x, y) &\sim T\varphi^{\text{inc}}(x, y) \text{ as } x \rightarrow \infty \end{aligned} \quad (10)$$

where R and T are the unknown complex reflection and transmission coefficients respectively to be determined.

3 Method of solution

By Havelock's expansion of water wave potential, the eigen function expansions of $\varphi_j(x, y)$ satisfying (3) to (9) in the different regions R_j , $j = 1, 2, 3, 4$ are given below.

$$\begin{aligned} \varphi_1(x, y) &= \{e^{i\alpha_0(x+b)} + Re^{-i\alpha_0(x-b)}\}\psi_0(y) \\ &\quad + \sum_{n=1}^{\infty} A_n e^{\alpha_n(x+b)} \psi_n(y) \\ \varphi_2(x, y) &= \{B_0 \cos \lambda_0 x + C_0 \sin \lambda_0 x\} \chi_0(y) \\ &\quad + \sum_{n=1}^{\infty} \{B_n \cosh \lambda_n x + C_n \sinh \lambda_n x\} \chi_n(y) \\ \varphi_3(x, y) &= \{D_0 \cos \lambda_0 x + E_0 \sin \lambda_0 x\} \chi_0(y) \\ &\quad + \sum_{n=1}^{\infty} \{D_n \cosh \lambda_n x + E_n \sinh \lambda_n x\} \chi_n(y) \\ \varphi_4(x, y) &= Te^{i\alpha_0(x-b)} \psi_0(y) + \sum_{n=1}^{\infty} F_n e^{-\alpha_n(x-b)} \psi_n(y) \end{aligned} \quad (11)$$

where $\{A_n\}_{n=1}^{\infty}$, $\{B_n\}_{n=0}^{\infty}$, $\{C_n\}_{n=0}^{\infty}$, $\{D_n\}_{n=0}^{\infty}$, $\{E_n\}_{n=0}^{\infty}$, $\{F_n\}_{n=1}^{\infty}$, R , T are unknowns to be determined. Here

$$\begin{aligned} \psi_0(y) &= \frac{\cosh \alpha_0(h-y)}{\cosh \alpha_0 h}, \psi_n(y) = \frac{\cos \alpha_n(h-y)}{\cos \alpha_n h}, n = 1, 2, \dots \\ \chi_0(y) &= \frac{\cosh \alpha_0(c-y)}{\cosh \alpha_0 c}, \chi_n(y) = \frac{\cos \alpha_n(c-y)}{\cos \alpha_n c}, n = 1, 2, \dots \end{aligned} \quad (12)$$

and $\alpha_0, \pm i\alpha_n$ are the roots of the Equation (2) and $\lambda_0, \pm i\lambda_n$ are the roots of the equation

$$\lambda \tanh \lambda c = K \quad (13)$$

For determination of the unknowns $A'_n, B'_n, C'_n, D'_n, E'_n, F'_n, R$ and T we proceed as follows. Let us define

$$\begin{aligned} f_j(y) &= \left[\frac{\partial \varphi_j}{\partial x} \right]_{x=-b, l, b} = \left[\frac{\partial \varphi_{j+1}}{\partial x} \right]_{x=-b, l, b} \quad j = 1, 2, 3 \\ g_j(y) &= [\varphi_{j+1} - \varphi_j]_{x=-b, l, b} \quad j = 1, 2, 3 \end{aligned} \quad (14)$$

where $y \in (0, h)$ for $j = 1, 3$ and $y \in (0, c)$ for $j=2$. Using the representation (11) in the Equation (14), we obtain

$$\begin{aligned} f_1(y) &= i\alpha_0 \{1 - R\} \psi_0(y) + \sum_{n=1}^{\infty} \alpha_n A_n \psi_n(y) \\ &= \lambda_0 \{B_0 \sin \lambda_0 b + C_0 \cos \lambda_0 b\} \chi_0(y) \\ &\quad + \sum_{n=1}^{\infty} \lambda_n \{-B_n \sinh \lambda_n b + C_n \cosh \lambda_n b\} \chi_n(y) \\ g_1(y) &= \{B_0 \cos \lambda_0 b - C_0 \sin \lambda_0 b\} \chi_0(y) \\ &\quad + \sum_{n=1}^{\infty} \{B_n \cosh \lambda_n b - C_n \sinh \lambda_n b\} \chi_n(y) \\ &\quad - \{1 + R\} \psi_0(y) - \sum_{n=1}^{\infty} A_n \psi_n(y) \\ f_2(y) &= \lambda_0 \{-B_0 \sin \lambda_0 l + C_0 \cos \lambda_0 l\} \chi_0(y) \\ &\quad + \sum_{n=1}^{\infty} \lambda_n \{B_n \sinh \lambda_n l + C_n \cosh \lambda_n l\} \chi_n(y) \\ &= \lambda_0 \{-D_0 \sin \lambda_0 l + E_0 \cos \lambda_0 l\} \chi_0(y) \\ &\quad + \sum_{n=1}^{\infty} \lambda_n \{D_n \sinh \lambda_n l + E_n \cosh \lambda_n l\} \chi_n(y) \\ g_2(y) &= \{D_0 \cos \lambda_0 l + E_0 \sin \lambda_0 l\} \chi_0(y) \\ &\quad + \sum_{n=1}^{\infty} \{D_n \cosh \lambda_n l + E_n \sinh \lambda_n l\} \chi_n(y) \\ &\quad - \{B_0 \cos \lambda_0 l + C_0 \sin \lambda_0 l\} \chi_0(y) \\ &\quad - \sum_{n=1}^{\infty} \{B_n \cosh \lambda_n l + C_n \sinh \lambda_n l\} \chi_n(y) \\ f_3(y) &= \lambda_0 \{-D_0 \sin \lambda_0 b + E_0 \cos \lambda_0 b\} \chi_0(y) \\ &\quad + \sum_{n=1}^{\infty} \lambda_n \{D_n \sinh \lambda_n b + E_n \cosh \lambda_n b\} \chi_n(y) \\ &= i\alpha_0 T \psi_0(y) - \sum_{n=1}^{\infty} \alpha_n F_n \psi_n(y) \\ g_3(y) &= T \psi_0(y) + \sum_{n=1}^{\infty} F_n \psi_n(y) \\ &\quad - \{D_0 \cos \lambda_0 b + E_0 \sin \lambda_0 b\} \chi_0(y) \\ &\quad - \sum_{n=1}^{\infty} \{D_n \cosh \lambda_n b + E_n \sinh \lambda_n b\} \chi_n \end{aligned} \quad (15)$$

Now applying the Havelock's inversion formula to Equation (15) we obtain the unknown constants as,

$$1 - R = \frac{-4i \cosh \alpha_0 h}{\delta_0} \int_0^h f_1(u) \cosh \alpha_0(h - u) du \quad (16)$$

$$A_n = \frac{4 \cos \alpha_n h}{\delta_n} \int_0^h f_1(u) \cos \alpha_n(h - u) du \quad (17)$$

$$B_0 = \frac{4 \cosh \lambda_0 c}{\gamma_0 \sin \lambda_0(b + l)} \left[\cos \lambda_0 l \int_0^h f_1(u) \cosh \lambda_0(c - u) du - \cos \lambda_0 b \int_0^c f_2(u) \cosh \lambda_0(c - u) du \right] \quad (18)$$

$$B_n = \frac{4 \cos \lambda_n c}{\gamma_n \sinh \lambda_n(b + l)} \left[\cosh \lambda_n l \int_0^c f_2(u) \cos \lambda_n(c - u) du - \cosh \lambda_n l \int_0^h f_1(u) \cos \lambda_n(c - u) du \right] \quad (19)$$

$$C_0 = \frac{4 \cosh \lambda_0 c}{\gamma_0 \sin \lambda_0(b + l)} \left[\sin \lambda_0 l \int_0^h f_1(u) \cosh \lambda_0(c - u) du + \sin \lambda_0 b \int_0^c f_2(u) \cosh \lambda_0(c - u) du \right] \quad (20)$$

$$C_n = \frac{4 \cos \lambda_n c}{\gamma_n \sinh \lambda_n(b + l)} \left[\sinh \lambda_n l \int_0^h f_1(u) \cos \lambda_n(c - u) du + \sinh \lambda_n b \int_0^c f_2(u) \cos \lambda_n(c - u) du \right] \quad (21)$$

$$D_0 = \frac{4 \cosh \lambda_0 c}{\gamma_0 \sin \lambda_0(b - l)} \left[\cos \lambda_0 b \int_0^c f_2(u) \cosh \lambda_0(c - u) du - \cos \lambda_0 l \int_0^h f_3(u) \cosh \lambda_0(c - u) du \right] \quad (22)$$

$$D_n = \frac{-4 \cos \lambda_n c}{\gamma_n \sinh \lambda_n(b - l)} \left[\cosh \lambda_n b \int_0^c f_2(u) \cos \lambda_n(c - u) du - \cosh \lambda_n l \int_0^h f_3(u) \cos \lambda_n(c - u) du \right] \quad (23)$$

$$E_0 = \frac{4 \cosh \lambda_0 c}{\gamma_0 \sin \lambda_0(b - l)} \left[\sin \lambda_0 b \int_0^c f_2(u) \cosh \lambda_0(c - u) du - \sin \lambda_0 l \int_0^h f_3(u) \cosh \lambda_0(c - u) du \right] \quad (24)$$

$$E_n = \frac{4 \cos \lambda_n c}{\gamma_n \sinh \lambda_n(b - l)} \left[\sinh \lambda_n b \int_0^c f_2(u) \cos \lambda_n(c - u) du - \sinh \lambda_n l \int_0^h f_3(u) \cos \lambda_n(c - u) du \right] \quad (25)$$

$$T = \frac{-4i \cosh \alpha_0 h}{\delta_0} \int_0^h f_3(u) \cosh \alpha_0(h - u) du \quad (26)$$

$$F_n = \frac{-4 \cos \alpha_n h}{\delta_n} \int_0^h f_3(u) \cos \alpha_n(h - u) du \quad (27)$$

where $\delta_0 = 2\alpha_0 h + \sinh 2\alpha_0 h$; $\delta_n = 2\alpha_n h + \sin 2\alpha_n h$; $\gamma_0 = 2\lambda_0 c + \sinh 2\lambda_0 c$; $\gamma_n = 2\lambda_n c + \sin 2\lambda_n c$ ($n = 1, 2, \dots$).

Thus all the unknown constants are expressed in terms of the function $f_j(y)$ and $g_j(y)$, $j = 1, 2, 3$ so that once $f_j(y)$ and $g_j(y)$, $j = 1, 2, 3$ are known then the problem is solved. In the next section we proceed to determine $f_j(y)$, $j = 1, 2, 3$ through integral equation formulation.

3.1 Formation of integral equation

In this section we will derive integral equations for $f_j(y)$, $j = 1, 2, 3$. For this we first consider the continuity of $\varphi_j(x, y)$ across the gaps in the boundaries of different regions as given in relation (9). This gives

$$\begin{aligned} \varphi_1(-b-0, y) &= \varphi_2(-b+0, y), 0 < y < h \\ \varphi_2(l-0, y) - \varphi_3(l+0, y) &= -\frac{i}{\alpha_0 G} f_2(y), 0 < y < a \\ \varphi_2(l-0, y) - \varphi_3(l+0, y) &= 0, a < y < c \\ \varphi_3(b-0, y) &= \varphi_4(b+0, y), 0 < y < h \end{aligned} \quad (28)$$

These provides the three integral equations given by

$$\begin{aligned} \int_0^h p_1(u) L_j(y, u) du + \int_0^c p_2(u) M_j(y, u) du \\ + \int_0^h p_3(u) N_j(y, u) du = X_j, j = 1, 2, 3 \end{aligned} \quad (29)$$

where $X_1 = \psi_0(y)$, $X_2 = q_2(y)$, $X_3 = 0$; $p_i(u) = \frac{g_i(u)}{1+R}$, $i = 1, 2, 3$.

$$q_2(u) = \frac{f_2(u)}{1+R} \quad (30)$$

$$\begin{aligned} L_1(y, u) &= -\sum_{n=1}^{\infty} \frac{4 \cos \alpha_n(h-y) \cos \alpha_n(h-u)}{\delta_n} \\ &+ \frac{4 \cosh \lambda_0(c-y) \cosh \lambda_0(c-u)}{\gamma_0 \tan \lambda_0(b+l)} \\ &- \sum_{n=1}^{\infty} \frac{4 \cos \lambda_n(c-y) \cos \lambda_n(c-u)}{\gamma_n \tanh \lambda_n(b+l)} \end{aligned} \quad (31)$$

$$\begin{aligned} M_1(y, u) &= -\frac{4 \cosh \lambda_0(c-y) \cosh \lambda_0(c-u)}{\gamma_0 \sin \lambda_0(b+l)} \\ &+ \sum_{n=1}^{\infty} \frac{4 \cos \lambda_n(c-y) \cos \lambda_n(c-u)}{\gamma_n \sinh \lambda_n(b+l)} \end{aligned} \quad (32)$$

$$N_1(y, u) = 0 \quad (33)$$

$$L_2(y, u) = -\frac{4 \cosh \lambda_0(c-y) \cosh \lambda_0(c-u)}{\gamma_0 \sin \lambda_0(b+l)} + \sum_{n=1}^{\infty} \frac{4 \cos \lambda_n(c-y) \cos \lambda_n(c-u)}{\gamma_n \sinh \lambda_n(b+l)} \quad (34)$$

$$M_2(y, u) = \frac{4 \cosh \lambda_0(c-y) \cosh \lambda_0(c-u)}{\gamma_0 \tan \lambda_0(b-l)} + \frac{4 \cosh \lambda_0(c-y) \cosh \lambda_0(c-u)}{\gamma_0 \tan \lambda_0(b+l)} - \sum_{n=1}^{\infty} \frac{4 \cos \lambda_n(c-y) \cos \lambda_n(c-u)}{\gamma_n \tanh \lambda_n(b-l)} - \sum_{n=1}^{\infty} \frac{4 \cos \lambda_n(c-y) \cos \lambda_n(c-u)}{\gamma_n \tanh \lambda_n(b+l)} \quad (35)$$

$$N_2(y, u) = -\frac{4 \cosh \lambda_0(c-y) \cosh \lambda_0(c-u)}{\gamma_0 \sin \lambda_0(b-l)} + \sum_{n=1}^{\infty} \frac{4 \cos \lambda_n(c-y) \cos \lambda_n(c-u)}{\gamma_n \sinh \lambda_n(b-l)} \quad (36)$$

$$L_3(y, u) = 0 \quad (37)$$

$$M_3(y, u) = -\frac{4 \cosh \lambda_0(c-y) \cosh \lambda_0(c-u)}{\gamma_0 \sin \lambda_0(b-l)} + \sum_{n=1}^{\infty} \frac{4 \cos \lambda_n(c-y) \cos \lambda_n(c-u)}{\gamma_n \sinh \lambda_n(b-l)} \quad (38)$$

$$N_3(y, u) = \frac{4 \cosh \lambda_0(c-y) \cosh \lambda_0(c-u)}{\gamma_0 \tan \lambda_0(b-l)} - \sum_{n=1}^{\infty} \frac{4 \cos \lambda_n(c-y) \cos \lambda_n(c-u)}{\gamma_n \tanh \lambda_n(b-l)} - \frac{4i \cosh \alpha_0(h-y) \cosh \alpha_0(h-u)}{\delta_0} - \sum_{n=1}^{\infty} \frac{4 \cos \alpha_n(h-y) \cos \alpha_n(h-u)}{\delta_n} \quad (39)$$

3.2 Solution of integral equation

We solve the integral Equations (29) using the multi-term Galerkin approximation method. For this we write the unknown functions $p_i(y)$, $i = 1, 2, 3$ as

$$p_1(y) = \sum_{n=0}^N a_n u_n(y), 0 < y < h \quad (40)$$

$$p_2(y) = \sum_{n=0}^N b_n z_n(y), 0 < y < c \quad (41)$$

where $z_n(y) = v_n(y)$ for $0 < y < a$ and $z_n(y) = x_n(y)$ for $a < y < c$.

$$p_3(y) = \sum_{n=0}^N c_n w_n(y), 0 < y < h \quad (42)$$

$$q_2(y) = \sum_{n=0}^N b_n v_n(y), 0 < y < a$$

where the basis functions $u_n(y)$ for $0 < y < h$ and $w_n(y)$ for $0 < y < h$ are chosen in terms of ultraspherical Gegenbauer polynomials of order $1/6$ and $z_n(y)$ for $0 < y < a$ are chosen in terms of orthogonal Chebyshev polynomial of order $2n$ with suitable weights respectively. The choice of basis functions depends on the types of singularities at the corners of the trench and at the submerged sharp edge of the barrier as given by the edge condition (7). Here we also consider simple polynomial as basis function to solve the integral equation. The basis function in various intervals are given below.

3.3 Basis functions

3.3.1 Approximation of basis functions in terms of orthogonal polynomials

In each integral Equation of (29) there are three integrals where the unknown functions are $p_1(y)$, $p_2(y)$, $p_3(y)$ in first, second and third integral respectively. As given in Equations (40) to (42), these unknown function are expanded in terms of suitable basis functions. The choice of basis functions are explained in section 3.2 and are given below.

For the first integral in (29) we write,

$$u_n(y) = -\frac{d}{dy} \left[e^{-Ky} \int_y^h e^{-Kt} \tilde{u}_n(t) dt \right], 0 < y < h \quad (43)$$

We choose the basis functions in terms of $\tilde{u}_n(y)$ as follows

$$\tilde{u}_n(y) = \frac{2^{\frac{7}{6}} \Gamma\left(\frac{1}{6}\right) (2n)!}{\pi \Gamma\left(2n + \frac{1}{3}\right) (h)^{\frac{1}{3}} (h^2 - y^2)^{\frac{1}{3}}} C_{2n}^{\frac{1}{6}}\left(\frac{y}{h}\right), 0 < y < h \quad (44)$$

where $C_{2n}^{\frac{1}{6}}\left(\frac{y}{h}\right)$ are Gegenbauer polynomials of order $1/6$.

For the second integral in (29) we write,

$$v_n(y) = -\frac{d}{dy} \left[e^{-Ky} \int_0^a e^{-Kt} \tilde{v}_n(t) dt \right], 0 < y < a$$

$$x_n(y) = -\frac{d}{dy} \left[e^{-Ky} \int_a^c e^{-Kt} \tilde{x}_n(t) dt \right], a < y < c \quad (45)$$

We choose the basis functions in terms of $\tilde{v}_n(y)$ as follows

$$\begin{aligned}\tilde{v}_n(y) &= \frac{2(-1)^n}{\pi(2n+1)ah} (a^2 - y^2)^{\frac{1}{2}} U_{2n}\left(\frac{y}{a}\right), 0 < y < a \\ \tilde{x}_n(y) &= \frac{2(-1)^n}{\pi\{(h-y)^2 - (h-a)^2\}^{\frac{1}{2}}} T_{2n}\left(\frac{h-y}{h-a}\right), a < y < c\end{aligned}\quad (46)$$

where $T_{2n}(y)$ and $U_{2n}(y)$ are Chebychev's polynomial of first and second kind of order $2n$.

For the third integral in (29) we write,

$$w(y) = -\frac{d}{dy} \left[e^{-ky} \int_y^h e^{-kt} \tilde{w}_n(t) dt \right], 0 < y < h \quad (47)$$

Here also we choose the $\tilde{w}_n(y)$ in terms of Gegenbauer polynomials of order $1/6$ as given below.

$$\tilde{w}_n(y) = \frac{2^{\frac{7}{6}} \Gamma\left(\frac{1}{6}\right) (2n)!}{\pi \Gamma\left(2n + \frac{1}{3}\right) (h)^{\frac{1}{3}} (h^2 - y^2)^{\frac{1}{3}}} C_{2n}^{\frac{1}{6}}\left(\frac{y}{h}\right), 0 < y < h \quad (48)$$

Thus the unknown functions satisfying the integral Equation (29) are expanded in terms of suitable orthogonal polynomials as shown in Equations (43) to (48). However the unknown functions satisfying the integral Equation (29) can also be expanded in terms of simple polynomials as shown below.

3.3.2 Approximation of basis functions in terms of simple polynomials

For the first integral in (29) we write,

$$u_n(y) = \left(\frac{h}{h-y}\right)^{\frac{1}{3}} \left(\frac{y}{h}\right)^n, 0 < y < h \quad (49)$$

For the second integral in (29) we write,

$$v_n(y) = \left(\frac{a}{a-y}\right)^{\frac{1}{2}} \left(\frac{y}{a}\right)^n, 0 < y < a \quad (50)$$

and

$$x_n(y) = \left(\frac{a}{y-a}\right)^{\frac{1}{2}} \left(\frac{y}{a}\right)^n, a < y < c \quad (51)$$

For the third integral in (29) we write,

$$w_n(y) = \left(\frac{h}{h-y}\right)^{\frac{1}{3}} \left(\frac{y}{h}\right)^n, 0 < y < h \quad (52)$$

3.4 Formation of linear system of equations

As the three integral equations given by (29) are in different ranges, they can be extended to the range $(0, c)$ as in (Morris, 1975) by multiplying with appropriate Heaviside unit functions. This gives the linear system of equations as follows

$$\sum_{n=0}^N a_n \mathcal{K}_{mn} + \sum_{n=0}^N b_n \mathcal{L}_{mn} = d_m, m = 0, 1, \dots, N \quad (53)$$

$$\sum_{n=0}^N a_n \mathcal{M}_{mn} + \sum_{n=0}^N b_n \mathcal{N}_{mn} + \sum_{n=0}^N c_n \mathcal{P}_{mn} = 0, m = 0, 1, \dots, N \quad (54)$$

$$\sum_{n=0}^N b_n \mathcal{Q}_{mn} + \sum_{n=0}^N c_n \mathcal{R}_{mn} = 0, m = 0, 1, \dots, N \quad (55)$$

where

$$\mathcal{K}_{mn} = \int_0^h \left(\int_0^h L_1(y, u) u_n(u) du \right) u_m(y) dy$$

$$\begin{aligned}\mathcal{L}_{mn} &= -i\alpha_0 G \int_0^a \left(\int_0^a M_1(y, u) v_n(u) du \right) v_m(y) dy \\ &\quad + \int_a^c \left(\int_a^c M_1(y, u) x_n(u) du \right) x_m(y) dy\end{aligned}$$

$$\mathcal{M}_{mn} = \int_0^h \left(\int_0^h L_2(y, u) u_n(u) du \right) u_m(y) dy$$

$$\begin{aligned}\mathcal{N}_{mn} &= -i\alpha_0 G \int_0^a \left(\int_0^a M_2(y, u) v_n(u) du \right) v_m(y) dy \\ &\quad + \int_a^c \left(\int_a^c M_2(y, u) x_n(u) du \right) x_m(y) dy \\ &\quad - \int_0^a v_n(y) v_m(y) dy\end{aligned}$$

$$\mathcal{P}_{mn} = \int_0^h \left(\int_0^h N_2(y, u) w_n(u) du \right) w_m(y) dy$$

$$\begin{aligned}\mathcal{Q}_{mn} &= -i\alpha_0 G \int_0^a \left(\int_0^a M_3(y, u) v_n(u) du \right) v_m(y) dy \\ &\quad + \int_a^c \left(\int_a^c M_3(y, u) x_n(u) du \right) x_m(y) dy\end{aligned}$$

$$\mathcal{R}_{mn} = - \int_0^h \left(\int_0^h N_3(y, u) w_n(u) du \right) w_m(y) dy$$

and

$$d_m = \int_0^h \frac{\cosh \alpha_0(h-u)}{\cosh \alpha_0 h} u_m(y) dy$$

with $m, n = 0, 1, 2, \dots, N$.

Substituting the expression of $f_1(y)$ in terms of $p_1(y)$

in (16), we obtain the reflection coefficient as follows

$$R = \frac{1 + C_s}{1 - C_s} \quad (56)$$

where

$$T = -\frac{4i(1 + R)\cosh \alpha_0 h}{\delta_0} \sum_{n=0}^N c_n \int_0^h \cosh \alpha_0(h - y) w_n(y) dy \quad (58)$$

3.5 Evaluation of coefficients of linear system of equations

The various coefficients $\mathcal{K}_{mn}$, $\mathcal{L}_{mn}$, $\mathcal{M}_{mn}$, $\mathcal{N}_{mn}$, $\mathcal{P}_{mn}$, $\mathcal{Q}_{mn}$, $\mathcal{R}_{mn}$, occurring in the linear system of algebraic Equations (53) to (55), can be evaluated by choosing the basis functions in terms of simple polynomial as given by equations

$$\begin{aligned} \mathcal{K}_{mn} = & \int_0^h \left(\int_0^h L_1(y, u) u_n(u) du \right) u_m(y) dy = - \sum_{r=1}^{\infty} \frac{4}{\delta_r} \left(\int_0^h \cos \alpha_r(h - y) u_m(y) dy \right) \left(\int_0^h \cos \alpha_r(h - u) u_n(u) du \right) \\ & + \frac{4}{\gamma_0 \tan \lambda_0(b + l)} \left(\int_0^h \cosh \lambda_0(c - y) u_m(y) dy \right) \left(\int_0^h \cosh \lambda_0(c - u) u_n(u) du \right) \\ & - \sum_{r=1}^{\infty} \frac{4}{\gamma_r \tanh \lambda_r(b + l)} \left(\int_0^h \cos \lambda_r(c - y) u_m(y) dy \right) \left(\int_0^h \cos \lambda_r(c - u) u_n(u) du \right) \end{aligned}$$

Now,

$$\int_0^h \cos \alpha_r(h - y) u_m(y) dy = \frac{2^{\frac{7}{6}} \Gamma\left(\frac{1}{6}\right) (2m)!}{\pi \Gamma\left(2m + \frac{1}{3}\right) (h)^{\frac{1}{3}}} \int_0^h \cos \alpha_r(h - y) \frac{1}{(h^2 - y^2)^{\frac{1}{3}}} C_{2m}^{\frac{1}{6}}\left(\frac{y}{h}\right) dy = \frac{2(-1)^m \cos(\alpha_r h) J_{2m + \frac{1}{6}}(\alpha_r h)}{(\alpha_r h)^{\frac{1}{6}}}$$

where J_n 's are Bessel functions of first kind.

Similarly other integrals in the expression for $\mathcal{K}_{mn}$ can

be evaluated. Using this result in the expression for $\mathcal{K}_{mn}$, we get

$$\begin{aligned} \mathcal{K}_{mn} = & - \sum_{r=1}^{\infty} \frac{16(-1)^{m+n} \cos^2(\alpha_r h)}{\delta_r (\alpha_r h)^{\frac{1}{3}}} J_{2m + \frac{1}{6}}(\alpha_r h) J_{2n + \frac{1}{6}}(\alpha_r h) + \frac{16 \cosh^2(\lambda_0 c)}{\gamma_0 (\lambda_0 h)^{\frac{1}{3}} \tan \lambda_0(b + l)} I_{2m + \frac{1}{6}}(\lambda_0 h) I_{2n + \frac{1}{6}}(\lambda_0 h) \\ & - \sum_{r=1}^{\infty} \frac{16(-1)^{m+n} \cos^2(\lambda_r c)}{\gamma_r (\lambda_r h)^{\frac{1}{3}} \tanh \lambda_r(b + l)} J_{2m + \frac{1}{6}}(\lambda_r h) J_{2n + \frac{1}{6}}(\lambda_r h) \end{aligned}$$

where I_n 's are modified Bessel function of first kind.

$$\begin{aligned} \mathcal{L}_{mn} = & i\alpha_0 G \left[\frac{16 \cosh^2(\lambda_0 c)}{\gamma_0 (\lambda_0 c) \sin \lambda_0(b + l)} I_{2m+1}(\lambda_0 a) I_{2n+1}(\lambda_0 a) - \sum_{r=1}^{\infty} \frac{16(-1)^{m+n} \cos^2(\lambda_r c)}{\gamma_r (\lambda_r c) \sinh \lambda_r(b + l)} J_{2m+1}(\lambda_r a) J_{2n+1}(\lambda_r a) \right] \\ & - \frac{16}{\gamma_0 (\lambda_0 c) \sin \lambda_0(b + l)} I_{2m}(\lambda_0(c - a)) I_{2n}(\lambda_0(c - a)) + \sum_{r=1}^{\infty} \frac{16(-1)^{m+n}}{\gamma_r (\lambda_r c) \sinh \lambda_r(b + l)} J_{2m}(\lambda_r(c - a)) J_{2n}(\lambda_r(c - a)) \\ \mathcal{M}_{mn} = & - \frac{16 \cosh^2(\lambda_0 c)}{\gamma_0 (\lambda_0 h)^{\frac{1}{3}} \sin \lambda_0(b + l)} I_{2m + \frac{1}{6}}(\lambda_0 h) I_{2n + \frac{1}{6}}(\lambda_0 h) + \sum_{r=1}^{\infty} \frac{16(-1)^{m+n} \cos^2(\lambda_r c)}{\gamma_r (\lambda_r h)^{\frac{1}{3}} \sinh \lambda_r(b + l)} J_{2m + \frac{1}{6}}(\lambda_r h) J_{2n + \frac{1}{6}}(\lambda_r h) \end{aligned}$$

$$\begin{aligned}
\mathcal{N}_{mn} = & i\alpha_0 G \left[-\frac{16 \cosh^2(\lambda_0 c)}{\gamma_0(\lambda_0 c) \tan \lambda_0(b-l)} I_{2m+1}(\lambda_0 a) I_{2n+1}(\lambda_0 a) - \frac{16 \cosh^2(\lambda_0 c)}{\gamma_0(\lambda_0 c) \tan \lambda_0(b+l)} I_{2m+1}(\lambda_0 a) I_{2n+1}(\lambda_0 a) \right. \\
& + \sum_{r=1}^{\infty} \frac{16(-1)^{m+n} \cos^2(\lambda_r c)}{\gamma_r(\lambda_r c) \tanh \lambda_r(b-l)} J_{2m+1}(\lambda_r a) J_{2n+1}(\lambda_r a) + \sum_{r=1}^{\infty} \frac{16(-1)^{m+n} \cos^2(\lambda_r c)}{\gamma_r(\lambda_r c) \tanh \lambda_r(b+l)} J_{2m+1}(\lambda_r a) J_{2n+1}(\lambda_r a) \left. \right] \\
& + \frac{16}{\gamma_0(\lambda_0 c) \tan \lambda_0(b+l)} I_{2m}(\lambda_0(c-a)) I_{2n}(\lambda_0(c-a)) - \sum_{r=1}^{\infty} \frac{16(-1)^{m+n}}{\gamma_r(\lambda_r c) \tanh \lambda_r(b+l)} J_{2m}(\lambda_r(c-a)) J_{2n}(\lambda_r(c-a)) \\
& + \frac{16}{\gamma_0(\lambda_0 c) \tan \lambda_0(b-l)} I_{2m}(\lambda_0(c-a)) I_{2n}(\lambda_0(c-a)) - \sum_{r=1}^{\infty} \frac{16(-1)^{m+n}}{\gamma_r(\lambda_r c) \tanh \lambda_r(b-l)} J_{2m}(\lambda_r(c-a)) J_{2n}(\lambda_r(c-a)) \\
& - \frac{4(-1)^{m+n}}{\pi^2 a^2 h^2 (2m+1)(2n+1)} \int_0^a (a^2 - y^2) U_{2m}\left(\frac{y}{a}\right) U_{2n}\left(\frac{y}{a}\right) dy \\
\mathcal{P}_{mn} = & -\frac{16 \cosh^2(\lambda_0 c)}{\gamma_0(\lambda_0 h)^{\frac{1}{3}} \sin \lambda_0(b-l)} I_{2m+\frac{1}{6}}(\lambda_0 h) I_{2n+\frac{1}{6}}(\lambda_0 h) + \sum_{r=1}^{\infty} \frac{16(-1)^{m+n} \cos^2(\lambda_r c)}{\gamma_r(\lambda_r h)^{\frac{1}{3}} \sinh \lambda_r(b-l)} J_{2m+\frac{1}{6}}(\lambda_r h) J_{2n+\frac{1}{6}}(\lambda_r h) \\
\mathcal{Q}_{mn} = & i\alpha_0 G \left[\frac{16 \cosh^2(\lambda_0 c)}{\gamma_0(\lambda_0 c) \sin \lambda_0(b-l)} I_{2m+1}(\lambda_0 a) I_{2n+1}(\lambda_0 a) - \sum_{r=1}^{\infty} \frac{16(-1)^{m+n} \cos^2(\lambda_r c)}{\gamma_r(\lambda_r c) \sinh \lambda_r(b-l)} J_{2m+1}(\lambda_r a) J_{2n+1}(\lambda_r a) \right] \\
& - \frac{16}{\gamma_0(\lambda_0 c) \sin \lambda_0(b-l)} I_{2m}(\lambda_0(c-a)) I_{2n}(\lambda_0(c-a)) + \sum_{r=1}^{\infty} \frac{16(-1)^{m+n}}{\gamma_r(\lambda_r c) \sinh \lambda_r(b-l)} J_{2m}(\lambda_r(c-a)) J_{2n}(\lambda_r(c-a)) \\
\mathcal{R}_{mn} = & -\frac{16 i \cosh^2(\alpha_0 h)}{\delta_0(\alpha_0 h)^{\frac{1}{3}}} I_{2m+\frac{1}{6}}(\alpha_0 h) I_{2n+\frac{1}{6}}(\alpha_0 h) \\
& - \sum_{r=1}^{\infty} \frac{16(-1)^{m+n} \cos^2(\alpha_r h)}{\delta_r(\alpha_r h)^{\frac{1}{3}}} J_{2m+\frac{1}{6}}(\alpha_r h) J_{2n+\frac{1}{6}}(\alpha_r h) \\
& + \frac{16 \cosh^2(\lambda_0 c)}{\gamma_0(\lambda_0 h)^{\frac{1}{3}} \tan \lambda_0(b-l)} I_{2m+\frac{1}{6}}(\lambda_0 h) I_{2n+\frac{1}{6}}(\lambda_0 h) \\
& - \sum_{r=1}^{\infty} \frac{16(-1)^{m+n} \cos^2(\lambda_r c)}{\gamma_r(\lambda_r h)^{\frac{1}{3}} \tanh \lambda_r(b-l)} J_{2m+\frac{1}{6}}(\lambda_r h) J_{2n+\frac{1}{6}}(\lambda_r h) \\
& d_m = \frac{2 I_{2m+\frac{1}{6}}(\alpha_0 h)}{(\alpha_0 h)^{\frac{1}{6}}}
\end{aligned}$$

$$C_f = -i\rho\sigma \int_0^a g_2(y) dy \quad (60)$$

The non-dimensional form of the horizontal force coefficient on the vertical porous barriers is given by

$$K_f = \frac{K|C_f|}{\rho g} \quad (61)$$

3.7 Energy identity relation

The energy identity plays an important role in the theoretical study of water waves scattering by barriers. A part of incident wave energy can be dissipated by the porous barriers. The absolute values of the reflection and transmission coefficients are connected by the relation

$$|R|^2 + |T|^2 = 1 - J \quad (62)$$

Here J signifies the amount of dissipated energy due to the permeability of barriers and its expression in terms of the potential differences across the barriers is found to be

$$J = 2K\text{Re}(G) \int_0^a |g_2(y)|^2 dy \quad (63)$$

where $\text{Re}(G)$ is the real part of the porous effect parameters. It is clearly noticed from the expression of J that the integrand in the right-hand side of (63) is always positive

3.6 Dynamic wave force

The dynamic pressure $P(x, y)$ can be obtained according to the Bernoulli equation (Li et al., 2015)

$$P(x, y) = -i\rho\sigma g_2(y) \quad (59)$$

where ρ is the fluid density. The magnitude of horizontal wave force acting on barriers can be obtained by integrating the dynamic pressure along the porous barriers as follows

for a non-zero $\text{Re}(G)$ so that $|R|^2 + |T|^2 < 1$ for the case of permeable barriers, whereas for impermeable barriers the energy identity relation satisfies $|R|^2 + |T|^2 = 1$, which provides the convergence of result in the present study.

4 Numerical results

In this section the numerical results of hydrodynamic quantities viz. reflection coefficient, transmission coefficient, energy dissipation coefficient and hydrodynamic force are depicted graphically. Here this quantities are non dimensionalized by the water depth h .

In Table 1, we display the numerical results for $|R|$ showing its convergence with the truncation number N in Equations (53) to (55), taking $N = 2, 3, 4, 5$ for different values of Kh and fixed values of $a/h = 0.5$, $b/h = 1.5$, $c/h = 1.5$, $l/h = 0.5$, $G = 0.5$. In this case we see that, for $N = 3, 4, 5$, the numerical values computed here coincide up to 5 decimal places. We have also checked that the present method converges for all other values of the parameters and the wave number when N is 3. So, for all the calculations we have chosen $N=3$.

Table 1 Convergence of $|R|$ with N

Kh	$N=2$	$N=3$	$N=4$	$N=5$
0.000 01	0.001 271	0.001 351	0.001 372	0.001 377
0.500 01	0.090 594	0.090 595	0.090 596	0.090 596
1.000 01	0.249 584	0.251 970	0.252 254	0.252 259
1.500 01	0.570 622	0.575 437	0.577 160	0.577 165
2.000 01	0.179 136	0.187 224	0.187 403	0.187 405
2.500 01	0.263 810	0.251 276	0.254 864	0.254 867

Then the accuracy of the numerical results are established in Table 2, by comparing our present result with Ray et al. (2021). In Table 2, the numerical values of $|R|$ computed by present method are compared with those given in Table 1 of Ray et al. (2021) for a rigid barrier placed along the y -axis, by considering $a/h = 0.5$, $b/h = 1.5$, $c/h = 1.5$, $l/h = 0$, $G = 0$. Ray et al. (2021) used Galerkin technique with simple polynomial as a basis function, whereas in the present work, suitable orthogonal polynomials (Gegenbauer polynomial for $1/3$ rd singularity and Chebyshev polynomial for half singularity) are chosen as a basis function. From Table 2, it is clearly noticed that the linear system of Equations (53) to (55), converges for $N = 5$ in Ray et al. (2021), where as in the present study the same converges for $N = 3$. Thus the rate of convergence of the linear system taking algebraic polynomials as basis function is slower than the orthogonal polynomials as basis function.

In Table 3, the values of reflection coefficient are presented for different wave numbers Kh and fixed values of

Table 2 Comparison of our results with the results of Ray et al. (2021)

Kh	$ R $ ($N=5$) (Ray et al., 2021)	$ R $ ($N=5$) (Present study)	$ T $ ($N=5$) (Ray et al., 2021)	$ T $ ($N=3$) (Present study)
0.001	0.013 015	0.013 019	0.999 915	0.999 915
0.491	0.132 204	0.132 209	0.991 213	0.991 221
0.981	0.047 433	0.047 441	0.998 867	0.998 874
1.471	0.315 932	0.315 937	0.947 309	0.948 780
1.961	0.636 024	0.636 030	0.783 752	0.771 664
2.451	0.873 816	0.873 824	0.474 271	0.486 242
2.941	0.966 781	0.966 789	0.262 009	0.255 576

$a/h = 0.6$, $b/h = 1.5$, $c/h = 1.3$, $l/h = 0.5$, $G = 0.5$, and compared by choosing simple polynomials and orthogonal polynomial as basis functions in Galerkin method. It is observed that rate of convergence of the method choosing basis function as orthogonal polynomial ($N=3$) is faster than taking basis function as simple polynomial ($N=5$). Also the Table 2 shows that the values of $|R|$ choosing orthogonal polynomial as basis function matches with values of $|R|$ choosing simple polynomial as basis function upto five places of decimal. Hence we may infer that the orthogonal polynomials as basis function is definitely a better choice.

Table 3 $|R|$ for orthogonal and simple polynomial

Kh	Orthogonal polynomials ($N=3$)	Simple polynomials ($N=5$)
0.000 01	0.000 948	0.000 945
0.500 01	0.024 745	0.024 751
1.000 01	0.344 937	0.344 935
1.500 01	0.379 658	0.379 654
2.000 01	0.956 727	0.956 721
2.500 01	0.265 245	0.265 241

The numerical values of the reflection coefficient, transmission coefficient, energy dissipation coefficients and the energy identity are presented in Table 4 for different values of Kh and fixed values of $a/h = 0.5$, $b/h = 1.5$, $c/h = 1.5$, $l/h = 0.5$, $G = 1.0$. In this table for all the values of Kh we observe that the energy identity relation $|R|^2 + |T|^2 + J = 1$ is satisfied. This tables provide a partial check on the correctness of the results obtained by the present method.

Table 4 Energy identity

Kh	$ R $	$ T $	J	$ R ^2 + T ^2 + J$
0.100 01	0.113 531 0	0.994 401	0.001 723 29	1.003
1.200 01	0.361 891 0	0.904 478	0.050 953 60	1.000
2.000 01	0.077 184 6	0.715 724	0.481 782 00	1.000
2.500 01	0.270 846 0	0.745 861	0.370 333 00	1.000
2.830 01	0.976 509 0	0.136 927	0.027 680 90	1.000

4.1 Validation of the results

To show the validity of the analytical solutions obtained in the present work, it is compared with the results of Lee and Chwang (2000) considering limiting values of certain parameters in our results. By taking $c/h = 1$, $b/h = 0.001$, i.e., in absence of trench, in the present configuration, the reflection coefficient $|R|$ is compared with the results of Lee and Chwang (2000) by taking $a/h = 0.5$ and $G (= 0.25, 0.5, 1)$ in Figure 3. A good matching of $|R|$ is observed in the figure.

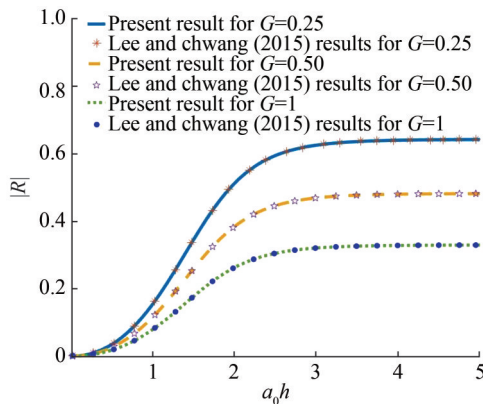


Figure 3 Comparison of our results to the Figure 6 of Lee and Chwang (2000) taking $\frac{a}{h} = 0.5$, $\frac{b}{h} = 0.001$, $\frac{c}{h} = 1$ and different values of G

In absence of barrier (i.e., making $a/h \rightarrow 0$, $G = 0$ in the present analysis), the problem reduces to study of wave propagation over a rectangular trench which was studied earlier by Lee and Ayer (1981). Choosing $a/h = 0.0001$, $b/h = 2.5$, $c/h = 2$ and $G=0$ in Figure 4 of the present analysis, if we plot reflection coefficient $|R|$ against $a_0 h/2\pi$ we see that our result almost match with the Figure 2 of Lee and Ayer (1981).

In Figure 5, $|R|$ for rigid barrier, obtained in the present analysis by making $a/h = 0.9$, $b/h = 1.5$, $c/h = 1.5$, $G = 0$,

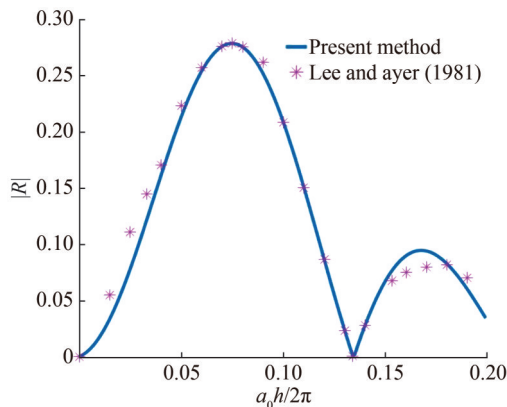


Figure 4 Comparison of our results to the Figure 2 of Lee and Ayer (1981) taking $\frac{a}{h} = 0.0001$, $\frac{b}{h} = 2.5$, $\frac{c}{h} = 2$ and $G=0$

is compared with the results given by Ray et al. (2021) (Figure 4a there). A good matching of the results are observed from Figure 5.

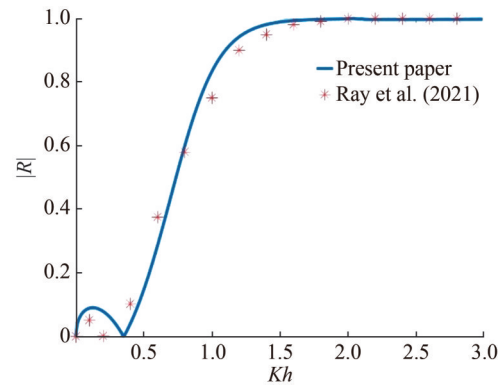


Figure 5 Comparison of our results to the Figure 4 a of Ray et al. (2021) taking $\frac{a}{h} = 0.9$, $\frac{b}{h} = 1.5$, $\frac{c}{h} = 1.5$ and $G=0$

4.2 Effect of various parameters on scattering coefficients ($|R|$ and $|T|$)

Figure 6 illustrates the effect of width of the trench on the reflection coefficients in presence of porous barrier. Here $|R|$ is plotted against the non-dimensional wave number Kh for three different width of the trench $b/h = 1.3, 1.4, 1.5$ and fixed values of $a/h = 0.7$, $c/h = 1.3$, $l/h = 0.5$ and $G=0.5$. We observe oscillatory behaviour of $|R|$ and the frequency of oscillation increases as the width of the trench b/h increases. Also we observe that for $Kh > 2$, larger width of the trench increases the amount of reflection and the same behavior have shown in Chakraborty and Mandal (2014).

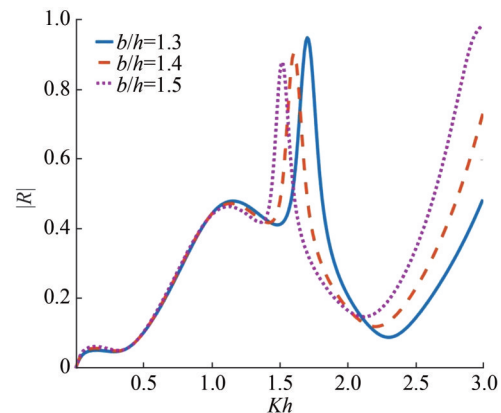


Figure 6 Reflection coefficient for different $\frac{b}{h}$ and fixed $\frac{a}{h} = 0.7$, $\frac{c}{h} = 1.3$, $\frac{l}{h} = 0.5$, $G = 0.5$

The effect of the length of the partially immersed porous barrier on the reflection coefficient $|R|$ is studied in Figure 7 where $|R|$ is plotted against the non-dimensional wave number Kh for various values of the length of the barrier

($a/h = 0.3, 0.6, 0.9$) and fixed values of $b/h = 1.5$, $c/h = 1.3$, $l/h = 0.5$ and $G=0.5$. Figure 7 shows that $|R|$ exhibits oscillatory behaviour and the amplitude of oscillation increases with increasing length of the porous barrier. For Kh near about 1.5 there occurs a sharp increase in amplitude of oscillation in $|R|$ showing resonating behaviour of the reflection coefficient.

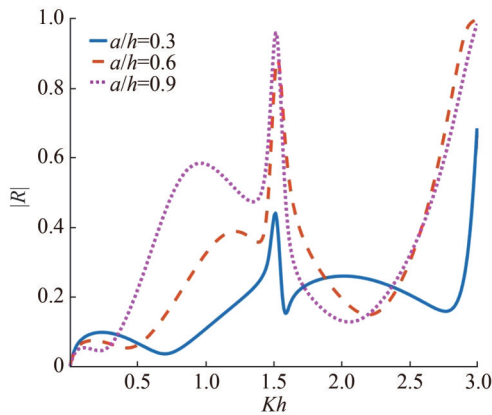


Figure 7 Reflection coefficient for different $\frac{a}{h}$ and fixed $\frac{b}{h} = 1.5$, $\frac{c}{h} = 1.3$, $\frac{l}{h} = 0.5$, $G = 0.5$

Figure 8 illustrates the effect of depth of the trench on the reflection coefficient. Here $|R|$ is plotted against the non-dimensional wave number Kh , for $a/h = 0.7$, $b/h = 1.4$, $l/h = 0.5$ and $G=0.5$ and for various depth of the trench viz, $c/h = 1.1, 1.2, 1.3$. It is observed that $|R|$ shows oscillatory behaviour and amplitude of oscillation increases with the increasing values of the trench depth c/h . Also for $Kh \sim 1.5$, there occur resonance in $|R|$.

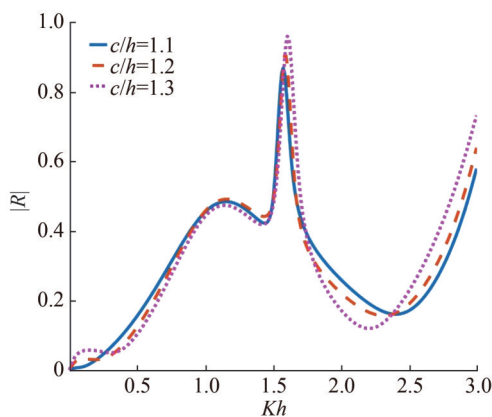


Figure 8 Reflection coefficient for different $\frac{c}{h}$ and fixed $\frac{a}{h} = 0.7$, $\frac{b}{h} = 1.4$, $\frac{l}{h} = 0.5$, $G = 0.5$

In Figure 9, reflection coefficient $|R|$ is plotted against the non-dimensional wave number Kh for different values of the porosity parameter G ($= 0.5, 1, 2$) and for $a/h = 0.7$, $b/h = 1.4$, $l/h = 0.5$, $c/h = 1.3$. It is seen that $|R|$ shows oscillatory behaviour with multiple peaks and for $Kh \sim 1.75$ a

resonance occurs in the reflection coefficient $|R|$ for all values of porosity parameter G . This occurs as a consequence of the interaction between porous barrier, trench and the waves. It is seen that with increasing porosity parameter reflection decreases. This is due to the porous effect of the barrier.

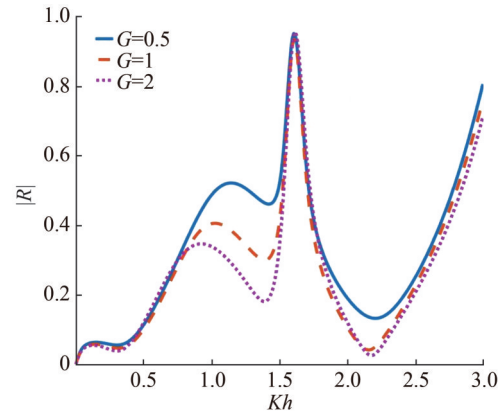


Figure 9 Reflection coefficient for different G and fixed $\frac{a}{h} = 0.7$, $\frac{b}{h} = 1.4$, $\frac{c}{h} = 1.3$, $\frac{l}{h} = 0.5$

In Figure 10, reflection coefficient $|R|$ is plotted against the non-dimensional wave number Kh for different values of l/h ($= 0.4, 0.5, 0.6$) and keeping $a/h = 0.7$, $b/h = 1.4$, $l/h = 0.5$, $c/h = 1.3$, fixed. Here $|R|$ shows oscillatory behaviour. For $Kh < 1.5$, $|R|$ almost coincide for all values of l/h . For $1.5 < Kh < 1.7$, a sharp peak is observed in $|R|$ for each value of l/h and the peaks shift towards left as l/h decreases. Thus as the barrier is placed away from the central line of the trench, there is a phase shift in the peak of $|R|$. Also $|R|$ increases with decreasing values of l/h . It means that the reflection coefficient increases when the porous barrier is shifted towards the centre of the trench from the right side of the trench. In all curves, $|R|$ starts from zero near $Kh=0$ and for large values of Kh , $|R|$ becomes unity.

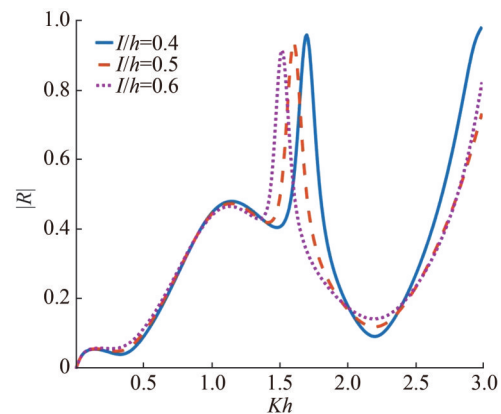


Figure 10 Reflection coefficient for different $\frac{l}{h}$ and fixed $\frac{a}{h} = 0.7$, $\frac{c}{h} = 1.3$, $\frac{b}{h} = 1.4$, $G = 0.5$

In Figure 11, amplitude of the energy dissipation coefficient J is plotted against the non-dimensional wave number Kh for different values of the porosity parameter G ($= 1, 2, 3$) and keeping $a/h = 0.7$, $b/h = 2.0$, $l/h = 0.5$, $c/h = 1.3$ fixed. From this figure it is seen that J exhibits oscillatory behaviour and J increases as Kh increases. Initially for small wave number $Kh < 1.2$, the energy dissipation increases with increasing porosity, then for $1.2 < Kh < 2$ energy dissipation decreases with increasing porosity. But for large wave number $Kh > 2$, energy dissipation again increases with increasing porosity parameter of the barrier. This may be due to the interaction of waves with the barrier and the trench.

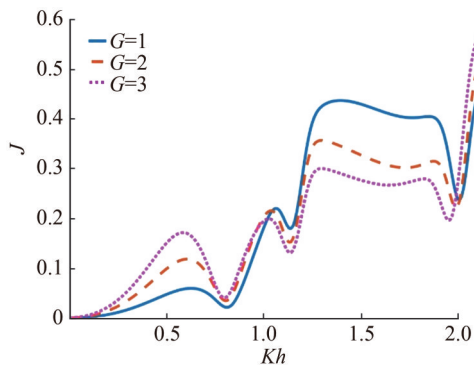


Figure 11 Energy dissipation for different G and fixed $\frac{a}{h} = 0.7$, $\frac{c}{h} = 1.3$, $\frac{b}{h} = 2.0$, $\frac{l}{h} = 0.5$

In Figure 12, non-dimensional horizontal wave force K_f is plotted against the non-dimensional wave number Kh for different values of the porosity parameter G ($= 1, 2, 3$) and keeping $a/h = 0.7$, $b/h = 1.5$, $l/h = 0.5$, $c/h = 1.3$ fixed. It is observed that the horizontal wave force increases with decreasing porosity G showing that with the decrease in G , the resistance force coefficient of the porous barrier decreases, allowing water to pass through the pores of the barrier and so the horizontal wave force increases.

4.3 Partially immersed porous barrier at middle of the trench

The porous barrier is along the y -axis, i.e., along the centre of the trench, when $l/h=0$. The Figures 13–18 depicts the behaviour of $|R|$, J , K_f for various values of the parameters a/h , b/h , c/h , G .

Figure 13 exhibits the effect of width of the trench on the reflection coefficients for three different widths of the trench ($b/h = 0.5, 1.5, 2.5$) and fixed values of $a/h = 0.6$, $c/h = 1.5$, $G = 0.5 + i$. In this figure we see that $|R|$ exhibits oscillatory behaviour and the number of oscillation increases as the width of the trench b/h increases. Also there occurs resonance in $|R|$ for a particular value of Kh and multiple resonance occurs as the width of the trench in-

creases when the barrier is along the central vertical line of the trench. This phenomena is not observed when the barrier is shifted to the right of the central line.

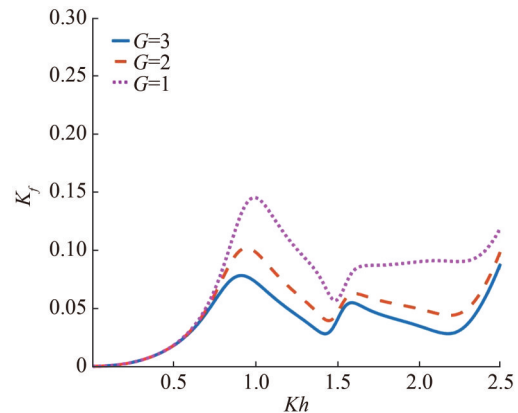


Figure 12 Hydrodynamic wave force for different G and fixed $\frac{a}{h} = 0.7$, $\frac{c}{h} = 1.3$, $\frac{b}{h} = 1.5$, $\frac{l}{h} = 0.5$

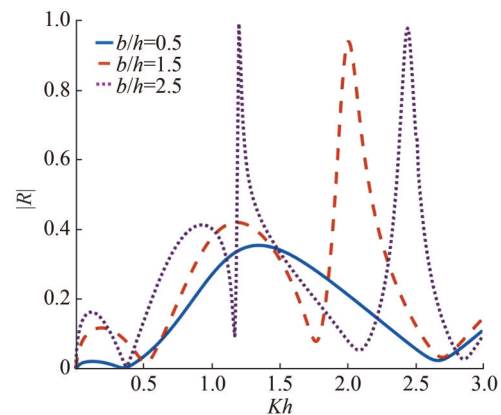


Figure 13 Reflection coefficient for different $\frac{b}{h}$ and fixed $\frac{a}{h} = 0.6$, $\frac{c}{h} = 1.5$, $G = 0.5 + i$

The effect of the length of the partially immersed porous barrier a/h on the reflection coefficient $|R|$ is studied in Figure 14. Here $|R|$ is plotted against the non-dimensional wave number Kh for various values of the length of the barrier ($a/h = 0.5, 0.8, 1.2$) and fixed values of $b/h = 1.5$, $c/h = 1.5$, $G = 0.5 + i$. Figure 14 shows oscillatory behaviour in $|R|$ exhibits and the amplitude of oscillation increases with increasing length of the porous barrier.

In Figure 15, $|R|$ is depicted against the non-dimensional wave number Kh , keeping $a/h = 0.6$, $b/h = 1.5$, $G = 0.5 + i$ fixed and taking $c/h = 1.1, 1.3, 1.5$ to visualize the effect of the trench depth on the reflection coefficient. Here also $|R|$ exhibits oscillatory behaviour. It is observed that the amplitude of the reflection coefficients increases as the depth of the trench increases. Also we observe that waves with large wave numbers, i.e., the short waves which are near the free surface do not feel much the presence of trench in bottom of the ocean.

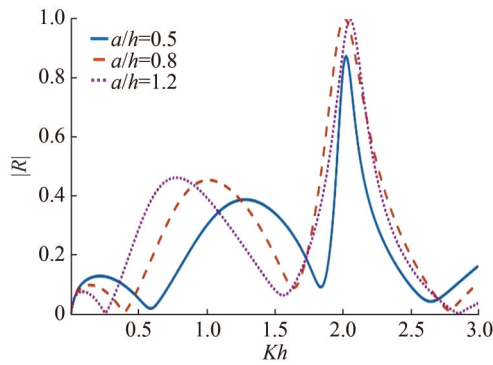


Figure 14 Reflection coefficient for different $\frac{a}{h}$ and fixed $\frac{b}{h} = 1.5$, $\frac{c}{h} = 1.5$, $G = 0.5 + i$

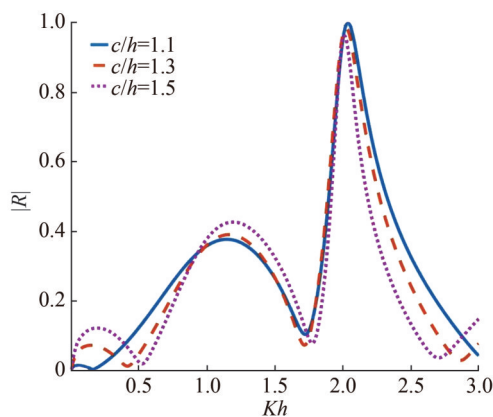


Figure 15 Reflection coefficient for different $\frac{c}{h}$ and fixed $\frac{a}{h} = 0.6$, $\frac{b}{h} = 1.5$, $G = 0.5 + i$

In Figure 16, reflection coefficient $|R|$ is plotted against the non-dimensional wave number Kh for different values of the porosity parameter G ($= 0.5, 1, 2 + i$) and keeping $a/h = 0.6$, $b/h = 1.5$, $c/h = 1.3$ fixed. It is seen that multiple peaks occur in the reflection coefficient. This phenomena occurs due to the interaction of wave with the porous barrier. It is seen that with as G increases from 0.5 to 1, the reflection coefficient decreases. Also it is seen that $|R|$ is least when $G=2+i$. This is due to the fact that presence of inertial force coefficient G_i in G allows the passage of water through the pores which diminishes reflection.

In Figure 17, coefficient of energy dissipation J is plotted against the non-dimensional wave number Kh for different values of the porosity parameter G ($= 1, 2, 3$) and keeping $a/h = 0.5$, $b/h = 1.5$, $c/h = 1.5$ fixed. From the figure it is visible that, for small wave numbers $Kh < 1$, energy dissipation increases as porosity parameter increases while for $Kh > 1$ energy dissipation decreases with increasing porosity parameter G . The occurrence of this phenomena is due to the fact that as Kh increases, the wavelength decreases and the short waves which are near the free surface interact

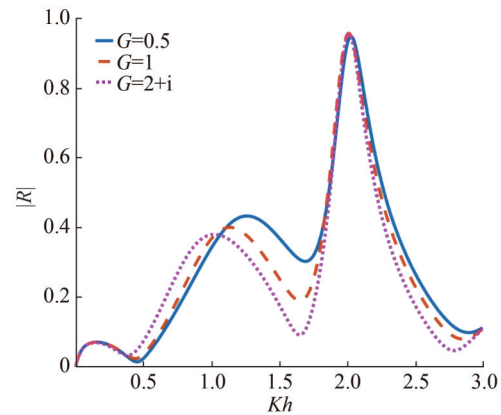


Figure 16 Reflection coefficient for different G and fixed $\frac{a}{h} = 0.6$, $\frac{b}{h} = 1.5$, $\frac{c}{h} = 1.3$

with the porous barrier. Here G is real $G = G^r$ which means that the inertial force coefficient G^i is much less than the resistance force coefficient G^r of the porous barrier and so as $G = G^r$ increases, the resistance force coefficient of the porous barrier resists the passage of water through the pores and reflects back the short waves thereby reducing the energy dissipation.

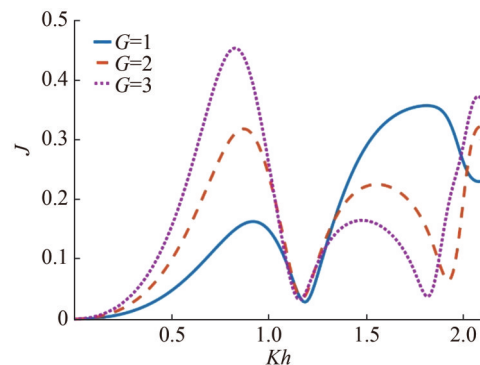


Figure 17 Energy dissipation for different G and fixed $\frac{a}{h} = 0.5$, $\frac{b}{h} = 1.5$, $\frac{c}{h} = 1.5$

In Figure 18, non-dimensional horizontal wave force K_f is plotted against the non-dimensional wave number Kh for different values of the porosity parameter G ($= 1, 2, 3$) and keeping $a/h = 0.7$, $b/h = 1.5$, $c/h = 1.3$ fixed. The coefficient of wave force increases with decreasing porosity parameters.

From all these oscillatory behavior of the curves, one can find the values of wave number in which maximum reflection and transmission occurs. Also full reflection of waves occurs for some wave numbers. This behavior may be due to the interaction of waves with the depression of the bottom and the porous barrier. These outcomes are very useful for marine engineers to construct breakwaters to protect sea shore areas.

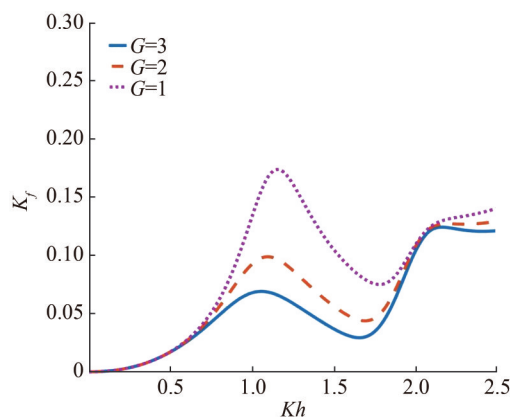


Figure 18 Hydrodynamic wave force for different G and fixed $\frac{a}{h} = 0.7$, $\frac{b}{h} = 1.5$, $\frac{c}{h} = 1.3$

5 Conclusions

The problem of water waves scattering by thin porous plate over a rectangular trench is discussed here considering linearized theory of water waves. The physical problem is mathematically modeled in terms of a boundary value problem for two different positions of the porous plate. Havelock's expansion of potential function together with its inversion formulae have been employed to the problem which then modified into a system of linear integral equations in terms of horizontal component of velocity function above the trench and below the edge of the vertical barrier. There are two types of integrable singularities occurred in the integral equations. One is square root singularity at the sharp edge of the thin barrier and the other type is $1/3$ rd singularity at the edges of the rectangular trench. To manage both types of singularities a multi term Galerkin approximation technique with appropriate basis functions is considered. For $1/3$ rd singularity Gegenbauer polynomial of order $1/6$ and Chebyshev polynomial for $1/2$ singularity are utilized here. Also we consider simple polynomials as basis function in mathematical computations and compare the rate of convergence of the method taking two types of basis functions in Table 3. It is observed that use of special function as basis function produces better rate of convergence of the method than simple polynomial as basis function. The numerical appraisal of reflection and transmission coefficients, energy dissipation and horizontal wave force are determined for different parameter values. These results are illustrated in a number of figures that some of them are quite well matched with earlier works. For both positions of the barrier, effect of porosity decreases the reflection coefficient but increasing length of the barrier reflect more energy and less transmit. Larger width of the trench transmit more energy. Hence from present study, it is clear that scattering nature of surface waves in presence

of bottom obstacle in form a rectangular trench is really influenced by thin porous barrier. These changes of surface waves play a crucial role in marine structures in coastal regions.

Nomenclature

a	Length of the partially immersed barrier
$2b$	Width of the trench
c	Depth of the trench from the free surface
h	Depth of the fluid region
l	Distance of the barrier from the middle of the trench on the free surface
σ	Angular frequency of incoming wave train
K	Wavenumber
φ^{inc}	Velocity potential of the incident wave
$\varphi_1, \varphi_2, \varphi_3, \varphi_4$	Velocity potential of resultant motion in the fluid subregions
r_1	Distance from the submerged left edge of the trench
r_2	Distance from the submerged edge of the barrier
r_3	Distance from the submerged right edge of the trench
G	Dimensionless porous parameter
G^r, G^i	Resistance force coefficient and inertial force coefficient of the porous material
R, T	Reflection and Transmission coefficient

Acknowledgement The authors thank Prof. Sudeshna Banerjee, Department of Mathematics, Jadavpur University, India, for providing helpful suggestions to improve the work. They also thank the referees for their valuable suggestions and comments that improved the presentation of the paper.

Competing interest The authors have no competing interests to declare that are relevant to the content of this article.

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