

Multiple Nonlinear Regression Analysis for the Stability of Non-overtopping Perforated Quarter Circle Breakwater

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Abstract

Breakwaters have been built throughout the centuries for the coastal protection and the port development, but changes occurred in their layout and criteria used for the design. Quarter circle breakwater (QBW) is a new type evolved having advantages of both caisson type and perforated type breakwaters. The present study extracts the effect of change in the percentage of perforations on the stable conditions of seaside perforated QBW by using various physical models. The results were graphically analyzed using dimensionless parameters and it was concluded that there is a reduction in dimensionless stability parameter with an increase in steepness of the wave and change in water depth to the height of breakwater structure. Multiple non-linear regression analysis was done and the equation for the best fit curve with a higher regression coefficient was obtained by using Excel statistical software—XLSTAT.

Keywords Breakwater · Perforations · Regression analysis · Stability · Wave steepness · Water depth

1 Introduction

Ports are considered to be the most important transit locations to carry out the world trade through seaways, which need to be protected from the disturbances due to the incoming waves. A seaport plays an important role in the sector of sea transportations, exports, imports, tourism, and travel, and thus, it is an important ingredient of economic growth. Water transport is a major economy input for a nation with over 82% of world trade in tons and 94% of world trade in tons-kilometers which are moved by shipping and thereby through ports.

Article Highlights

- Stability of perforated quarter circle breakwater based on physical models and empirical approaches is studied.
- An equation is developed for the stability parameter using the multiple nonlinear regression analysis.

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The basic requirement of any port, harbor, or marina is a sheltered area free from the sea waves. In the coastal areas where natural protection from sea waves is not available, the development of harbor requires artificial protection for the creation of calm areas. Large structures such as rubble mound breakwaters or vertical wall breakwaters were constructed in major harbors and nearer to other marine structures where more tranquility conditions are essential for its sustainability. Breakwaters are also provided in the lagoon and at the entrance channel of ports, for maneuvering of ships and port operations. Sometimes, they are used to protect beaches from erosion due to the destructive wave forces.

Recent advances in science and technology led to the development in various types of composite type breakwater structures with different layouts and using modern construction techniques. The design of breakwater depends on the wave conditions, locality, and their appearance. As a result, variety of breakwater structures has been evolved in the past few decades. Innovative concepts for types of breakwater are in progress as a measure for calming down the destructive effect caused by sea waves. Semicircular breakwater is a composite structure first developed in Japan at the beginning of the 1990s and first adopted for the formation of the harbor in Miyazaki Port, Japan. Xie et al. (2006), developed the concept of QBW, and has conducted studies comparing the performance of semicircular and QBW. Quarter circle breakwater (QBW) has various advantages such

as lightweight and cheaper to construct. It is also found to be favorable in all soil conditions, can be easily transported and placed, provide better esthetic view, sustainable, and can withstand severe wave conditions.

Recent studies in this field are related to the effect of earthquake and tsunami waves on the stability of the breakwater foundation. Chaudhary et al. (2017) investigated various effective reinforcement techniques for the foundation of the breakwater in order to provide resiliency to overcome earthquake and tsunami. It was found that the duration and magnitude of acceleration of earthquake loadings had significant impacts on the settlement and horizontal displacement of the breakwater. Chaudhary et al. (2018a, b) evaluated the effectiveness of the reinforced foundation by conducting a series of shaking table tests and hydraulic model tests, and comparisons were made between the conventional and reinforced foundations. Numerical analyses were also performed to clarify the mechanism of the soil–breakwater–reinforcement–fluid system. Chaudhary et al. (2018a, b) carried out physical model studies using geo-grid, gabions, and sheet piles for reinforcing a foundation for reducing the damage of the breakwater brought by the earthquake and tsunami. Chaudhary et al. (2019) conducted studies on the development of reinforcing countermeasures for a breakwater foundation that can produce a resilient breakwater against earthquakes and tsunamis, such as foundations reinforced with sheet piles and gabions. Physical model tests were carried out for scaled-down breakwater models to examine the performance of the reinforcing countermeasures under an earthquake and tsunami.

The present study investigates the stability of an emerged seaside perforated quarter circle breakwater with varying percentage of perforations, for different water depths and wave conditions. The major objectives include the study of the effect of incident wave steepness, S/D (spacing between perforations/diameter of perforations) ratio or percentage perforations, and water depth on the stability of quarter circle breakwater with varying seaside perforations.

2 Experimental Investigations

The experimental studies were carried out in a 2D wave flume existing in the Marine Structures Laboratory

under Applied Mechanics and Hydraulics Department, National Institute of Technology Karnataka, Mangalore, India (Hughes 1993).

2.1 Wave Flume and Instrumentation

The physical model study for regular waves was conducted in a 2D wave flume having a length of 50 m long, width 0.71 m, and depth of 1.1 m (refer Figure 1). Ranges of wave height varying from 0.08 to 0.24 m and periods from 0.8 to 4.0 s can be generated with this facility (Shi et al. 2011).

2.2 Breakwater Test Model

The model for the quarter circle breakwater was constructed as two parts: the topmost part is a caisson having a quarter-circular shape and a bottom part consisting of base slab constructed with precast concrete placed on rubble mound foundation (Luwen et al. 2013).

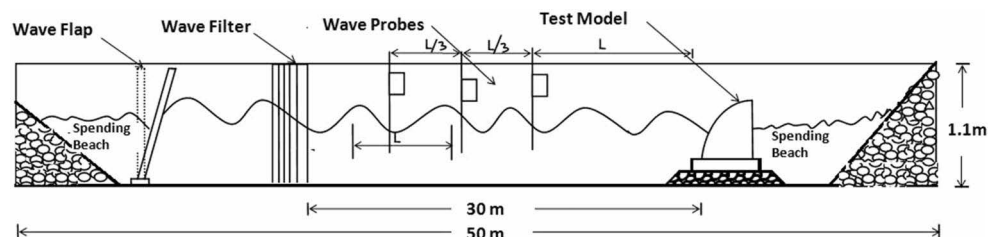
Quarter circular caisson is made-up of the galvanized iron (GI) sheets having 2 mm thickness and is joined together. For the experimental study, physical models with geometrical similarity having a scale of 1:30 are used to simulate the present wave conditions of the Mangalore coast and suitable size of perforation in accordance with the Froude's law.

Based on the wave conditions and water depth available for the present study, QBW having a radius of 0.55 m radius was selected so that there will be no wave overtopping and the model serves as the emerged type QBW under all adverse conditions. Stiffeners are used for fixing the breakwater model and the base slab. The perforations are made in the quarter circle shaped front part of the QBW model, with the help of the drilling machine. The typical cross-section of QBW considered for the study is shown in Figure 2.

3 Methodology

Rubble mound foundation having a slope 1:2 and thickness 0.05 m was laid with stones having weight 50 to 100 g and the QBW model after casting was placed over it. The entire QBW model with the foundation was placed at 30 m distance from the wave flap (Binumol et al. 2017).

Figure 1 Longitudinal section of wave flume



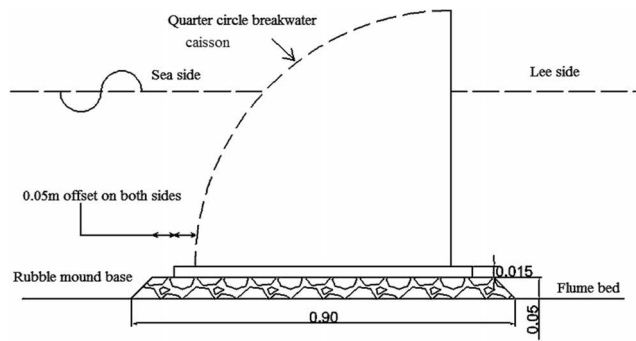


Figure 2 Typical cross-section of perforated QBW

For measuring the wave height for the incident and reflected waves, the 3 probe method proposed by Isaacson (1991) was used. The distance between the first probes from the center of the model is kept as one wavelength L , but for other probes, space between the probes was maintained as $L/3$. To avoid continuous reflection, a burst of five waves was generated for each trial used. The elevation of the wave surface obtained by the wave probes was recorded by the wave recorder and the voltage signals are transformed into wave heights and wave period with the help wave recorder software developed by EMCON (Environmental Measurements and Controls), Kochi, India. The list of experimental variables and the ranges used for the present study are tabulated as shown in Table 1.

The experimental investigations carried out are based on the following assumptions:

- The sea bed is rigid and horizontal and it is assumed that the sediment movement does not interfere with the wave motion and does not affect the model performance.
- The waves are periodic and monochromatic.
- There is no interference between the incident waves and the structure since the waves are generated by the flume as bursts.
- The generation of secondary waves is not taken into account.
- Reflection caused by the waves on the flume bottom or flume side walls is avoided.

Table 1 Experimental variables

Parameters	Range
Wave specific parameters	
Incident wave height, H_i (m)	0.03, 0.06, 0.09, 0.12, 0.15, 0.18
Water depth, d (m)	0.35, 0.40, 0.45
Time period, T (s)	1.2, 1.4, 1.6, 1.8, 2.0, 2.2
Structure specific parameters	
Radius of the structure (m)	0.55
S/D ratio	2, 2.5, 3, 4, 5
Diameter of perforation (m)	0.016, 0.020

- The difference between the density of freshwater and sea water is neglected.
- The frictional effects between the base slab and structure have not been accounted

4 Results and Discussions

Studies are conducted on emerged seaside perforated QBW with 0.55 m radius and different spacing to diameter (S/D) ratios having the depth of water 0.35, 0.40, and 0.45 m under varying wave conditions. Variations of minimum weight to prevent sliding for different wave conditions and structural parameters were studied, and the variations were then presented in graphical format using dimensionless parameters obtained from a dimensional analysis by Buckingham's π theorem. The stability based on sliding performance was denoted by a dimensionless stability parameter ($W/\gamma H_i^2$), where W is the minimum weight of the QBW required (including the additional weight) to resist the sliding per unit length of the breakwater, γ is the specific weight of water, and H_i is the incident wave height. The sliding of QBW occurs when the wave force exceeds the frictional resistance offered by the QBW. The stability of QBW against wave force increases with an increase in the total weight of the structure. Therefore, the experiment is repeated by the addition of weight in increments until the structure stops sliding.

4.1 Influence of Incident Wave Steepness on $W/\gamma H_i^2$

The graphs of non-dimensional stability parameter ($W/\gamma H_i^2$) against the incident wave steepness, H_i/gT^2 (H_i = incident wave height in m, g = acceleration due to gravity in m/s^2 , and T = wave period in sec), are plotted for different values of relative water depth, d/h_s , and a constant spacing to diameter of perforation ratio (S/D).

4.1.1 Seaside Perforated QBW with $S/D = 2$

The variation of $W/\gamma H_i^2$ with H_i/gT^2 at different water depths for QBW radius 0.55 m and S/D ratio equal to 2 was plotted as shown in Figure 3.

It is clear from the graphs that $W/\gamma H_i^2$ decreases with increase in H_i/gT^2 for all values of d/h_s . This may be due to the reason that waves of higher steepness (lower wave period for a given wave height) interact with the QBW surface only for short interval of time causing lesser wave force. Therefore, lesser weight has to be added to prevent sliding and hence lower values of $W/\gamma H_i^2$.

Considering all values d/h_s , $W/\gamma H_i^2$ varies from 2.225 to 10.532 for $6.24 \times 10^{-4} < H_i/gT^2 < 6.4 \times 10^{-3}$ and $S/D = 2$. The maximum value for $W/\gamma H_i^2$ observed is 10.532 for $H_i/gT^2 =$

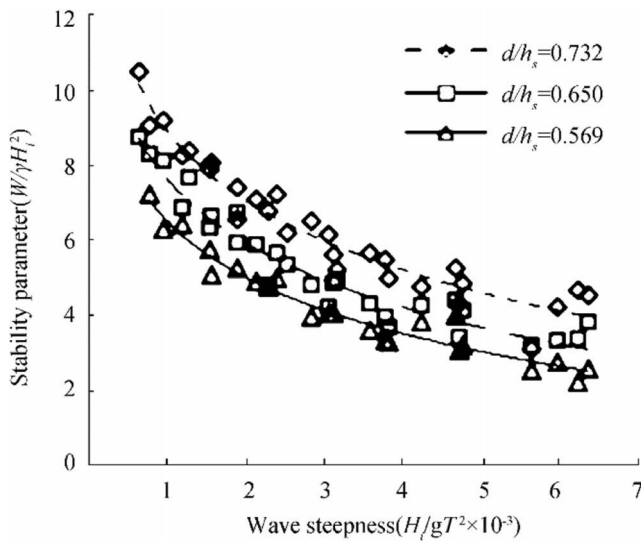


Figure 3 Influence of H_i/gT^2 on $W/\gamma H_i^2$ for $S/D=2$

6.318×10^{-4} and at 0.45 m water depth (d/h_s equal to 0.732). The minimum value for $W/\gamma H_i^2$ observed is 2.225 for $H_i/gT^2 = 6.241 \times 10^{-3}$ and at water depth equal to 0.35 m ($d/h_s = 0.569$).

4.1.2 Seaside Perforated QBW with $S/D = 2.5$

When S/D ratio is increased to 2.5, the observed values for $W/\gamma H_i^2$ vary from 2.110 to 10.269 for $6.24 \times 10^{-4} < H_i/gT^2 < 6.4 \times 10^{-3}$. The maximum value for $W/\gamma H_i^2$ observed is 10.269 for $H_i/gT^2 = 6.318 \times 10^{-4}$ and at water depth equal to 0.45 m (d/h_s equal to 0.732). The minimum $W/\gamma H_i^2$ observed is 2.110 for $H_i/gT^2 = 6.241 \times 10^{-3}$ and at water depth equal to 0.35 m ($d/h_s = 0.569$) (refer Figure 4).

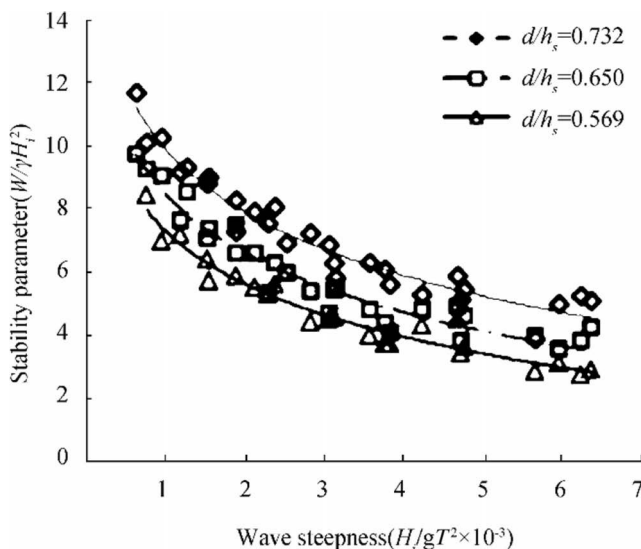


Figure 4 Influence of H_i/gT^2 on $W/\gamma H_i^2$ for $S/D=2.5$

4.1.3 Seaside Perforated QBW with $S/D = 3$

For $S/D=3$, the values obtained from the experiments for $W/\gamma H_i^2$ are observed to be varying from 2.472 to 11.223 for $6.24 \times 10^{-4} < H_i/gT^2 < 6.4 \times 10^{-3}$ (Refer Figure 5). The maximum $W/\gamma H_i^2$ observed is 11.223 for $H_i/gT^2 = 6.318 \times 10^{-4}$ and at water depth equal to 0.45 m. The minimum $W/\gamma H_i^2$ observed is 2.472 for $H_i/gT^2 = 6.241 \times 10^{-3}$ and at 0.35 m water depth.

4.1.4 Seaside Perforated QBW with $S/D = 4$

From Figure 6 showing the variation of $W/\gamma H_i^2$ with H_i/gT^2 for different water depths with radius of breakwater constant ($R = 0.55$ m or $h_s = 0.615$ m) and S/D ratio equal to 4, the values of $W/\gamma H_i^2$ are found to be varying from 2.729 to 11.668 for $6.24 \times 10^{-4} < H_i/gT^2 < 6.4 \times 10^{-3}$. The maximum $W/\gamma H_i^2$ observed is 11.688 for wave height of 0.03 m and a wave period of 2.2 s ($H_i/gT^2 = 6.318 \times 10^{-4}$) and at water depth equal to 0.45 m (d/h_s equal to 0.732). The minimum $W/\gamma H_i^2$ observed is 2.729 for wave height of 0.12 m and a wave period of 1.4 s ($H_i/gT^2 = 6.241 \times 10^{-3}$) and at water depth equal to 0.35 m (d/h_s equal to 0.569).

4.1.5 Seaside Perforated QBW with $S/D = 5$

From Figure 7, it is observed that for S/D equal to 5 and for all values d/h_s and QBW of 0.55 m radius, $W/\gamma H_i^2$ varies from 3.006 to 12.850 for $6.24 \times 10^{-4} < H_i/gT^2 < 6.4 \times 10^{-3}$. The highest value for $W/\gamma H_i^2$ observed is 12.850 for $H_i/gT^2 = 9.439 \times 10^{-4}$ and at water depth equal to 0.45 m. The lowest $W/\gamma H_i^2$ observed is 3.006 for wave height of 0.12 m and a wave period of 1.4 s ($H_i/gT^2 = 6.241 \times 10^{-3}$) and at water depth equal to 0.35 m.

For all models, it was concluded that when H_i/gT^2 increases, the dimensionless stability parameter decreases. Hence, for a given wave height, considering waves of long period (lower values of H_i/gT^2) impart a larger force on the caisson resulting in more minimum weight and short period

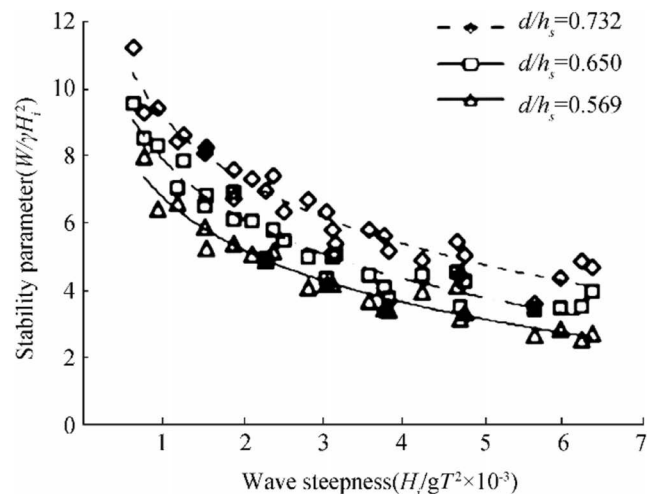


Figure 5 Influence of H_i/gT^2 on $W/\gamma H_i^2$ for $S/D=3$

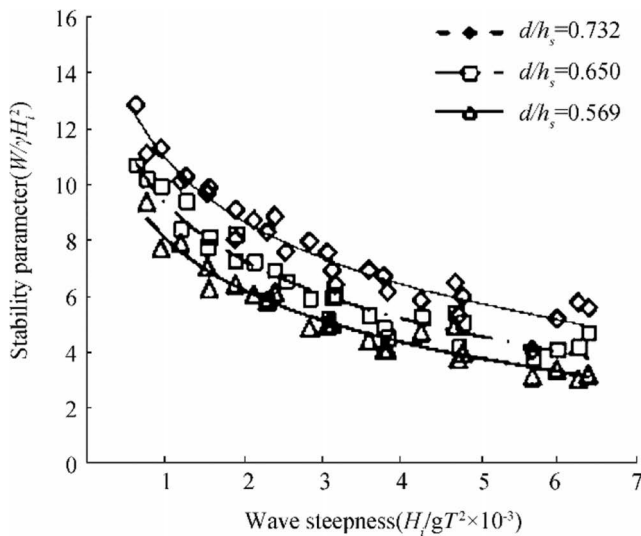


Figure 6 Influence of H_i/gT^2 on $W/\gamma H_i^2$ for $S/D = 4$

(steep) waves transfer low force, hence minimum structural weight. The safety due to sliding for higher values of wave force is increased by providing more weight into the QBW caisson thereby increasing the structural weight. The variation of $W/\gamma H_i^2$ with H_i/gT^2 for different water depths and S/D ratio for seaside perforated QBW was summarized in Table 2.

4.2 Influence of Water Depth on $W/\gamma H_i^2$

From Figure 8, it is concluded that by increasing the water depth, the values of $W/\gamma H_i^2$ will be more and the lesser value for $W/\gamma H_i^2$ is obtained for 0.35 m water depth. If water depth changes from 0.35 to 0.40 m, the values for $W/\gamma H_i^2$ were found to be increased by 17.30% to 31.24%. When water depth increases from 0.35 to 0.45 m, there is an increase in $W/\gamma H_i^2$ by 31.17% to 33.28%. Therefore, it is clear that the structure is safe against sliding with minimum weight including additional

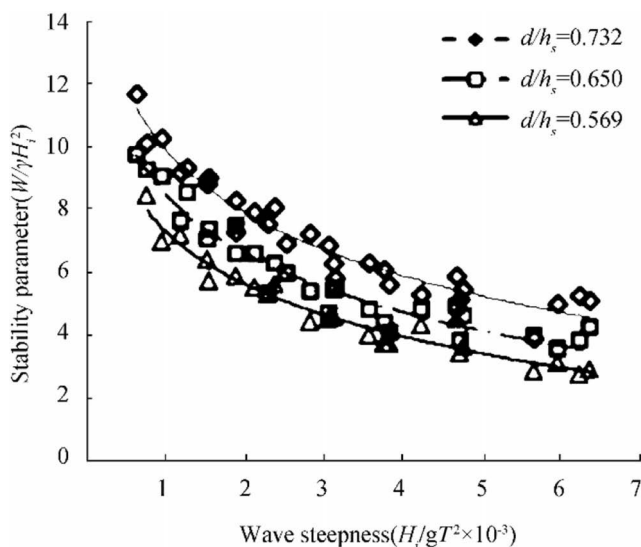


Figure 7 Influence of H_i/gT^2 on $W/\gamma H_i^2$ for $S/D = 5$

Table 2 Variation of $W/\gamma H_i^2$ with H_i/gT^2

S/D ratio	Water depth (cm)	d/h_s	Variation of $W/\gamma H_i^2$
2	0.45	0.732	3.335–10.532
	0.40	0.650	3.236–8.766
	0.35	0.569	2.225–7.249
2.5	0.45	0.732	3.198–10.269
	0.40	0.650	3.155–8.546
	0.35	0.569	2.110–6.967
3	0.45	0.732	3.566–11.223
	0.40	0.650	3.385–9.566
	0.35	0.569	2.472–7.989
4	0.45	0.732	3.882–11.668
	0.40	0.650	3.555–9.711
	0.35	0.569	2.729–8.412
5	0.45	0.732	4.056–12.850
	0.40	0.650	3.779–10.695
	0.35	0.569	3.006–9.355

weight at lower water depths. For deep water conditions, more area of the QBW will be subjected to wave action resulting in greater force and higher values of $W/\gamma H_i^2$. Hence, for a constant S/D ratio, heavier structure is required for sliding stability in more water depths compared with lower values for water depth.

4.3 Influence of S/D Ratio on $W/\gamma H_i^2$

Figure 9 shows the variation on the values of $W/\gamma H_i^2$ for different values of S/D and H_i/gT^2 . From dimensionless graphs, it was concluded that the values of $W/\gamma H_i^2$ are more for higher values of the S/D ratio. When depth of water

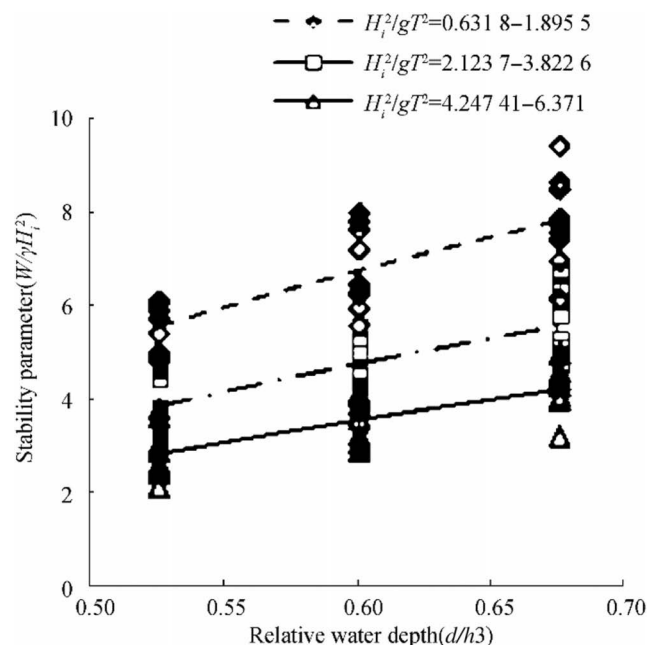


Figure 8 Influence of d/h_s on $W/\gamma H_i^2$

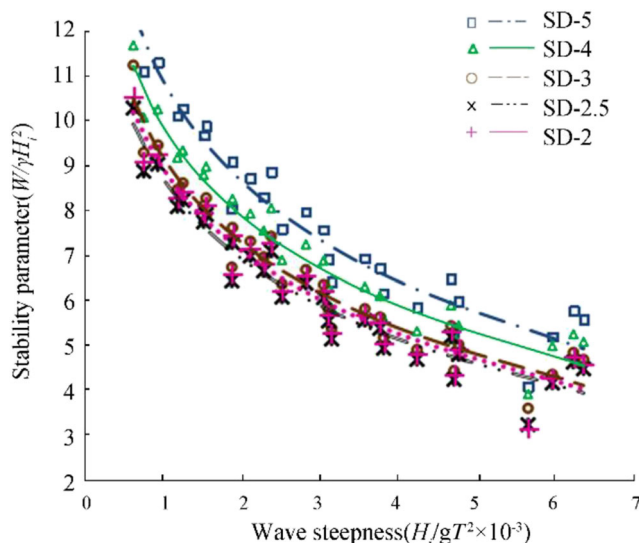


Figure 9 Influence of S/D values on $W/\gamma H_i^2$ for different H_i/gT^2

changes to 0.40 m, there will be decrease in $W/\gamma H_i^2$ from 5.92% to 9.20%, 10.42% to 10.55%, 16.51% to 20.09%, and 14.36% to 18.04% for $S/D = 4, 3, 2.5$, and 2 when compared with S/D equal to 5. Similarly, when the depth of water changes to 0.45 m, the percentage of $W/\gamma H_i^2$ reduces for $S/D = 4$ from 4.3% to 9.2% compared with $S/D = 5$.

The percentage reduction in $W/\gamma H_i^2$ for $S/D = 3, 2.5$, and 2 is 12.3% to 12.7%, 20.08% to 21.2%, and 17.8% to 18.1% with respect to $S/D = 5$. When values of S/D are decreased, $W/\gamma H_i^2$ was found to lower when compared with higher values of S/D except for the QBW model having $S/D = 2$.

For lower values of S/D , more perforations will be affected causing the absorption of a significant amount of energy due to incoming waves, and therefore, force exerted on the caisson will be less. Hence, lesser additional weight to be added for resisting sliding stability and hence the values for $W/\gamma H_i^2$ will be lower.

4.4 Equations Developed for Stability Parameter

The results for the experimental studies on stability characteristics for impermeable QBW for different breakwater radius at different water depths and wave conditions are combined into

Table 3 Summary of statistics

Variable	Minimum	Maximum	Mean	Standard deviation
$W/\gamma H_i^2$	1.967	11.668	5.515	1.850
H_i/gT^2	0.632	6.371	3.141	1.666
d/h_s	0.526	0.732	0.626	0.067
S/D	2.000	5.000	3.298	1.076

Table 4 Fitness of statistics

Observations	1254.000
Degrees of Freedom, DF	1247.000
Regression coefficient, R^2	0.909
Sum of squares of the errors, SSE	393.927
Mean of squares of the errors, MSE	0.316
Root mean squares of the errors, RMSE	0.562
Iterations	53.000

suitable dimensionless terms (Jiang et al. 2008). The regression analysis is done by using the Excel statistical software—XLSTAT—and the equation for the best fit curve is obtained.

Nonlinear regression analysis was conducted using the XLSTAT software with 1254 sets of data's for $W/\gamma H_i^2$ corresponding to different values of H_i/gT^2 , d/h_s , and S/D . Model parameters are assigned according to the limitations while selecting dependent and independent variables. For this case, $W/\gamma H_i^2$ is the dependent variable which depends on values of H_i/gT^2 , d/h_s , and S/D . Summary of the statistics is shown in Table 3 and the fitness of the statistics is shown in Table 4.

From the above table, it is clear that regression coefficient R^2 is 0.909 which means the curve obtained is the best fit curve with minimum error. The resulting equation is obtained as the output from the XLSTAT software is noted (Misra 2001).

The equation for $W/\gamma H_i^2$ for seaside perforated QBW was obtained as follows:

$$W/\gamma H_i^2 = 6.813(H_i/gT^2) - 0.503 + 31.580(d/h_s)8.357 + 0.0008(d/h_s)4.524 - 0.0841$$

Figure 10 shows the residual of the errors obtained after the regression analysis. Residual errors are obtained due to the difference of the measured results from the actual values.

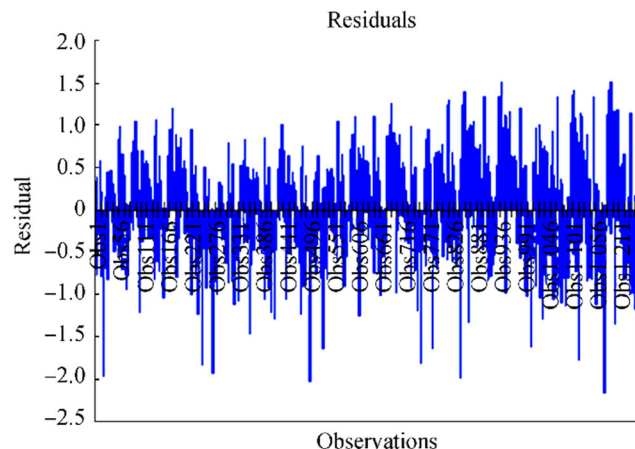


Figure 10 Residuals values of $W/\gamma H_i^2$ for perforated QBW

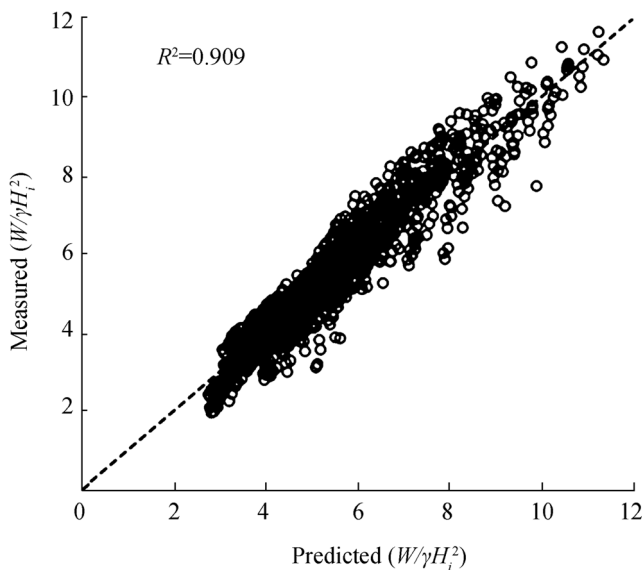


Figure 11 Comparison between measured and predicted values of $W/\gamma H_i^2$ for perforated QBW

Figure 11 shows the comparison between the measured and predicted values of stability parameter $W/\gamma H_i^2$ for perforated QBW obtained using the XLSTAT software.

5 Conclusions

Based on the model studies conducted on seaside perforated QBW, following are the conclusions obtained:

- 1) For varying d/h_s and S/D ratio, the values of $W/\gamma H_i^2$ will be lesser for higher values of H_i/gT^2 .
- 2) For different values for S/D and H_i/gT^2 , the values for stability parameter ($W/\gamma H_i^2$) are found to be more for higher values of water depth when compared with lower water depth.
- 3) The stability parameter $W/\gamma H_i^2$ is found to be reduced for lower S/D ratio with different values of H_i/gT^2 and d/h_s , but a reverse trend was obtained for seaside perforated QBW having $S/D=2$. When S/D is reduced, there will be more dissipation of wave energy and QBW is subjected to less wave force resulting in less weight of QBW to prevent sliding or lesser values of $W/\gamma H_i^2$. Further decreasing S/D to 2 causes back propagation of wave and hence exerts more force on QBW resulting in higher values for $W/\gamma H_i^2$.
- 4) Nonlinear regression analysis was carried out for the various tests conducted and the equation for the best fit curve with a good regression coefficient (R^2 value) was found out.

- 5) From the experiments conducted and the comparative analysis, it is clear that the perforated QBW of $S/D=2.5$ has better performance characteristics and can be adopted in the site with the favorable condition.

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Nomenclature

S/D	Spacing between perforations/diameter of perforations
H_i	Incident wave height
D	Depth of water
g	Acceleration due to gravity
T	Wave period
L	Wave length
W	Weight of QBW per unit length
h_s	Height of structure
γ	Weight density
d/h_s	Relative water depth
H_i/gT^2	Incident wave steepness parameter
$W/\gamma H_i^2$	Stability parameter

References

- Binumol S, Rao S, Hegde AV (2017) Wave reflection and loss characteristics of an emerged quarter circle breakwater with varying sea-side perforations. *J Inst Eng (India)*: Ser A Springer 98(3):311–315
- Chaudhary B, Hazarika H, Nishimura N (2017) Effects of duration and acceleration level of earthquake ground motion on the behavior of unreinforced and reinforced breakwater foundation. *Soil Dyn Earthq Eng* 98:24–37
- Chaudhary B, Hazarika H, Murakami A, Fujisawa K (2018a) Geosynthetic-sheet pile reinforced foundation for mitigation of earthquake and tsunami induced damage of breakwater. *Geotext Geomembr* 46:597–610
- Chaudhary B, Hazarika H, Murakami A, Fujisawa K (2018b) Countermeasures for enhancing the stability of composite breakwater under earthquake and subsequent tsunami. *Acta Geotech* 13(4): 997–1017
- Chaudhary B, Hazarika H, Murakami A, Fujisawa K (2019) Development of resilient breakwater against earthquake and tsunami. *Int J Geomech, ASCE* 19 (1): 04018188–1 to 17
- Hughes SA (1993) Physical models and laboratory techniques in coastal engineering advanced series on Ocean Engineering. World Scientific Singapore
- Isaacson M (1991) Measurement of regular wave reflection. *J Waterw Port Coast Ocean Eng* 117(6):553–569
- Jiang XL, Gu HB, Li YB (2008) Numerical simulation on hydraulic performances of quarter-circular breakwater. *China Ocean Eng* 22(4):585–594
- Luwen Q, Zhang X, Jiang X, Qin Y (2013) Research on partial coefficients for the design of quarter-circular caisson breakwater. *J Mar Sci Appl* 12:65–71

- Misra SC (2001) Uncertainty analysis in hydrodynamic tests. Proceedings, International Conference in Ocean Engineering, 200-207
- Shi Y-J, Mi ling W, Jiang X, Li Y-b (2011) Experimental researches on reflective and transmitting performances of quarter-circular breakwater under regular and irregular waves. China Ocean Eng 25:469–478
- Xie SL, Li YB, Wu YQ, Gu HB (2006) Preliminary research on wave forces on quarter circular breakwater. Ocean Eng J 24(1):14–18