

Fractional-Order Super-Twisting Sliding-Mode Procedure Design for a Class of Fractional-Order Nonlinear Dynamic Underwater Robots

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Abstract

The purpose of this study is to design a fractional-order super-twisting sliding-mode controller for a class of nonlinear fractional-order systems. The proposed method has the following advantages: (1) Lyapunov stability of the overall closed-loop system, (2) output tracking error's convergence to zero, (3) robustness against external uncertainties and disturbances, and (4) reduction of the chattering phenomenon. To investigate the performance of the method, the proposed controller is applied to an autonomous underwater robot and Lorenz chaotic system. Finally, a simulation is performed to verify the potential of the proposed method.

Keywords Underwater robot · Fractional-order system · Sliding-mode control · Super-twisting algorithm · Lyapunov function

1 Introduction

In the last two decades, autonomous underwater vehicles (AUVs) have performed general and inclusive tasks in investigations and research (Wang et al. 2014). These vehicles have been used in military, commercial, and scientific research and mapping actions, hence eliminating the need for human machinists. The high capacity of AUV robots and their human superiority in oceanic missions, especially at high depths, has enabled them to play a key role in underwater industries.

The main limitation of these robots is the existence of various conditions in the modeling of systems. The dynamics and control of these vehicles are highly attractive yet complicated because of nonlinear equations, structural and nonstructural uncertainties, and the presence of external disturbances caused by ocean flows (Cajo et al. 2019). These challenges and the

vast autonomous underwater robot applications are the main inspiration of this work (Wang et al. 2012).

In recent years, fractional calculus has received increasing attention (Kilbas et al. 2006). About 300 years ago, fractional calculus was a mathematical topic, but it is rapidly developing nowadays. Fractional calculus has many applications, mainly in the field of mathematical sciences and engineering, such as electromagnetism, optics, electrochemistry, controllers (Birs et al. 2019), fluid mechanics, signal processing (Aslam and Raja 2015), biological population dynamics models (Magin 2006), chaos (Senejohnny and Delavari 2012; Tavazoei et al. 2008), and viscoelasticity (Ibrir and Bettayeb 2015). To model physics and engineering processes, fractional differential equations are used because fractional-order systems are more accurate than classical integer-order dynamic systems in describing and modeling real objects (Podlubny 1998; Valdes-Parada et al. 2007). In the past decades, surveys on fractional-order controller design methodologies for some systems have been a special research topic. Fractional-order $PI^{\lambda}D^{\mu}$ controller (Birs et al. 2016) and fractional-order PD path-following control (Cajo et al. 2018) are some of the well-known fractional-order robust controllers.

Sliding-mode (SM) controller has been widely used to design robust controllers. It is a variable control structure with a simple structure, fast response, and no sensitivity to internal parameters and external disturbances (Utkin 1992). However, the traditional SM controller suffers from

Article Highlights

- The proposed methodology developed the Lyapunov theorem for fractional-order nonlinear system.
- Robustness of the proposed fractional-order sliding mode is guaranteed.
- Super-twisting sliding-mode controller decreases the chattering phenomena.

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chattering due to its discrete nature. A high-order SM controller decreases chattering and preserves the advantages of a conventional SM controller (Munoz et al. 2007; Lan et al. 2008). One of the algorithms that are widely applied in high-order SM controllers is the super-twisting algorithm. Rooka and Ghasemi (2018) presented a fuzzy fractional SM observer for a class of nonlinear dynamics for the diagnosis of cancer. Sharafian and Ghasemi (2017) studied the fractional neural network observer for a class of chaotic systems. The proposed method in the present study ensures the stability of the closed-loop system and a zero tracking error convergence even under disturbances.

In this paper, a fractional super-twisting controller design procedure for the nonlinear fractional dynamics of an underwater robot and the Lorenz system with guaranteed stability is proposed. Compared with other studies, this work focuses on the super-twisting SM controller design for a disturbed nonlinear system. Our methodology has the following merits: (1) A fractional class of super-twisting SM controller was derived to reduce the chattering phenomena. (2) The stability of the closed-loop system and the convergence of the tracking error to zero are both guaranteed even under disturbances.

The rest of this paper is presented as follows: Section 2 includes the basic definitions of fractional integral and differential. Section 3 provides a problem formulation on the AUV system. Section 4 presents a problem formulation on the Lorenz system. Section 5 illustrates the simulation results of the method on chaotic fractional systems. Finally, Section 6 presents some brief conclusions.

2 Principles

Differentiation and integration are generalized to a non-integer-order fundamental operator, which is defined in fractional calculus as follows (Petrás 2010):

$$D_t^q = \begin{cases} \frac{d^q}{dt^q} & q > 0 \\ I & q = 0 \\ \int_a^t (d\tau)^{-q} & q < 0 \end{cases} \quad (1)$$

where a and t are the operation bounds and $q \in \mathbb{R}$ is the order.

These definitions are widely used for fractional derivative and fractional integral as described in this section.

Definition 1 The Riemann–Liouville (RL), Grunwald–Letnikov (GL), and Caputo (C) derivatives of order q of function $f(t)$ are given in the following (Chen et al. 2011; Yan et al. 2009):

$${}^{GL}D_t^q f(t) = \lim_{N \rightarrow \infty} \left[\frac{t-a}{N} \right]^{-q} \sum_{j=0}^{N-1} (-1)^j \binom{q}{j} f\left(t - j \left[\frac{t-a}{N} \right]\right) \quad (2)$$

$${}^{RL}D_t^q f(t) = \frac{1}{\Gamma(1-q)} \frac{d}{dt} \int_a^t (t-\tau)^{-q} f(\tau) d\tau \quad (3)$$

$${}^CD_t^q f(t) = \frac{1}{\Gamma(1-q)} \int_a^t (t-\tau)^{-q} \dot{f}(\tau) d\tau \quad (4)$$

where $0 < q < 1$ and the Gamma function is $\Gamma(\cdot)$.

Definition 2 The RL fractional integral of order q is defined as

$$D_t^{-q} f(t) = \frac{1}{\Gamma(q)} \int_a^t (t-\tau)^{q-1} f(\tau) d\tau \quad (5)$$

3 Dynamics Model of AUVs

The motion of an AUV is designated in 6 degrees of freedom (DOF), such as the rotations and displacements that label the orientation and position of the vehicle, including the heave, surge, sway, pitch, roll, and yaw (Yuh 2000). The underwater robot is a type of moving robots, and two special reference frames represent the study of the motion of AUVs. The earth-fixed $\{n\}$ frames are considered to be inertial with a fixed origin at the center of the frame, and the body-fixed $\{b\}$ frame is a frame fixed to the vehicle that moves with its origin at the center of mass of the vehicle, as shown in Figure 1.

In this study, the robot movements are evaluated only on the horizontal plane and the roll and pitch movements are neglected (Wang et al. 2012). The kinematic equations of this model are as follows:

$$\begin{cases} D^q x = u \cos\psi - v \sin\psi \\ D^q y = u \sin\psi + v \cos\psi \\ D^q \psi = r \end{cases} \quad (6)$$

The surge and sway linear velocities of the AUV are represented by u and v , respectively. The yaw angular velocity of the vehicle is presented by r , x and y are the coordinates of the AUV's center of mass, and the orientation of the vehicle is represented by ψ states. The earth-fixed frame $\{n\}$ defines the position and orientation of the robot (i. e., (x, y, ψ)), and the body-fixed frame $\{b\}$ defines the linear and angular velocities (i. e., (u, v, r)).

I_z refers to the AUV's moment of inertia about the z -axis; m indicates the robot's mass; X_u , Y_v , and N_r illustrate the negative terms, including linear damping impacts; and X_u , Y_v , and N_r represent the hydrodynamic additional mass terms in the surge, sway, and yaw directions of motion in turn. The vertical dynamics of an AUV is as follows:

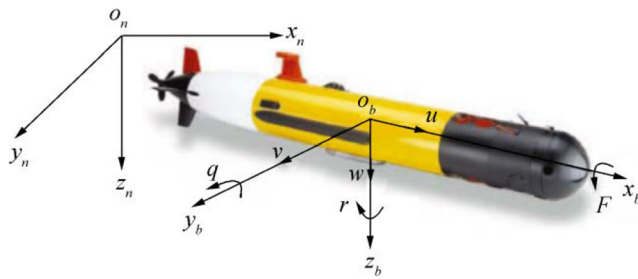


Figure 1 Earth-fixed and body-fixed reference frames for an AUV

$$\begin{aligned} D^q u &= A_1(X_u u + B_1 v r + f_u) \\ D^q v &= A_2(Y_v v + B_2 u r + f_v) \\ D^q r &= A_3(N_r u + B_3 u v) \end{aligned} \quad (7)$$

where $A_1 := 1/(m - X_{\dot{u}})$, $A_2 := 1/(m - Y_{\dot{v}})$, $A_3 := 1/(I_z - N_{\dot{r}})$, $B_1 := m - Y_{\dot{v}}$, $B_2 := X_{\dot{u}} - m$, and $B_3 := Y_{\dot{v}} - X_{\dot{u}}$. The control inputs f_u and f_v are the surge and sway force, respectively.

To formulate the trajectory tracking controller design methodology, the following position tracking errors are considered:

$$\begin{aligned} e_x &= x - x_d \\ e_y &= y - y_d \end{aligned} \quad (8)$$

where x_d and y_d stand for the coordinates of the desired trajectory.

By substituting Eq. (6) in the time q -order derivative of Eq. (8), the position dynamics of error is obtained as follows:

$$\begin{bmatrix} D^q e_x \\ D^q e_y \end{bmatrix} = \begin{bmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} - \begin{bmatrix} D^q x_d \\ D^q y_d \end{bmatrix} \quad (9)$$

Moreover, the velocity tracking errors are defined as

$$\begin{aligned} e_u &= u - u_d \\ e_v &= v - v_d \end{aligned} \quad (10)$$

where u_d and v_d are the desired surge and sway velocities, respectively. With the q -order derivative of Eq. (10) and using Eq. (7), the following equations are obtained.

$$\begin{aligned} D^q e_u &= A_1(X_u u + B_1 v r + f_u) - D^q u_d \\ D^q e_v &= A_2(Y_v v + B_2 u r + f_v) - D^q v_d \end{aligned} \quad (11)$$

In a recent project on an underwater robot controller design, the following reference velocities (u_d and v_d) on the time q -order derivatives of the desired trajectory are chosen:

$$\begin{aligned} u_d &= \begin{pmatrix} D^q x_d \cos\psi + D^q y_d \sin\psi \\ +l_x \tanh(-k_x e_x) \cos\psi \\ +l_y \tanh(-k_y e_y) \sin\psi \end{pmatrix} \\ v_d &= \begin{pmatrix} D^q y_d \cos\psi - D^q x_d \sin\psi \\ -l_x \tanh(-k_x e_x) \sin\psi \\ +l_y \tanh(-k_y e_y) \cos\psi \end{pmatrix} \end{aligned} \quad (12)$$

where $k_x, k_y > 0$ are the controller gains and $l_x, l_y > 0$ are the saturation constants.

To demonstrate the stability of the method, the Lyapunov function was

$$V = \frac{1}{2} e_x^2 + \frac{1}{2} e_y^2 \quad (13)$$

Taking the q -order fractional derivative of the Lyapunov function in Eq. (13) yields the following:

$$\begin{aligned} D^q V &< (e_x D^q e_x + e_y D^q e_y) \\ &= \begin{pmatrix} -l_x e_x \tanh(-k_x e_x) \\ -l_y e_y \tanh(-k_y e_y) \end{pmatrix} < 0 \end{aligned} \quad (14)$$

4 Super-Twisting SM Controller Design

4.1 AUV Systems

To design the SM tracking controller, the subsequent sliding surfaces were considered:

$$\begin{aligned} s_1 &= e_u \\ s_2 &= e_v \end{aligned} \quad (15)$$

Taking the q -order fractional differential of Eq. (15) yields

$$\begin{aligned} D^q s_1 &= D^q e_u = A_1(X_u u + B_1 v r + f_u) - D^q u_d \\ D^q s_2 &= D^q e_v = A_2(Y_v v + B_2 u r + f_v) - D^q v_d \end{aligned} \quad (16)$$

Then, the control inputs by the super-twisting algorithm were introduced as

$$\begin{aligned} f_u &= \frac{1}{A_1} (D^q u_d - X_u u - B_1 v r + u_{r1}(t)) \\ f_v &= \frac{1}{A_2} (D^q v_d - Y_v v - B_2 u r + u_{r2}(t)) \end{aligned} \quad (17)$$

where $u_{ri}(t)$ is the reaching controller part that is defined as follows:

$$\begin{aligned} u_{ri}(t) &= \frac{1}{\lambda_i} [-\alpha |s_i|^{\rho_i} \text{sgn}(s_i) - \beta] \text{sgn}(s_i) dt \\ i &= 1, 2, \dots \end{aligned} \quad (18)$$

where $0 < \rho < 1$, $\alpha, \beta > 0$.

Theorem 1 Consider the nonlinear fractional-order dynamical AUV system designated by Eqs. (6) and (7). Suppose the

velocity tracking errors as mentioned in Eq. (10) with the desired velocities in Eq. (12). When the control inputs for the surge and sway planned in Eqs. (17) and (18) are used to the dynamics model of the AUV, then the asymptotic convergence of the velocity and trajectory tracking errors to the neighborhood of zero are guaranteed.

Proof: Candidate the Lyapunov function to investigate the closed-loop stability as

$$V = \frac{1}{2}s_1^2 + \frac{1}{2}s_2^2 \quad (19)$$

Take the q -order fractional derivative of a proposed Lyapunov function to have

$$D^q V < s_1 D^q s_1 + s_2 D^q s_2 \quad (20)$$

Thus, by using Eqs. (16), (17), and (18), the above equation is rewritten as follows:

$$D^q V < \left(s_1 \frac{1}{\lambda_1} \left[\frac{-\alpha|s_1|^{\rho_1} \text{sgn}(s_1)}{-\beta \int \text{sgn}(s_1) dt} \right] + s_2 \frac{1}{\lambda_2} \left[\frac{-\alpha|s_2|^{\rho_2} \text{sgn}(s_2)}{-\beta \int \text{sgn}(s_2) dt} \right] \right) \quad (21)$$

Thus,

$$D^q V < -\alpha|s_1|^{\rho_1+1} - \beta \int |s_1|^{\rho_1+1} - \alpha|s_2|^{\rho_2+1} - \beta \int |s_2|^{\rho_2+1} < 0 \quad (22)$$

According to the standard Lyapunov theorem, the control inputs proposed in Eqs. (17) and (18) guarantee the asymptotic convergence of (e_x, e_y) to the neighborhood of zero, hence completing the proof.

4.2 Lorenz System

In this section, the super-twisting SM controller is designed for a class of nonlinear fractional-order systems as (Aghababa 2012)

$$\begin{cases} D^q x = F_1(x, y, z) \\ D^q y = F_2(x, y, z) + u(t) + \sigma(t) \\ D^q z = F_3(x, y, z) \end{cases} \quad (23)$$

where $X = [x, y, z]^T$ is the state variable; F_1 , F_2 , and F_3 show the nonlinear function; u represents the control input; and $\sigma(t)$ shows the bounded uncertainties. The Chen, Liu, Lu, and Lorenz chaotic systems (Senejohnny and Delavari 2012) are presented in Eq. (23).

The conventional SM controller design uses a linear sliding-mode (LSM) surface to track the desired trajectory, which causes an inherently nonlinear control. The main challenge in the LSM controller design is the sliding surface selection based on the system performance requirements.

The first step in controller design is assigning a sliding surface, such that

$$s = x + \lambda_1 y + \lambda_2 z \quad (24)$$

where λ_1 and λ_2 are the positive constants. Taking the q -order fractional differential of the above equation yields

$$D^q s = D^q x + \lambda_1 D^q y + \lambda_2 D^q z \quad (25)$$

The control objectives are (1) tracking of the desired trajectories, (2) convergence of the sliding surface to zero without chattering, and (3) closed-loop system stability. Without loss of generality, the desired trajectories assume as zero.

The sliding surface has been shown to achieve control objectives. Here, we show how to derive the control input. The system's total input can be proposed as follows:

$$u(t) = u_{eq}(t) + u_r(t) \quad (26)$$

where $u_{eq}(t)$ and $u_r(t)$ are the equal and reaching controller parts that are defined as follows:

$$u_{eq}(t) = \frac{1}{\lambda_1} [-F_1(x, y, z) - \lambda_1 F_2(x, y, z) - \lambda_2 F_3(x, y, z) - \lambda_1 \sigma(t)] \quad (27)$$

and

$$u_r(t) = \frac{1}{\lambda_1} [-\alpha|s|^{\rho} \text{sgn}(s) - \beta \int \text{sgn}(s) dt] \quad (28)$$

The first term compensates the uncertainties of the model and the second one reduces the chattering phenomena in which the parameters mentioned in Eq. (28) satisfy the following inequalities.

$$0 < \rho < 1, \alpha, \beta > 0$$

To facilitate the super-twisting SM controller, we derived the theorem below.

Theorem 2 The nonlinear fractional-order dynamics system given in Eq. (23) and the controller structure proposed in Eqs. (26), (27), and (28) based on the sliding surface mentioned in Eq. (24) make the closed-loop system stable according to the Lyapunov. Furthermore, both the convergence of the error trajectory to zero and boundedness of all signals involved in the closed-loop system are guaranteed.

Proof: We candidate the following Lyapunov function to investigate the closed-loop stability.

Table 1 REMUS AUV model parameters

Parameter	Value
M (kg)	30.48
I_z (kg·m ²)	3.45
X_u (kg/s)	-8.8065
Y_v (kg/s)	-65.5457
N_r (kg/s)	-6.7352
$X_{\dot{u}}$ (kg)	-0.93
$Y_{\dot{v}}$ (kg)	-35.5
$N_{\dot{r}}$ (kg·m ²)	-35.5

$$V = \frac{1}{2} s^2 \quad (29)$$

Taking the q -order fractional derivative of a proposed Lyapunov function leads to

$$D^q V = \sum_{i=0}^{\infty} \left(\frac{q}{i} \right) s^{(i)} D^{(q-i)} s < s D^q s \quad (30)$$

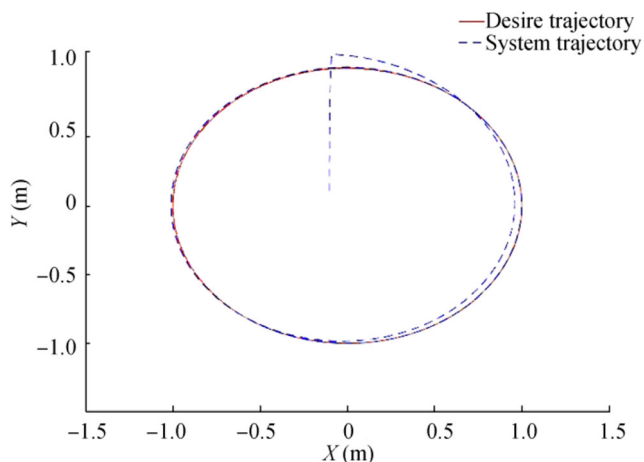
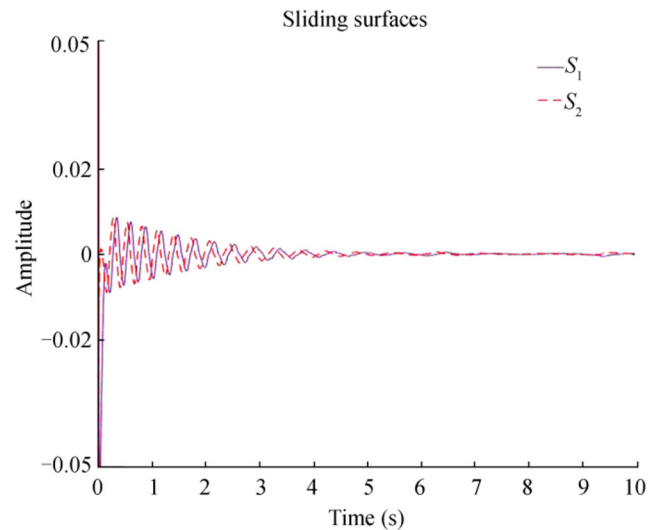
Therefore, the above equation is rewritten as follows:

$$\begin{aligned} D^q V &< s D^q s = s [D^q x + \lambda_1 D^q y + \lambda_2 D^q z] \\ D^q V &< s [F_1(x, y, z) + \lambda_1 F_2(x, y, z) + \lambda_1 \sigma(t) + \lambda_1 u + \lambda_2 F_3(x, y, z)] \end{aligned} \quad (31)$$

In accordance with Eq. (26), Eq. (31) can be rewritten as

$$D^q V < -\alpha |s|^{\rho+1} - \beta |s| < 0 \quad (32)$$

Lyapunov theorem suggests that the closed-loop system is stable and the sliding surface converges to zero. The

**Figure 2** Motion path of the robot and reference path**Figure 3** Sliding surfaces of the robot

boundedness of the signals in the closed-loop system is also guaranteed, hence completing the proof.

This approach is used to a class of chaotic systems to demonstrate its potentials.

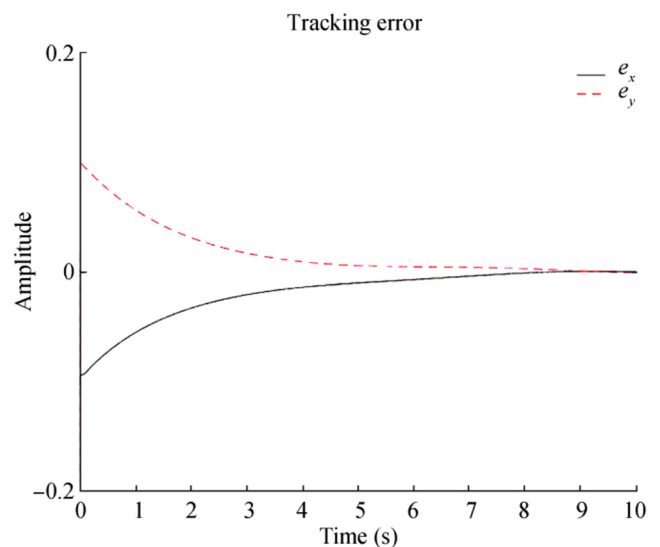
5 Simulation Results

In this section, the proposed control scheme is applied in the simulation test cases to investigate its efficacy and capability.

5.1 AUV Systems

Consider the following nonlinear q -order fractional AUV system.

The suggested control structure is applied to an example of 3-DOF AUVs, which is labeled by the Eqs. (6) and (7).

**Figure 4** Tracking error of the robot

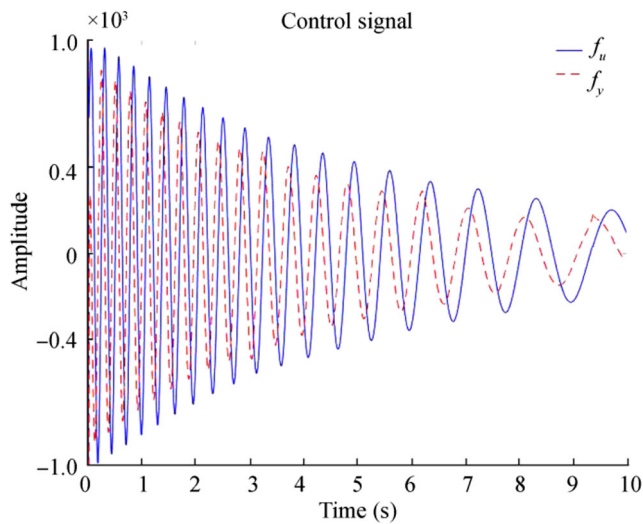


Figure 5 Surge and sway control laws of the robot

Considering the REMUS AUV, the values of the hydrodynamic vehicle's parameters and additional masses are presented in Table 1. The simulation results are derived using circular desired paths as $x_d = \sin t$, $y_d = \cos t$.

The parameters of the design are considered as follows: $l_x = l_y = 2$, $k_x = k_y = 0.5$. Figure 2 through Figure 5 represents the simulation results: Figure 2 shows the real and desired trajectories of the vehicle, and Figure 3 shows how the sliding surfaces S_1 and S_2 are enforced by the controller toward zero. Figure 4 represents how the tracking errors e_x and e_y of the AUV's trajectory converge asymptotically toward zero. Finally, the surge and sway control inputs are represented in Figure 5.

The simulation outcomes show the capable performance of the suggested methodology. Comparing our research with that of Tabataba'i-Nasab et al. (2018), the tracking error is shown in Figures 4 and 6, and the proposed methodology reduces chattering and extends the stability criteria.

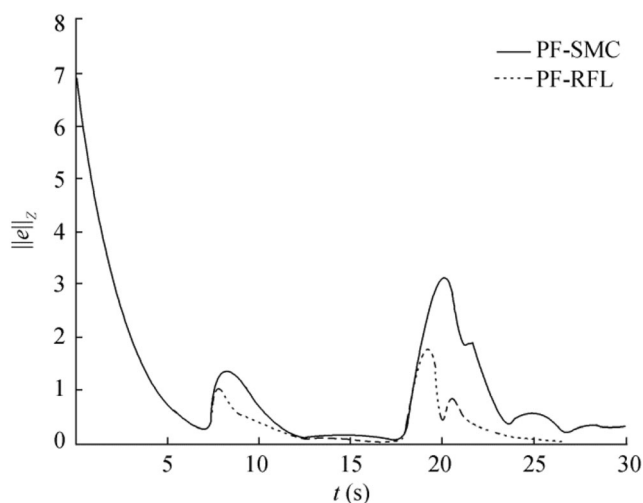
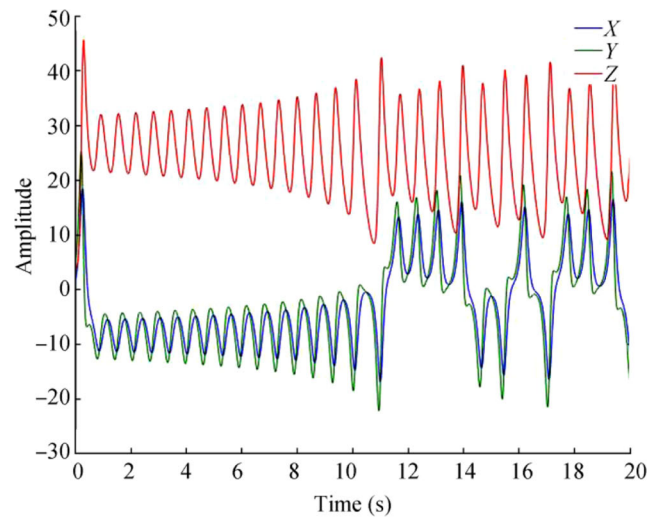
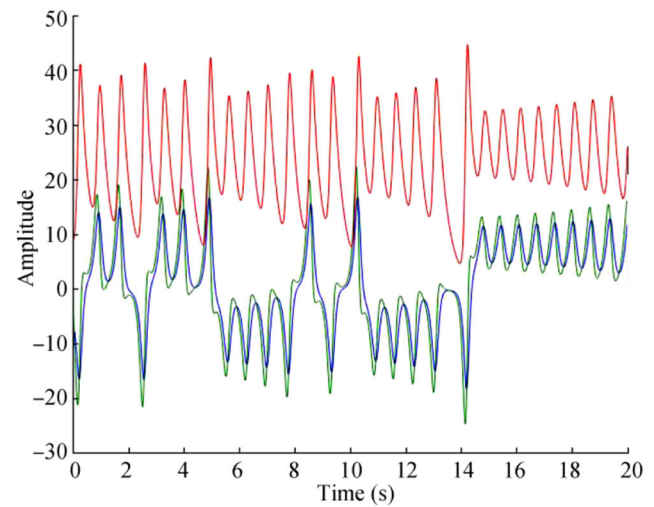


Figure 6 Tracking error of the robot in Tabataba'i-Nasab et al. (2018)



(a) $[x(0), y(0), z(0)] = [-9, -1, 9]$



(b) $[x(0), y(0), z(0)] = [1, 5, 4]$

Figure 7 State variable of the original system

5.2 Lorenz Systems

Consider the following nonlinear q -order fractional Lorenz system.

$$\begin{cases} D^q x = a(y-x) \\ D^q y = cx - xz - y + u(t) + \sigma(t) \\ D^q z = xy - bz \end{cases} \quad (33)$$

where $(a, b, c) = (10, \frac{8}{3}, 28)$ and $\sigma(t) = \sin(\omega t)$ and considered an external disturbance.

To show the extremely chaotic performance of Eq. (33), Figure 6 is derived for the initial conditions $[x(0), y(0), z(0)] = [-9, -1, 9]$ and $[x(0), y(0), z(0)] = [1, 5, 4]$. Using the proposed controller in Eq. (26), the states of the system are shown in Figure 7. Figures 6 and 7 clearly

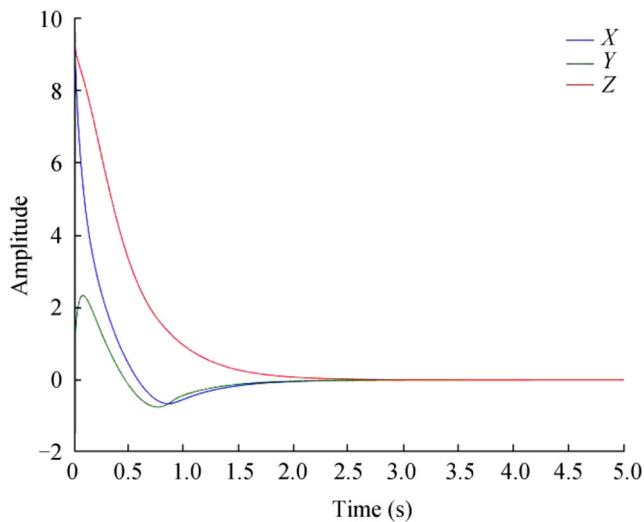


Figure 8 State variable with the proposed method with the initial condition $[x(0), y(0), z(0)] = [9, 1, 9]$

show the convergence of the states of the system to the neighborhood of zero without chattering. Figure 8 demonstrates that the smoothness of the sliding surface tends to zero. Figure 9 shows the control effort without any chattering phenomenon.

The simulation results show the promising performance of the proposed controller in terms of tracking and stability and its successful attempt in chattering reduction (Figure 10).

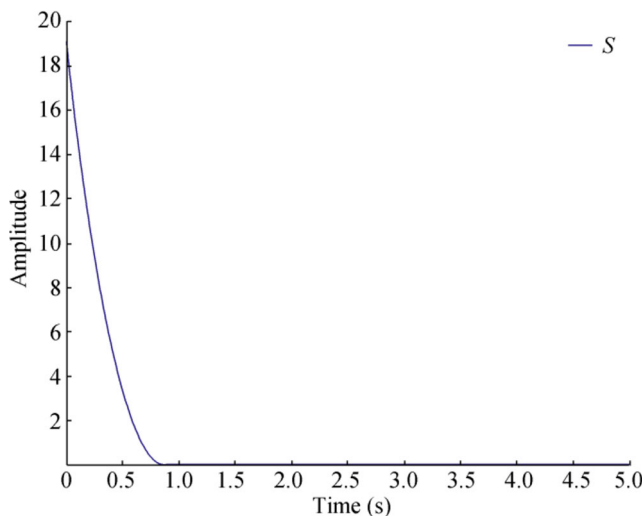


Figure 9 Proposed sliding surface

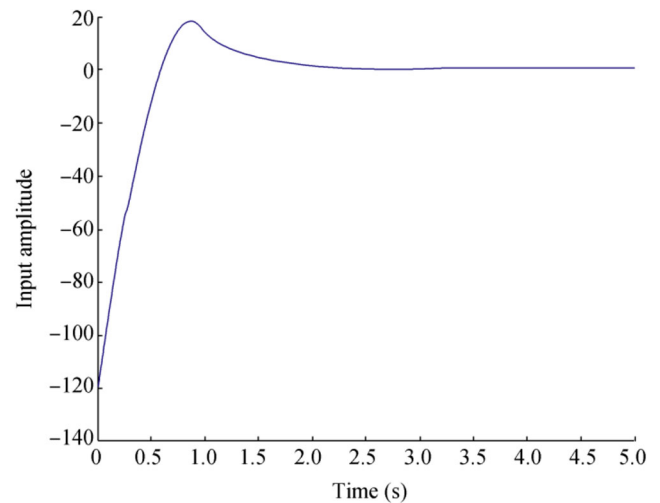


Figure 10 Proposed control signal

6 Conclusions

A super-twisting SM controller for a class of fractional-order nonlinear systems is designed for a class of underwater robots, in this study. The tracking error convergence to zero and boundedness of all closed-loop signals are ensured through fractional-order Lyapunov stability theorem. In addition, the proposed method is robust against external disturbances and uncertainties. The super-twisting approach decreases the chattering phenomena to reduce the simulation of the actuator. The proposed approach is used to a chaotic system and an underwater vehicle to verify its performance. Accordingly, the simulation results validate the performance of the proposed method.

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