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Performance Characterisation for Risk Assessment of Striking Ship Impacts Based on Struck Ship Damaged Volume

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Abstract: Ship collision accidents are rare events but pose huge threat to human lives, assets, and the environment. Many researchers have sought for effective models that compute ship stochastic response during collisions by considering the variability of ship collision scenario parameters. However, the existing models were limited by the capability of the collision computational models and did not completely capture collision scenario, and material and geometric uncertainties. In this paper, a novel framework to performance characterisation of ships in collision involving a variety of striking ships is developed, by characterising the structural consequences with efficient response models. A double-hull oil carrier is chosen as the struck ship to demonstrate the applicability of the proposed framework. Response surface techniques are employed to generate the most probable input design sets which are used to sample an automated finite element tool to compute the chosen structural consequences. The resulting predictor-response relationships are fitted with suitable surrogate models to probabilistically characterise the struck ship damage under collisions. As demonstrated in this paper, such models are extremely useful to reduce the computational complexity in obtaining probabilistic design measures for ship structures. The proposed probabilistic approach is also combined with available collision frequency models from literature to demonstrate the risk tolerance computations.

Keywords: ship collision, hull damage, numerical simulation, structural reliability, risk assessment

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1 Introduction

Ship accidents are recorded annually despite the proactive efforts to prevent their occurrence. Ships may be subjected to a variety of accidents such as collision, contact, grounding, foundering, fire and explosion. According to the statistical data provided by the International Tanker Owners Pollution Federation (ITOPF), ship-ship collision is one of the most catastrophic accidents among tankers resulting in oil spills (ITOPF, 2014). Spill from ship collision poses a great threat to marine biodiversity, human lives, structural assets and the reputation of the companies involved. Statistics on ship collision accidents between 1990 and 2011 are represented using a bar chart in Fig. 1. The ships involved in collisions are divided into six categories which are also considered later in the present study. It is observed that an average of about 20 ship collisions occurs annually (IHS Fairplay, 2012). Due to the recurring ship collision accidents, several researchers have attempted to analyse the associated impact on ship structural integrity (Béghin, 2013; Ringsberg, 2010; Samuelides, 2015). Also, ship collision accidents and their consequences motivated the introduction of the Formal Safety Assessment (FSA) procedures by the International Maritime Organisation (IMO) which involve the use of Quantitative Risk Assessment (QRA) to enhance maritime and environmental safety (IMO, 2002).

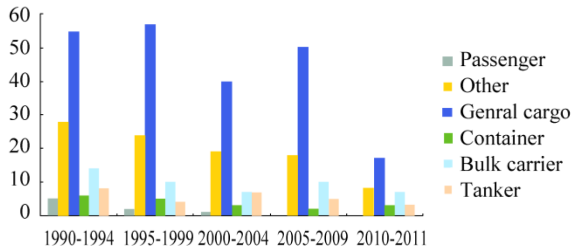

Fig. 1 Number of ship collision accidents by ship categories

Ship collision analysis is performed by estimating the likelihood of ship collisions and assessing the magnitude of the consequences. These consequences could be structural, environmental and economical which may lead to fatalities. The frequency of ship collisions and the consequences are quantified by considering the parameters that influence relevant collision scenarios, such as ship velocity, collision angle, water depth, sea state and crew competence. These parameters may be deterministic or stochastic in nature. Stochastic modelling of ship collisions involves the consideration of uncertainties stemming from the input random variables influencing the collision scenario. The uncertainties may be quantified by utilising data obtained from historical ship accidents to determine suitable probability density functions and other probabilistic characteristics of the random variables (Brown, 2002; Lutzen, 2001; Rawson *et al*., 1998; Tuovinen, 2006; Youssef *et al*., 2013).

The guidance notes for collision analysis by Lloyd's

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Register (2014) suggests the requirement for a wide variety of collision risk assessments to determine the damage susceptibility and criticality for ships and offshore installations. Hence, the assessment of ship damage in terms of the consequences can be used as the basis for setting structural performance targets. In the context of structural damage, risk is evaluated as:

$$\text{Risk} = \sum_{i=1}^{n} P_i\left(\text{Damage}\right) \times \text{Consequence}_i \qquad (1)$$

where P(Damage) is the probability of damage that considers all potential accident sequences. In the context of a ship collision event:

$$P\left(\text{Damage}\right) = P(\text{Damage}\middle|\text{Collision}) \times P(\text{Collision}) \qquad (2)$$

where P(Damage|Collision) is the probability of damage given a collision and P(Collision) is the probability of a collision, a function of the geometric and causation probabilities. The geometric probability is the number of ship collision candidates per unit of time if no aversive action is taken and the causation probability is the probability of failing to avoid the collision while on a collision course. In the presence of uncertainties in the parameters of ship collisions, ship structures are expected to achieve specific performance targets. These performance targets can be compared with the failure criteria associated with ship structural damage. A limit state function can then be developed, with respect to a failure criterion, to estimate the probability of ship structural failure during collisions. In the context of the present work, a damage event refers to the elements of a ship structure that have experienced a loss of load carrying capacity, that is the stage of deformation has exceeded the initiation of material fracture. To assess ship structural consequences with respect to ship collision accidental limit states, decisions need to be made about the failure criteria associated with ship structural damage. The extent of ship collision assessment is usually influenced by at least one of the four structural failure criteria: onset of the outer hull fracture; onset of the inner hull fracture; equivalence of the bow penetration to the double-hull depth and the dissipation of total kinetic energy (Haris and Amdahl, 2012; Paik *et al*., 2009; Wang *et al*., 2000; Youssef *et al*., 2014a).

There have been great achievements in assessing structural performances in ship collision analysis. However, the analysis requires extensive resources that involve multiple simulation of stochastic numerical models influenced by collision scenarios. Efforts are put into finding ways of simplifying the task. Recent stochastic modelling approaches have involved the evaluation of damage distributions using stochastic numerical models that are criteria-based, hence may not require the input of the external dynamics of the collision process (Ståhlberg *et al*., 2013; Youssef *et al*., 2014a). Existing computational models derive the damage and energy distributions of sampled collision scenarios based on the corresponding total collision energy loss and this approach will result in the evaluation of failure probabilities for a specified collision scenario only (e.g. Brown, 2002; Lutzen, 2001). There is the need to make informed decisions from the stochastic modelling of ship collisions that will ensure the paradigm shift to a risk-based ship design. A more recent effort looked at ways to develop mathematical relationships that govern the performance of ships during collisions while being suitable for structural reliability analysis (Obisesan *et al*., 2015). So far, this effort has been limited to the characterisation of material and geometric uncertainties in simplified analytical models. Furthermore, the capabilities of the existing collision computation models are limited due to cost of multiple simulation evaluations which may make tasks such as performance assessment, sensitivity analysis and design optimisation highly challenging.

Considering the highlighted limitations of existing models, the present study proposes a novel stochastic modelling framework for collision performance assessment in risk-based ship designs. Essentially, the framework investigates the structural behaviour of ships involved in collisions and shifts the deterministic analytical models of ship structural members to a stochastic space by including the probabilistic characteristics of their input random variables. The resulting stochastic response can then be used in Structural Reliability Assessments (SRA) to determine the probabilistic conditionality of risk components in ship collisions. In this paper, performance targets and ship responses are considered at the onset of hull fracture. The response of interest in this study is the structural damaged volume of the double-hull and it is linked to the economical consequence of repairing the damaged section, to compute the risk in monetary terms. The response can also be linked to other consequences such as compartment flooding and loss of containment due to the breach of the ship hull structure. The methodology of the present framework can be extended to consider these consequences in risk computations. However, the scope of the present study considers risk in terms of structural and economical consequences to serve as a decision support tool for ship design processes.

2 Proposed ship performance characterisation model for risk computation

The developed framework is described by categorising its components into four main modules; ship mechanics module, uncertainty characterisation module, response surface modelling module and performance and risk computation module, as shown in Fig. 2. The goal of the ship mechanics module is to identify the reference collision scenario and set the performance targets expected of ship structures in collisions. These targets can be linked with ship responses to develop the failure criteria against which the ship structural performance can be assessed. The objective/performance function of the ship collision analysis can then be formulated from either Simplified Analytical Models (SAM) or Response Surface Modelling (RSM). The SAM models refer to the direct collision analysis by categorising the response in stages with respect to the contribution of structural members to the total ship response (Obisesan *et al*., 2015).

Having a closed form objective function from SAM for the reference collision scenario analysis is advantageous. Although this is usually not the case as the analysis of ship collisions may involve a complex network of structural members with different deformation mechanisms. Also, the evaluated objective function may be implicitly defined due to features such as high nonlinearity and correlation between input variables, which could make SAM objective function unsuitable for performance measure computations. In such cases, RSM techniques are applied and this involves the generation and propagation of input design sets, representing the most probable collision scenarios, through an Automated Computational Model (ACM). The ACM allows for a seamless process of stochastic modelling through the integration of Python scripting, nonlinear finite element analysis software (Abaqus®) and MATLAB® for automated input design set propagation, efficient response generation and visualisation of various performance measures. The Python script written for a particular collision study creates the Finite Element Model (FEM) of collision scenarios, submits the corresponding jobs, collates the corresponding responses from the output database and saves the data in a readable format. The output data are then processed in the MATLAB® platform to gather the input/output relation for subsequent surrogate model evaluations.

There is a possibility of encountering cases where it is not practical to derive closed-form objective functions and surrogate models to characterise the reference collision scenario. In such cases, response data from the propagation of uncertainty characteristics through the ACM can be directly applied in sampling-based methods, such as Monte Carlo Simulations, for the computation of performance measures. Therefore, there are three possible paths that can be followed in the framework for the derivation of essential performance measures required for risk computations. The first path involves the utilisation of possible closed-form objective function evaluated from SAM analysis, the second is the direct utilisation of ACM outcomes in MCS while the third path involves the development of efficient response models from stochastic response surface modelling techniques. The automation and linking of deterministic nonlinear finite element analysis software and stochastic characterisation of ship structural performance are the bed rock of the proposed framework making it possible to transform ship collision analysis outcomes to compute performance measures and risks.

To demonstrate the functionality of the proposed framework, the assessment of the structural performance and asset risk of a double-hull crude oil carrier in a variety of ship collisions is carried out. Six reference collision scenarios are considered with respect to six striking ship categories. The consideration of various striking ship categories is to ensure that all possible ship encounters are considered for risk-based ship design. The most probable collision scenarios are represented by the input design sets generated from key random variables associated with load characteristics and strength variables: velocity, draught, collision angle, impact location, yield stress, ultimate stress and thickness. The response of the reference ship structure in these collision scenarios is evaluated using the ACM. The corresponding responses represent the structural performance targets of the collision scenarios and are recorded at the onset of outer hull fracture.

Based on the proposed framework, the path for the development of surrogate model objective functions is followed. The utilisation of two response surface modelling techniques are considered; polynomial regression and Kriging models. The developed objective functions are used in SRA in First Order Reliability Method (FORM) to measure the failure responses of the most probable ship collision scenarios against the reference ship performance targets (Ang and Tang, 2007; Bourinet *et al*., 2009). The resulting probability values define the likelihood of the reference ship structure missing its performance target. Sensitivity analyses are also performed to determine the influence of the input random variables on the respective hull structural capacity in collisions and to highlight how the magnitude and direction of variable sensitivity levels can be used to achieve optimum ship structural designs. The probabilistic values evaluated from SRA are combined with available collision frequency models in the literature (Montewka *et al*., 2012; Pedersen, 1995), to evaluate asset risks to the reference ship with respect to the six striking ship categories. The causation probability used in this study is based on the observations from Montewka *et al*. (2012), focussing on restricted waterways. As approximately 90% of the maritime accidents occur in restricted waterways (MacElrevey and MacElrevey, 2004), the obtained risk outcomes are more practically relevant.

To present the asset risks in a way that would be useful for decision making in risk-based ship design, asset risks to the reference ship structure are also presented in monetary values. A frequency exceedance curve is also constructed in terms of the consequential factor and collision frequency. The exceedance curve is generated using twenty probable collision scenarios generated from the probabilistic characteristics of the input random variables using the Latin hypercube sampling technique. The decision on the number of collision scenario sets to be generated for the risk computation is influenced by historical collision statistics (Brown, 2002; Youssef *et al*., 2013). The structural consequences of the accidents are evaluated for the collision scenario sets using the derived surrogate models, with respect to the striking ships. The exceedance curve provides useful information for ship designers on the minimum collision load to design a double-hull crude oil carrier with respect to a selected collision frequency.

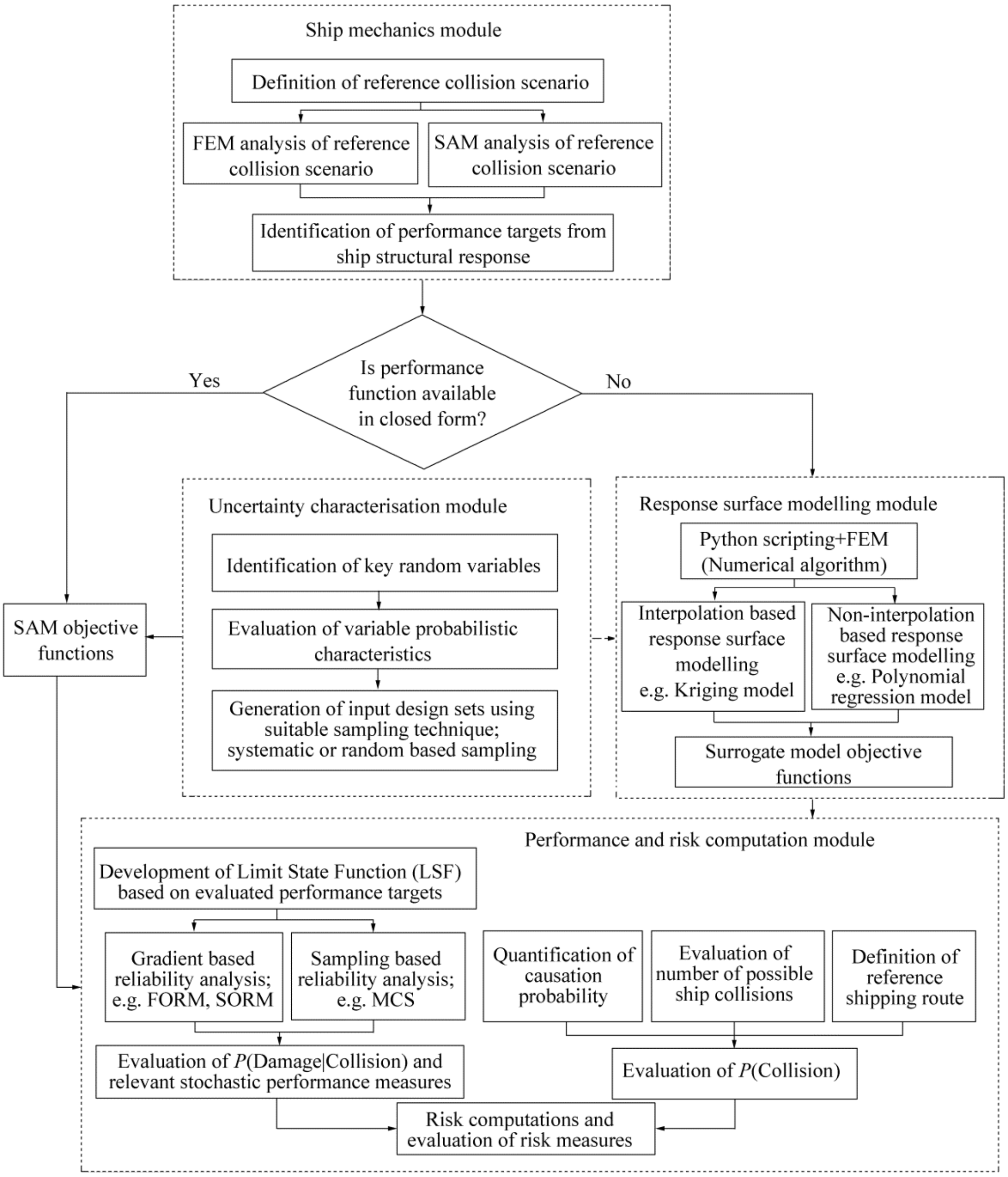


Fig. 2 Proposed framework for stochastic performance and risk assessment of ship collisions

3 Case study: collision scenario with double-hull oil carrier and various striking ships

3.1 Modelling of reference collision scenarios

This study is performed with the aim of evaluating the asset risk of ships and characterising their stochastic performance in collisions involving a variety of striking ships to demonstrate their relevance in risk-based ship designs. Six different collision scenarios are defined for a reference struck ship collision with respect to each of the striking ship categories. The reference struck ship is a 105400 DWT double-hull crude oil carrier presented in the study of Lutzen (2001). To reduce computational complexities, the full-scale double-hull support structure of the cargo hold section between two transverse bulkheads spaced at 22.2 m is modelled in this study. The dimensions of the considered structural members are shown in Fig. 3, and further geometrical data are presented in Table 1.

Table 1 Geometrical data for the double-hull oil carrier (Lutzen, 2001)

Frame spacing/m	Stiffener spacing/m	Stiffeners size/mm
3.70	0.81	400×100×13/18

To capture a wide range of collision possibilities, the striking ship types are categorised into six types, which include a number of more specific ship types (Brown, 2002; Youssef *et al*., 2014b; Brown and Sajdak, 2004):

- Tankers (Tkr): include crude and product tankers, ore/oil carriers, LPG tankers, LNG tankers and chemical carriers.

• Bulk carriers (BkC): include dry bulkers and coal carriers.

• Cargo vessels (GCo): include general cargo and refrigerated vessels.

• Passenger ships (Psg): include passenger vessels and ferries.

• Container vessels (Ctr): include container ships, car carriers, container/RO-ROs and RO-ROs.

• Other (Otr): include service vessels, fishing vessels, barges, dredgers, factory vessels, heavy lift vessels, pleasure boats and yachts.

The reference double-hull structure is impacted by the striking ships at a constant velocity of 6.7 m/s, at a right angle and at the midpoint with respect to the longitudinal direction of the double-hull structure. However, the impact location in the transverse direction is given as an offset point at a distance of 6.05 m from the midpoint to prevent initial overclosure between the colliding objects. An example model of a reference collision scenario is shown in Fig. 4, highlighting the cargo hold section considered in the present study. The response of interest in this study is the structural damaged volume of the double-hull due to impact by the striking ships. The metrics derived from the analysis of the collision scenarios represent the performance targets and also the measure for the structural consequence as a result of the collisions.

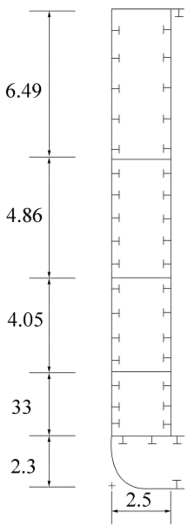


Fig. 3 Section view and dimensions (in m) of the double-hull oil carrier

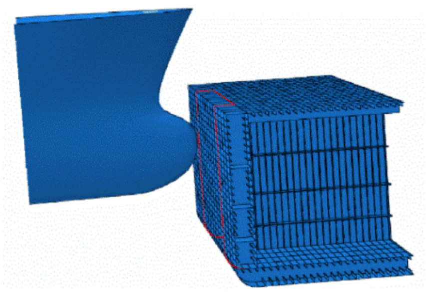


Fig. 4 An example of the collision scenario model

3.2 Numerical simulation

The explicit nonlinear finite element software Abaqus® is used to carry out the collision simulations. All members of the double-hull structure are modelled using an S4R element, which is a quadrilateral shell element with linear interpolation and reduced integration. An element size of 0.015 m is used for the double-hull structural members, in the region of contact, to account for the nonlinear structural deformation. The element size to thickness ratio is equal to 10 and this is sufficient for achieving consistent internal energy estimates (Alsos and Amdahl, 2007). The mesh size of stiffeners and the web plates follow the same meshing pattern as the shell plating. Mild steel is considered as the material of construction for all members. The plasticity model of the material is assumed to have linear isotropic hardening. This hardening model is chosen to consider the uncertainties from basic strength variables of the material in the response surface models. Hence, the plasticity model is characterised by the yield stress and the ultimate stress. Two damage models are defined in order to model the progressive damage of the material until failure and to capture the genuine fracture incident in the collision analysis response; ductile criterion and damage evolution. Due to limited information on the double-hull crude oil carrier presented by Lutzen (2001), the nominal values of the material properties considered by Obisesan *et al*. (2015) are used for the double-hull material as given in Table 2. A fracture strain value of 0.445 is defined in the model and is found to be valid for the mesh size (Levanger, 2012). The degradation of material stiffness as damage evolves is assumed to be linear. Hence, the evolution is modelled by the plastic displacement at full degradation with a value of 0.175 mm. The boundary conditions are represented by fully fixing all four edges of the double-hull to allow displacements only in the direction of the bulbous bow of the striking ships.

Table 2 Material properties of reference ship model

Parameter	Value	Reference
Density/(t·m^{-3})	7.85	
Yield stress/MPa	340	(Levanger, 2012)
Ultimate stress/MPa	590	(Levanger, 2012)
Effective plastic strain	0.445	(Levanger, 2012)
Displacement at fracture/mm	0.175	(Levanger, 2012)

The ductility design principle is adopted for the simulation, which implies that the struck object undergoes significant deformation and dissipates a major part of strain energy in the collision while the striking object remains undamaged (DNV, 2010; NORSOK, 2004). Hence, the representations of the striking ships are assumed to be rigid in the present study. It is acknowledged that the upper part of the bow section of a striking ship may make an early contact with the struck ship resulting in structural deformation that is significant enough to be considered in consequence analysis. However, the bow section of the striking ships is represented by the bulbous bow in the present study. This simplification is considered to reduce the computational cost of the simulation. The bulbous bow is modelled as a parabola with the following bow parameter (Zhang, 1999):

$$R_b = \frac{R_V^2}{R_L} \tag{3}$$

where R_V is the radius of the circular part of the parabola and R_L is the bow length. The bulbous bows for the six striking ship categories are modelled from the bow data available in Lutzen (2001) and are presented in Table 3.

Table 3 Bulbous bow parameters of the striking ships (Lutzen, 2001) m

Ship	R_V	R_L
Tkr	1.90	2.20
Psg	1.41	1.80
Otr	4.50	8.50
GCo	2.48	2.90
Ctr	5.10	7.50
BkC	5.90	7.50

An example of the simulation model involving the reference double-hull and the bulbous bow of the container ship is shown in Fig. 5. The indentation of the double-hull is displacement-controlled and the damaged volume of the ship at the onset of outer hull fracture is the response metric. The damaged volume (Vol_D) is estimated from the simulation results as the summation of the volume of double-hull elements with strain values (ε_i) exceeding that at yield point (ε_y) as (Minorsky, 1959):

$$\mathrm{Vol}_D = \sum_{i=1}^{n} \mathrm{Vol}_i \left(\varepsilon \geq \varepsilon_y \right) \tag{4}$$

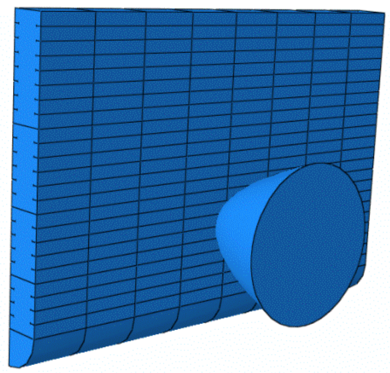


Fig. 5 A reference collision scenario model of the numerical simulation

The response of the double-hull structure at the onset of outer hull fracture for the collision scenario involving the container ship is shown in Fig. 6. It is observed that the structural deformation of the double-hull crude oil carrier is localised in the contact region with all structural members in this region, except the inner hull plating, experiencing plastic deformation. The cut-out view of the damaged sections of the double-hull from which damaged volume is computed, with respect to the striking ship categories, are shown in Fig. 7. The calculated damaged volume represents the required response from the simulations. The resulting values of the structural damaged volume considering the six striking ship categories are presented in Table 4. These values define the performance targets ($\mathrm{Vol}_{D,cr}$) and the asset consequences for assessing the performance and asset risks, respectively, of the double-hull in collisions.

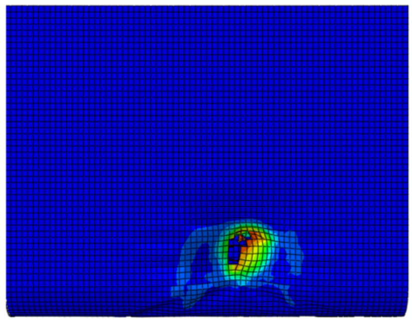
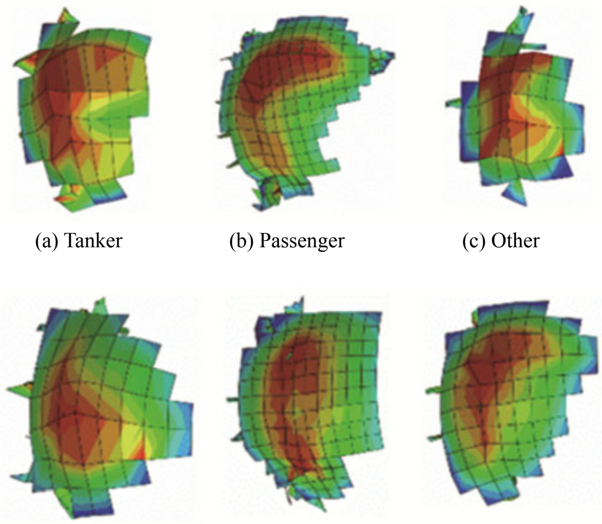


Fig. 6 The deformed double-hull showing outer hull fracture



(a) Tanker (b) Passenger (c) Other

(d) General Cargo (e) Container (f) Bulk Carrier

Fig. 7 Damaged sections of the double-hull after collision by the six striking ships

Table 4 Structural performance criteria for reference ship model

Ship	Tkr	Psg	Otr	GCo	Ctr	BkC
$\mathrm{Vol}_{D,cr}/\mathrm{m}^3$	0.123	0.401	0.091	0.210	0.520	0.298

4 Response surface modelling

4.1 Automated computational model

The behaviour of ship structures during collisions is dependent on certain factors such as the load characteristics, and material and geometrical properties of the structure, which are uncertain. These uncertainties therefore need to be considered during ship collision analysis. A statistical collision model, based on the data obtained from historical ship collision accidents, is adopted for the current study and the considered random variables represent parameters influencing collision scenarios, and material and geometric properties of the struck ship structural members.

Suitable probability density functions have been used to model collision scenario parameters in the literature, based on historical data on ship collision accidents occurring both in inland and international waters. The proposed model by Lutzen (2001) used 610 data points of ship-ship collision cases from classification societies and IMO damage database to derive the probabilistic characteristics of collision scenario parameters. The study by Brown (2002) collated historical ship collision data from USCG commercial vessel casualty file, ECO world tanker accidents and specific ship collision data. The model proposed by Youssef *et al*. (2014b) derived suitable distribution functions for collision scenario parameters based on the historical data collected from accident investigation boards of 14 countries. They considered data of both ship-ship collisions and near-collision cases. The considered collision scenario parameters are: impact location; speed of colliding ships; striking ship type and collision angle. The location of impact is defined by struck ship length, distance from the foremost point of the struck ship to the impact point and the draught of colliding ships. The probability characteristics of the collision scenario parameters estimated by Youssef *et al.* (2014b) are used in the present study due to the proactive approach of including near misses in the collision data. Also, the normalisation approach to characterising the parameters reduces computation costs and the number of variables in the evaluation of mathematical models. The probabilistic parameters of the considered load characteristics are presented in Table 5.

As mentioned earlier, the material and geometric properties of ship structures are important factors influencing ship structural behaviour during collisions. It was observed by Obisesan *et al*. (2015) that yield stress, ultimate stress and plate thickness are the most influential variables for studying the struck ship structural response during collisions. The consideration of these variables is also justified by the evaluation of damaged volume from the double-hull elements that have undergone plastic deformation during collision. The probabilistic characteristics of these random variables for shipbuilding steel are available in the literature (Atua *et al*., 1996; Ayyub and Assakkaf, 2000; Guo *et al*., 2012; Hess *et al*., 2002), and these are given in Table 5 for the considered variables. With the use of a suitable sampling technique, the combination of the random variables from the collision scenario and the material and geometrical properties can be used to generate the input design space required for stochastic ship collision analysis.

Table 5 Probability characteristics of the input random variables (Youssef *et al*., 2014b; Hess *et al*., 2002; Ayyub and Assakkaf, 2000)

Variable	Distribution	Min	Max	Mean	COV
V_R	Normal	0	1.63	0.79	0.65
D_R	Weibull	0.36	1.92	0.93	0.47
I_L	Normal	0.01	0.75	0.36	0.65
θ	Normal	15	160	77.14	0.58
σ_y	Lognormal	$1.00\sigma_y$	$1.20\sigma_y$	$1.11\sigma_y$	0.07
σ_u	Normal	$1.01\sigma_u$	$1.09\sigma_u$	$1.05\sigma_u$	0.05
t	Lognormal	$0.6t$	$1.67t$	t	0.02

To ensure that the developed mathematical formulation is valid for all possible scenarios, the most probable sampling range of the input random variables is identified and grouped into sets using the Design of Experiment (DOE) technique. The common DOE methods are the Central Composite Design (CCD) and Box-Behnken design (BBD) (Haldar and Mahadevan, 2000; Kaymaz and McMahon, 2005; Myers *et al*., 2009; Liu and Moses, 1994). Using the BBD, the most probable sampling range of the numeric factors (i.e. the input random variables) are used to construct input design sets for the model. The Design-Expert® software is used to carry out the BBD experimental design in the present study. The experimental design varies each numeric factor over the three most probable levels to obtain all possible combinations. The common three levels are the low, medium and high values (Haldar and Mahadevan, 2000). The number of possible combinations will then be 3^n, where n is the number of numeric factors. Hence, the higher the number of random variables, the higher the number of design sets and computations required to evaluate the mathematical models. However, BBD reduces the number of design sets required by selecting numeric factor combinations that emphasise more on the centre points and the neighbouring regions where considerable interest lies thereby reducing computational costs (Liu and Moses, 1994). The three levels of the input random variables are derived from their probabilistic characteristics presented in Table 5. It is worth noting that the value of collision angles is represented in radian in the study. Also, the yield and ultimate stresses are both represented in the models by flow stress (σ_0), calculated as the corresponding mean (Jones, 1998; Wierzbicki, 1983).

The construction of input design sets from the numeric factors generate 62 input sampling sets for each striking ship categorisation, thereby resulting in 372 collision scenario computations. These sets are propagated through the developed ACM based on the numerical simulation of the reference collision scenario. It should be noted that the length of the double-hull structure was modelled beyond that defined for the reference collision scenarios as it is observed that the bulbous bow penetration of some input design sets extends beyond the transverse bulkhead. This is done to reduce the effect of the defined boundary conditions on the structural responses of the ACM. The resulting response metrics are presented in the histogram plots of Fig. 8 and the corresponding characteristics of the resulting response metrics are presented in Table 6. The minimum and maximum damaged volume at the onset of hull fracture is recorded as 0.024 8 and 5.437 6 for scenarios involving the bulk carrier and the passenger ship categories, respectively. It is observed that some responses recorded from the ACM are higher than those of the reference collision scenarios. This is due to the representation of some input variables by their random extreme values, influencing the extent of impact in the double-hull structure.

Table 6 Characteristics of metrics from ACM considering striking ship types

Ship	Min	Max	Mean	COV
Tkr	0.060	0.997	0.290	0.063
Psg	0.026	5.438	1.160	1.518
Otr	0.035	0.546	0.146	0.017
GCo	0.102	1.382	0.408	0.117
Ctr	0.170	4.224	1.200	1.230
BkC	0.025	3.767	0.777	0.769

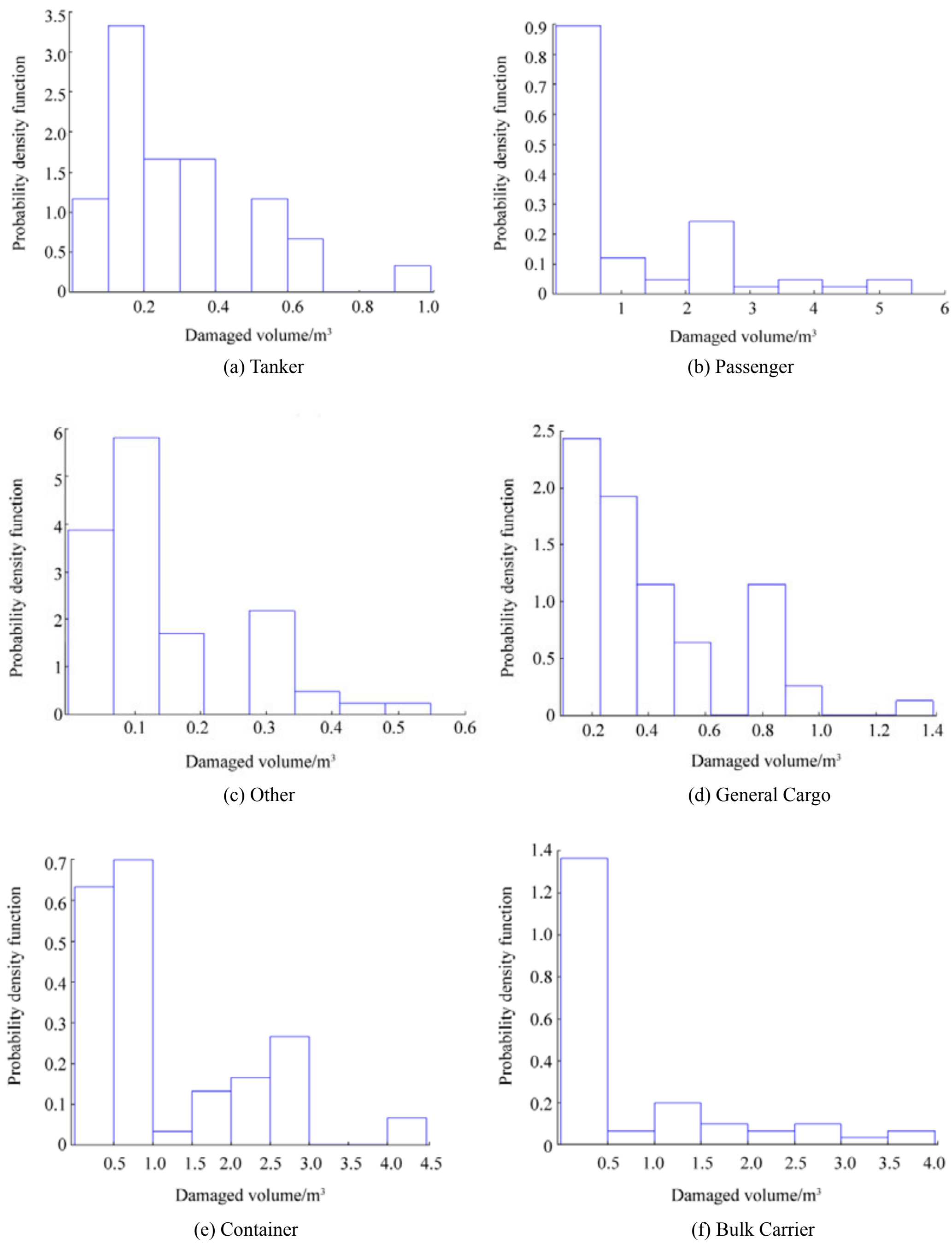


Fig. 8 Damaged volume distribution from ACM considering

4.2 Evaluation of surrogate models

Mathematical functions are fitted to the predictor-response of the collision scenario simulations to develop surrogate models that will represent the double-hull. There are several types of models available in the literature and they can be categorised as interpolating (e.g. Radial basis functions and Kriging models) and non-interpolating functions (e.g. Polynomial regression models and multivariate adaptive regression splines). These two models are considered in the present study with the application of the polynomial regression and Kriging models to fit the design points, as appropriate. The polynomial regression models are the most common form of surrogate models for fitting data in engineering problems (Bucher and Bourgund, 1990; Das and Zheng, 2000; Kim and Na, 1997). Usually, the first or second order polynomials are adopted as they are capable of accounting for linear and nonlinear effects as well as the correlation between random variables (Gaspar *et al*., 2014). Parameters of the polynomials can be derived by using multiple regression analysis technique. Given that n is the number of random variables, k is the number of coefficients, the response $\boldsymbol{Y}$ of n input design sets is an $n\times1$ vector, the basis function $f(\boldsymbol{X})$ of the input design sets is an $n\times k$ vector, the regression coefficient a is a $k\times1$ vector and the random error $\boldsymbol{e}$ is an $n\times1$ vector, the multivariate polynomial can be shown to be:

$$\boldsymbol{Y}(\boldsymbol{X}) = f(\boldsymbol{X})a + \boldsymbol{e} \tag{5}$$

The matrix representation of the above equation is (Haldar and Mahadevan, 2000):

$$\begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix} = \begin{pmatrix} 1 & f_1(\boldsymbol{X}^1) & \cdots & f_k(\boldsymbol{X}^1) \\ \vdots & \vdots & \ddots & \vdots \\ 1 & f_1(\boldsymbol{X}^n) & \cdots & f_k(\boldsymbol{X}^n) \end{pmatrix} \begin{pmatrix} a_0 \\ \vdots \\ a_k \end{pmatrix} + \begin{pmatrix} e_1 \\ \vdots \\ e_n \end{pmatrix} \tag{6}$$

A second order polynomial will take the form:

$$\boldsymbol{y}(x) = a_0 + \sum_{i=1}^{n} a_i x_i + \sum_{i=1}^{n-1}\sum_{j<i}^{n} a_{ij} x_i x_j + \sum_{i=1}^{n} a_{ii} x_i^2 + \boldsymbol{e} \tag{7}$$

where the constants, a_0, a_i, a_{ij} and a_{ii} are the regression coefficients to be calculated. The coefficients of polynomial regression models are evaluated using the least squares technique. The number of coefficients required can be estimated from n using the expression:

$$k = 0.5n^2 + 1.5n + 1 \tag{8}$$

The Kriging model uses the method of interpolation based on the assumption that there is a spatial correlation between the model predictions. The models have been proven to be useful in the approximation of both deterministic and stochastic simulations by fitting explicit functions to their predefined input data and the corresponding output data (Couckuyt *et al*., 2013; Lophaven *et al*., 2002). Unlike polynomial regression models, the Kriging models do not assume an underlying polynomial function rather, they estimate an optimal interpolation using surrounding data points which are weighted with respect to their respective correlation values. A two variable function is used as an example in Fig. 9 to discuss the functionality. Five surrounding input design sets are presented as dots while the unknown prediction is located at an unmeasured point and represented by the cross symbol. The value of the prediction will be a function of the distances between the surrounding input design sets and the distances from the prediction to each input design set. Hence, the basic form of Kriging model for the prediction of values is:

$$\boldsymbol{Y}(\boldsymbol{X}) - f(\boldsymbol{X}) = \boldsymbol{\Psi}^{-1}\boldsymbol{r}\left[\boldsymbol{Y}(\boldsymbol{X}') - f(\boldsymbol{X}')\right] \tag{9}$$

where $\boldsymbol{\Psi}^{-1}$ and $\boldsymbol{r}$ are correlation matrices associated with the function of the distance between surrounding data point pairs and the distance between data points and the predictor point, respectively (see Fig. 9), and $\boldsymbol{X}$ and $\boldsymbol{X}'$ are the point location vectors for the predictor point and the surrounding data points, respectively (note that these locations are determined by the input design space and they are $n\times k$ vectors because of the size of generated design space). The product of the two matrices is known as the Kriging weight. Based on the knowledge of the Kriging weight, it can be shown that a predictor evaluated at a known location x' will be equal to the observed simulation output. Hence, the Kriging predictor is an 'exact' interpolator unlike the polynomial regression model. The polynomial regression and Kriging models are evaluated in MATLAB®.

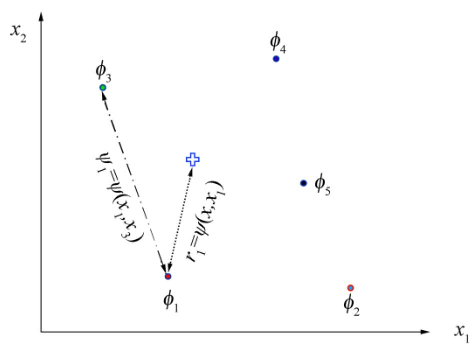
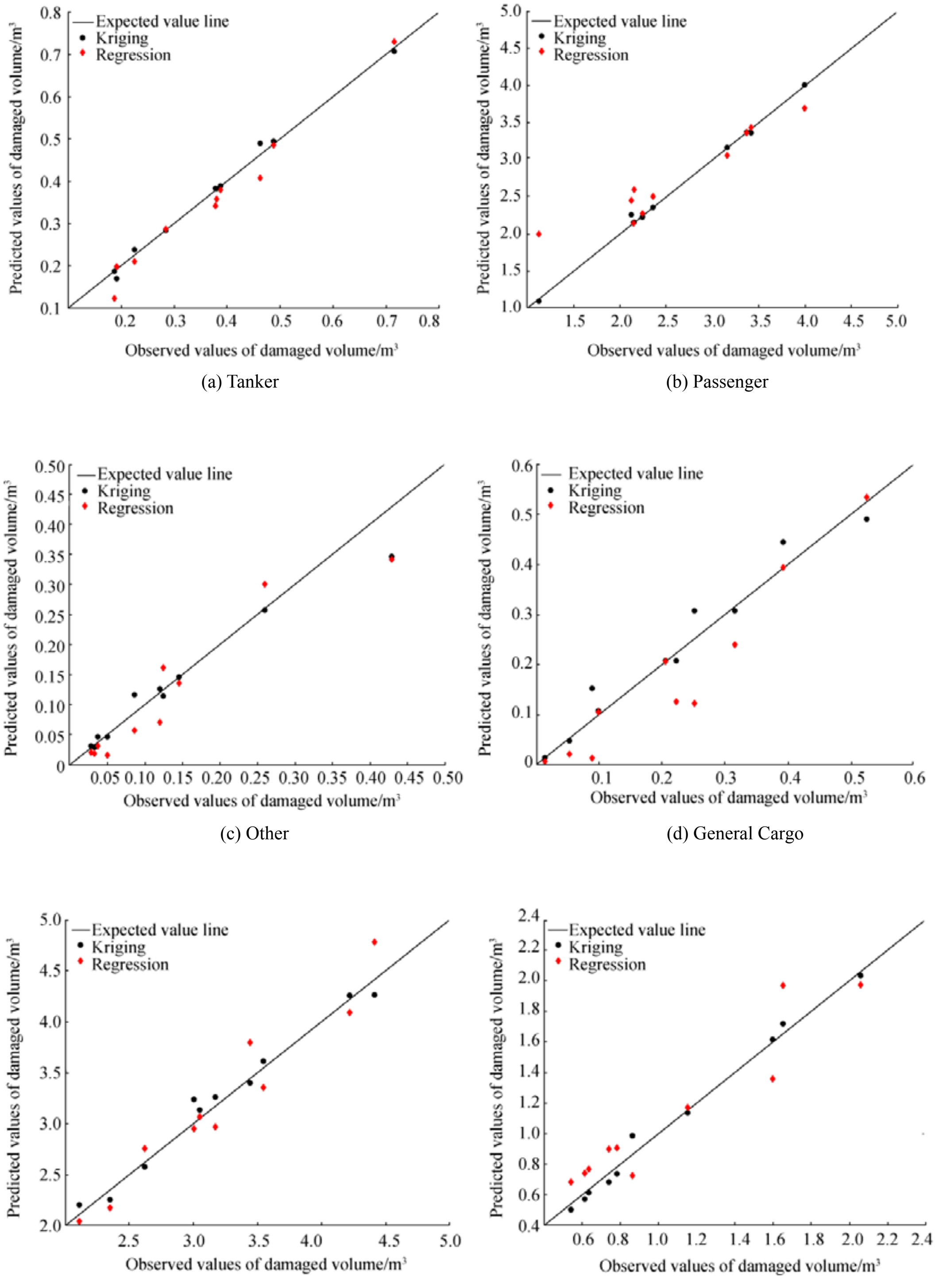


Fig. 9 Interpolation functionality of Kriging model

Based on the generated input design sets and the observed metric of the ACM, the available data considered for response modelling are 62 sets of six predictor variables and the corresponding structural damaged volume. Each of the response surface models produced six surrogate models with respect to the striking ship categories. For the polynomial regression approach, the second and third order regression models are fitted to the available data. Based on Eq. (8), 28 and 78 coefficients are evaluated for the second and third order regression models, respectively. There was no observed difference between the results of capturing the predictor-response relationships with the second and third order regression models. Hence, the polynomial regression approach is represented by the second-order regression model in this study.



(a) Tanker (b) Passenger

(c) Other (d) General Cargo

(e) Container (f) Bulk Carrier

Fig. 10 Predicted and observed damaged volume of reference ship considering

For the purpose of validating and comparing the application of the two response surface modelling approaches in surrogate model development, 10 input design sets, presented in Table 7, are generated using the LHS technique and are propagated through the ACM and the surrogate models. The resulting damaged volume observed from the six ACMs are compared with those predicted by the surrogate models, in Fig. 10, with respect to the striking ship categories. Although the two surrogate models give good prediction of the ACM outcomes, the results of the Kriging model are observed to provide better approximation of the reference ship response than the regression model. The minimum and maximum mean percentage errors of the Kriging model are 1.23% and 14.27% for cases involving the Passenger and General Cargo striking ship types, respectively; while those recorded for the polynomial regression model are 5.14% and 32.89% for cases involving the Container and General Cargo striking ship types, respectively. Based on these results, it is concluded that the Kriging model is the better response surface model capable of providing efficient description of the relationship between the six random variables and the response. The coefficients of the regression parameters for second-order polynomial models are presented in Table A1. The Kriging surrogate models are not presented in the study due to the large mathematical equations derived. However, the derived surrogate models from Kriging are used to represent the double-hull structural response for the rest of the present study.

Table 7 Input design sets for validating the surrogate models

Sets	V_R	D_R	I_L	θ/rad	σ_0/MPa	t/m
1	1.387	1.390	0.707	1.903	502.877	0.020
2	0.868	1.618	0.367	2.074	481.194	0.017
3	1.253	0.946	0.188	0.282	497.261	0.016
4	0.365	1.214	0.029	1.401	473.551	0.020
5	0.234	1.460	0.640	0.698	505.447	0.012
6	0.743	1.100	0.113	2.374	499.103	0.015
7	0.607	0.636	0.250	1.042	489.750	0.010
8	1.015	1.776	0.586	2.664	518.651	0.023
9	0.017	0.508	0.411	0.775	506.897	0.024
10	1.507	0.784	0.521	1.730	496.996	0.013

5 Performance measures based on FORM

Structural reliability assessment is performed by defining the LSF with respect to the input random variables characterising the structural damaged volume and the performance targets. The LSF is then expressed as:

$$G = \mathrm{Vol}_{D,cr} - \mathrm{Vol}_D\left(V_R, D_R, I_L, \theta, \sigma_0, t\right) \tag{10}$$

where $\mathrm{Vol}_D(\cdot)$ represent surrogate models for damaged volume and $\mathrm{Vol}_{D,cr}$ is the performance target represented by deterministic values defined in Table 4. The deterministic values are performance metrics from the damage assessment of the reference collision scenarios. Hence, the structural reliability assessment is performed by considering hull rupture criteria for an existing ship design model. The developed LSFs are solved using both First/Second order reliability methods (FORM/SORM), however as the estimates are very close FORM results are presented here and taken forward for risk quantification. The design points for the LSF evaluation are derived from the probabilistic characteristics of the input random variables (Table 5). FERUM Version 4.1 (Ang and Tang, 2007; Bourinet *et al*., 2009) is used to evaluate the reliability analysis outcomes and they are presented in Tables 8 and 9. It is observed that the probability of the reference hull missing the performance target during impact by the striking ships is within the range of 0.501 and 0.627, with the highest failure probability coming from the collision involving the 'Tanker' category. Hence, the results suggest that there is approximately 63% chance that the reference ship hull would not meet its performance expectation when in a collision with the 'Tanker' ship type. As shown in Table 8, smaller reliability indices (β) are recorded for the reference ship in collision involving all striking ship categories, with the highest value being 0.324 for collisions involving the bulk carrier ship type. More importantly, the failure probability values can represent the probability of the hull damage given a collision (P(Damage|Collision) or $P(D|C)$), a key component required for the computation of asset risks to make informed decisions on the design process of the reference ship.

The sensitivity indices (α) are measures of the influence of the input random variables on the reference ship performance in collisions. These indices are presented in Table 9 for the random variables considered in the study with respect to the striking ship categories. The values of α with the negative sign suggest that a better ship structural resistance is achievable by reducing the design value of the respective variable, thereby improving the reliability of the double-hull structure. The input variables are also ranked according to their respective contribution to the performance of the double-hull structure, and this is evident in the magnitude of their values. The variables relative velocity, relative draught and collision angle are identified to be influential in the analysis, so they could be adjusted to optimise the reliability of the double-hull structure. The sensitivity result also showed that the influence of the randomness of the material and geometric properties on the hull rupture is minimal. It is also observed that the impact location variable has less influence on the hull rupture. These observations suggest that variables with little influence on the result can be represented by their deterministic values when assessing the performance of the double-hull structure.

Table 8 Performance measures for reference double-hull based on FORM analysis

Ship	Tkr	Psg	Otr	GCo	Ctr	BkC
$P(D\|C)$	0.627	0.552	0.541	0.607	0.501	0.54
β	−0.324	−0.130	−0.102	−0.271	−0.003	−0.10

Table 9 Sensitivity indices of random variables

Ship	Tkr	Psg	Otr	GCo	Ctr	BkC
V_R	−0.03	0.02	0.03	0.012	−0.70	0.01
D_R	0.82	4E-3	0.00	−0.01	0.26	0.59
I_L	0.05	4E-3	0.016	0.024	−0.09	4E-3
θ	−0.56	−1.0	−0.99	−0.79	−0.65	−0.80
σ_y	−0.09	2E-3	6E-3	0.40	−0.09	−0.02
σ_u	−0.11	2E-3	7E-3	0.47	−0.10	−0.03
t	0.014	2E-3	5E-3	4E-4	0.05	6E-3

In a FORM analysis, the linear function derived at the most probable design coordinate of the standard normal space is used to determine β (Koutsourelakis *et al*., 2004; Liu and Der Kiureghian, 1991). The values of the input random variables within this space are the most probable failure points that is, the points with the highest failure density. It is possible to identify these points from the FORM analysis and they are presented for the reference ship collision involving all striking ship categories in Table 10. It is observed that the design values vary for different striking ship types. The variation relates to the solutions of the six different LSFs in the iterative algorithm utilised in FORM analysis with respect to the striking ship categories. However, the design values are observed to be close to the mean values of distributions representing the random variables. It is worth noting that these identified points may vary with the failure criteria and the probabilistic characteristics of the input random variables considered for structural reliability analysis. The most probable design points and the reliability analysis outcomes are crucial to ship designers as they serve as reference values for improving the reliability of the double-hull structure.

Table 10 Most probable design value for the input random variables

Ship	Tkr	Psg	Otr	GCo	Ctr	BkC
V_R	1.192	1.20	1.20	1.20	1.20	1.20
D_R	0.94	1.077	1.077	1.078	1.076	1.05
I_L	0.60	0.69	0.69	0.69	0.34	0.67
θ/(°)	85.17	82.88	81.64	86.63	77.10	80.72
σ_y /MPa	377.26	376.47	376.46	373.64	376.49	376.54
σ_u/MPa	618.21	617.13	617.12	613.2	617.15	617.23
t/mm	0.015	0.015	0.015	0.015	0.015	0.015

6 Collision probability and risk quantification

6.1 Ship collision probability

For the present study, the collision scenarios are assumed to occur between two crossing waterways linking harbours in the Gulf of Finland. Waterway 1 is represented by the North-South (N-S) traffic while waterway 2 is represented by the East-West (E-W) traffic, as discussed in the study by Montewka *et al*. (2010). The estimation of the probability of a collision requires the knowledge of the frequency of ship collision (F_N) for a specified shipping route. The value of F_N is estimated as the multiple of the geometric and causation probabilities (Fujii and Tanaka, 1971; Macduff, 1974). Hence, P(Collision) is the probability of a collision which can be evaluated from (Lutzen, 2001):

$$P(\text{Collision}) = 1 - e^{-P_C N_A} \tag{11}$$

where N_A is the geometric probability and P_c is the causation probability. The geometric probability is estimated from marine traffic based on the trajectory of ships plying two or more crossing routes, hence it is a function of the shipping route, size, speed, type of ships and the studied time frame (Goerlandt and Kujala, 2014). Numerous methods are available in the literature for the evaluation of the geometric probability (e.g. (Baris and Otay, 1999; Debnath and Chin, 2009; Fujii and Tanaka, 1971; Geng *et al*., 2009; Kaneko, 2002; Macduff, 1974; Pedersen, 1995; Roeleven *et al*., 1995; Søfartsstyrelsen, 2008)), as discussed by Li *et al*. (2012). Pedersen's model (Pedersen, 1995) is the most common among the models as it has been applied in numerous studies; for example, in Pedersen and Zhang (2000), Otto *et al*. (2002), Kujala *et al*. (2009), Montewka *et al*. (2010), Silveira *et al*. (2013) and Youssef *et al*. (2014a). Pedersen's model is also applied in this study to calculate the geometric probability. The model assumes that collisions occur at the intersection of two waterways given that the traffic flow (i.e. number of vessels passing a route per unit time), relative velocity, collision angle and particulars of vessels on collision course in the crossing routes are all known. If a vessel belonging to a class j in waterway 2 is on a collision course with a vessel belonging to a class i in waterway 1, the associated relative velocity can be calculated using:

$$V_{ij} = \sqrt{\left(V_i^{(1)}\right)^2 + \left(V_j^{(2)}\right)^2 - 2V_i^{(1)}V_j^{(2)}\cos\theta} \tag{12}$$

and the geometric collision diameter is estimated from the following equation:

$$D_{ij} = \frac{L_i^{(1)}V_j^{(2)} + L_j^{(2)}V_i^{(1)}}{V_{ij}}\sin\theta + B_j^{(2)}\left\{1 - \left(\sin\theta\frac{V_i^{(1)}}{V_{ij}}\right)^2\right\}^{\frac{1}{2}} + B_i^{(1)}\left\{1 - \left(\sin\theta\frac{V_j^{(2)}}{V_{ij}}\right)^2\right\}^{\frac{1}{2}} \tag{13}$$

where $L_i^{(1)}$, $B_i^{(1)}$ and $V_i^{(1)}$ are the length, breadth and velocity of vessel in class i in waterway 1; $L_j^{(2)}$, $B_j^{(2)}$ and $V_j^{(2)}$ are the length, breadth and velocity of vessel in class j in waterway 2, and θ is the crossing angle. The geometric probability can then be represented for crossing situation as:

$$N_A = \sum_{i,j}\frac{Q_i^{(1)}Q_j^{(2)}}{V_i^{(1)}V_j^{(2)}}D_{ij}V_{ij}\frac{1}{\sin\theta} \tag{14}$$

where Q is the number of passing vessels in a waterway per unit time (i.e. traffic flow).

The study by Montewka *et al*. (2010) performed a two-month vessel traffic analysis of the waterways considered in the present study, which closely reflects traffic profile during the winter and summer months, respectively. However, the result showed no significant difference between the studied months for the East-West traffic. Hence, N_A is deduced from the observed months for the study and the results are presented in Table 11 with respect to the striking ship categories. It should be noted that N_A differs for each scenario featuring respective striking ships as the geometric collision diameter (D_{ij}) is a function of the length and breadth of the colliding ships (see Eq. (13)).

Table 11 Traffic flow per ship year for the striking ship categories

Ship	Tkr	Psg	Otr	GCo	Ctr	BkC
E-W	7 284	6 024	4 800	11 148	7 752	1 740

Causation probability can be defined from the fractional estimation of the statistical number of ships involved in collision to the total number of ship passages. However, the analysis of P_c can be achieved by linking in a logical network the basic events leading to the occurrence of intermediate events, such as human factors, environmental conditions, instrumentation and mechanical status of ships, traffic type and management of the shipping route. The probabilistic parameters of these factors are usually estimated from historical data collected for a specific location and should be particular to an accident scenario; intersection, head-on or crossing scenarios (Otto *et al*., 2002). Hence, the calculated causation probability for a particular location will remain constant when there are future changes to traffic and geometrical parameters because causation probability is independent of these parameters.

The causation probability of crossing scenarios has been evaluated in the literature with respect to different locations (Fowler and Sørgård, 2000; Hänninen and Kujala, 2009; Macduff, 1974; Montewka *et al*., 2012). A summary of these results is presented in the works of Kujala *et al*. (2009) and Youssef *et al*. (2014a). It was observed that the calculated causation probabilities for crossing scenarios are within the interval (1.04×10^{-5}–6×10^{-4}. The causation probability, 1.04×10^{-5}, derived by Montewka *et al*. (2012) is used because the study considered the same waterways in the GoF as those in the present study. The causation probability was estimated from the ratio of the number of recorded collisions to the number of modelled collision candidates. A weighting factor called the ship handling factor was applied to adjust the probability result, in order to account for scenarios such as evasive manoeuvres (near collisions), blind navigation and encounter scenario. It is to be noted that information on the estimation of the number of modelled collision candidates is limited. Considering that harbour traffic is operated on restricted waterways, it is expected that the number is influenced by the uncertainty in parameters such as sea state, water depth, vessel traffic systems, crew experience, team management, size, type and frequency of vessels that operate on traffic lanes.

6.2 Quantitative risk assessment

With the availability of key sub-components required for computing risk that is, N_A, P_c and P(Damage|Collision), the asset risk to the double-hull structure with respect to the striking ship categories can be determined. The probability P(Collision), also $P(C)$, is evaluated using Eq. (11), P(Damage|Collision) has been evaluated in Section 5 and the consequence is represented by the structural damaged volume estimated from the numerical simulation of the reference collision scenarios (see Table 4). The collision frequency (F_N) is evaluated as:

$$F_N = P_C N_A \tag{15}$$

The result of these evaluations and the computed asset risks are presented in Table 12. It is observed that the collision scenario involving ships in 'Container' category has the highest risk while that involving ships in 'Other' category has the lowest risk value. The computed asset risk values for the collision scenarios compare well with observations from the literature (Youssef *et al*., 2014a). It should be noted that there is a possibility of occurrence of other consequences as a result of the asset damage, e.g. environmental consequence due to loss of containment. The consideration of these consequences in risk computations are not to be overlooked. As mentioned earlier, the scope of the present study is for the consideration of asset consequences only. It is also important to present the computed asset risks in ways that meet the understanding of ship designers. The graphical visualisation of asset risks and their definition in monetary terms are discussed in the next sections, as appropriate.

Table 12 Asset risk evaluation in m³/ship year for the reference ship collision scenario

Ship	Tkr	Psg	Otr	GCo	Ctr	BkC
N_A	7 284	6 024	4 800	11 148	7 752	1 740
F_N	0.075 8	0.062 6	0.049 9	0.115 9	0.080 6	0.018 1
$P(C)$	0.073 0	0.060 7	0.048 7	0.109 5	0.077 5	0.017 9
$P(D\|C)$	0.627 0	0.552 0	0.541 0	0.607 0	0.501 0	0.540 0
$\mathrm{Vol}_{D,cr}$	0.123 2	0.401 1	0.091 4	0.209 5	0.520 0	0.298 4
Risk	0.005 6	0.013 4	0.002 4	0.013 9	0.020 2	0.002 9

6.2.1 Ship collision exceedance curve

Risk is often characterised using either qualitative or quantitative approach to develop acceptance criteria that are weighted against consequential factors such as asset damage, cost and societal influence. Ultimately, the resulting criteria can be used to classify risk into acceptable or unacceptable regions, as in a risk matrix and demonstration of as Low as Reasonably Practicable (ALARP). A valuable measure in collision risk-based design is to determine the maximum tolerable/acceptable risk for ships and this may be achieved by a frequency diagram (Guarin *et al*., 2009).

Frequency diagrams, or exceedance curves, provide a probabilistic representation of the cumulative distribution of F_N against a consequential factor. The area under the curve mostly represents the risk tolerability region of the curve (Sames and Hamann, 2008). To create the frequency diagram for the reference collision scenarios, collision frequencies and consequences are calculated for a set of credible collision scenarios. As discussed in Section 1, an average of about 20 ship-ship collision accidents occur annually (see Fig. 1). It is then deduced that a minimum of 20 ship collision scenarios can be randomly generated using probable points of the input random variables to evaluate collision frequencies and structural consequences for a reference ship per ship year. The structural damaged volume is considered as the consequence of the collision scenarios and it is evaluated using the developed surrogate models with respect to the striking ship categories. The parameters, N_A and P_c, are evaluated following the procedure described in Section 6.1.

The Latin Hypercube Sampling (LHS) technique is used to generate 20 collision scenarios in MATLAB®. The generated input design sets are used to sample the surrogate models derived for the reference collision scenarios. Collision frequencies are estimated in conjunction with the particulars and the traffic profile of the striking ship categories. The estimated average and total collision frequencies per ship year for the reference struck ship with respect to the striking ship categories are presented in Table 13. It is observed that the total collision frequencies are within the range of 5.9×10^{-3}–2.1×10^{-2} per ship year. In the study of Eliopoulou and Papanilolaou (2007), the estimated collision frequencies for different size of tankers are within a range of 1.56×10^{-3}–3.05×10^{-3} per ship year. Deriving estimations from historical statistics, the IMO (2007) reported collision frequencies for double-hull oil carriers to be within a range of 8.6×10^{-3}–4.6×10^{-2} per ship year. Hence, the computed total frequencies in the present study compare well with estimations from the literature.

Table 13 Average collision frequencies (per ship year) of a double-hull oil carrier

Ship	Total freq.	Avg. freq.
Tkr	0.009 6	0.0004 8
Psg	0.014 1	0.000 71
Otr	0.005 0	0.000 25
GCo	0.015 9	0.000 6
Ctr	0.021	0.001 1
BkC	0.005 9	0.000 3

The exceedance curve with respect to the structural damaged volume of the reference ship at the onset of hull fracture is shown in a semi-logarithmic scale in Fig. 11, considering the six striking ship categories. Although the exceedance curve is limited to the considered data sets and collision scenarios, it can be used to determine the minimum structural damaged volume of the reference ship hull that would lead to outer hull fracture in a collision event when a particular collision frequency is chosen. For example, the selection of 0.001 per ship year as the frequency of double-hull crude oil carrier collision against a bulk carrier means that the hull structure is designed with the capacity of 1.78 m^3 volume of damage before the onset of outer hull rupture.

The area under the exceedance curve, that is the corresponding asset tolerable risk, can be further classified to influence the selection of the collision frequency by ensuring that the selected frequency is either within the ALARP or acceptable region. The demonstration of ALARP is beyond the scope of the present study, but in the context of the derived frequency diagram, the ALARP region will be influenced by the exceedance curves of the double-hull structure in relation to the striking ship categories.

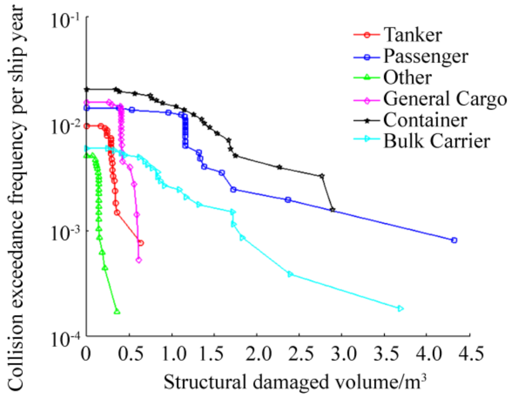


Fig. 11 Ship collision exceedance curve of the double-hull crude oil carrier

6.2.2 Risk computation in monetary units

The evaluation of the asset risk as units of volume may be less informative to ship designers. Hence, the monetary equivalent of the consequences is explored in the study. The study by Otto *et al*. (2002) estimated ship structural repair jobs to cost around €6 000 per unit tonnage (approximately $6750 USD using the average exchange rate between the months of August and October 2015), by considering the full repair process of cutting, building and fitting. The study by Youssef *et al.* (2014a) further argued that a full repair process of a damaged ship would likely cost twice as much as the estimate proposed by Otto *et al*. (2002). The former based their argument on important service works overlooked by the latter, which include the preparation of the damaged area for high temperature jobs and thickness measurement procedures. The value of the repair cost proposed by Youssef *et al*. (2014a) is used in the present study, that is $13500 USD per unit tonnage, due to the consideration of additional factors influencing ship repair costs.

The asset risk to the reference hull structure in collision is evaluated in monetary terms by considering the 20 collision scenarios generated in Section 6.2.1. The minimum and maximum total asset risks to the hull structure are estimated to be $42 USD and $1 591 USD per unit tonnage for collision scenarios involving 'other' and 'Container' ship types, respectively. As an example, the asset risk to the reference hull structure when in collision with a bulk carrier is presented in Table 14 for the 20 collision scenarios. As observed from the results of the reference struck ship reliability analysis in Table 8, *P*(Damage|Collision) is equal to 0.54 for collision involving a bulk carrier ship type. The multiplication of this value with the evaluated *P*(Collision) for each collision scenario gave the results of *P*(Damage), also *P*(*D*), as presented in Table 14. The resulting risk values for the collision scenarios are between $4 and $50 USD per unit tonnage with the total asset risk estimation being $370 USD per unit tonnage.

Table 14 Asset risk evaluation in USD of the reference ship-bulk carrier collision scenarios

Set	$P(D)$ ($\times10^{-4}$)	Vol_D/m^3	Risk	Set	$P(D)$ ($\times10^{-4}$)	Vol_D/m^3	Risk
1	1.36	0.41	5.84	11	1.30	0.92	12.66
2	1.72	1.17	21.37	12	1.11	2.39	28.01
3	1.09	0.46	5.33	13	1.47	0.25	3.91
4	1.39	0.87	12.76	14	1.35	1.32	18.83
5	1.58	0.33	5.57	15	1.00	3.68	39.11
6	2.41	0.62	15.89	16	1.84	1.09	21.24
7	1.48	0.71	11.19	17	2.56	1.83	49.59
8	1.91	0.77	15.71	18	1.81	0.83	15.96
9	1.59	0.69	11.54	19	1.47	0.84	13.00
10	1.85	1.72	33.81	20	1.55	1.72	28.25

The computed asset risk values for the collision scenarios compare well with observations from the literature (Youssef *et al*., 2014a). However, it should be noted that the computed risk values may be larger in real life due to possible costs for products and services overlooked by the estimated cost implication for ship repairs, such as salvage operations, renewal of damaged pipes and painting. As mentioned earlier, the monetisation of the computed risk is done with the consideration of structural and economical consequences only. The estimation of risk in monetary values ensure that outcomes of risk-based analysis of ships in collision events can be effectively presented to the ship operators and designers to make informed decisions for the improvement of the ship structural design.

7 Conclusions

This study presented a novel framework for stochastic performance characterisation in risk-based ship designs. The performance metrics are evaluated by following at least one of the three paths; the utilisation of objective functions developed from SAM, surrogate models, and the direct application of outcomes from the ACM. The stochastic response modelling at the onset of failure, as identified in this study, resulted in the evaluation of the probability of ship structures missing their set design targets. The resulting probability values are measures of the structural performance and provide an avenue for optimum ship structural design.

The possibility of automating the response distribution computations in various collision scenarios made it possible to apply the framework to a full scale double-hull crude oil carrier in collisions involving six typical striking ship categories. The utilisation of the Kriging approach to develop efficient surrogate models gave a better representation of the input-response mathematical relationships than the polynomial regression models. The probability of the ship structure exceeding the set design specifications, with respect to considered collision scenarios, was found to be within a range of 50%–63%. This showed the need to improve the capacity of the ship structure to achieve a desired performance. The most probable design points were identified to emphasise their use by ship designers to improve the reliability. These design points may vary with the choice of failure criteria and identified uncertainty in parameters of assessed collision scenarios. The contribution of the impact location and the material and geometric properties were found to be less significant to the performance results, hence they can be represented by their deterministic values during stochastic collision damage assessment.

The asset risks to the ship structure were computed in the units of damaged volume and the values compared well with those available in the literature. The computation of the asset risks in monetary units was demonstrated and it allowed for the evaluated risks to be presented in a measure understandable to ship designers and decision makers. Frequency exceedance curves were constructed in terms of the structural damaged volume. The exceedance curves provide useful information for ship designers on the minimum collision load to design a double-hull crude oil carrier with respect to a selected collision frequency. It is envisaged that the proposed approach would be a valuable tool for optimal performance characterisation and risk-based ship design. Although, a double-hull crude oil carrier is presented as the struck ship, the approach can be extended to characterise the performance and risk of other ship structures in collisions.

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Nomenclature

$B_i^{(1)}$	breadth of vessel in class i in waterway 1
$B_j^{(2)}$	breadth of vessel in class j in waterway 2
$L_i^{(1)}$	Length of vessel in class i in waterway 1
$L_j^{(2)}$	Length of vessel in class j in waterway 2
$Q_i^{(1)}$	Traffic flow of vessel in class i in waterway 1
$Q_j^{(2)}$	Traffic flow of vessel in class j in waterway 2
$V_i^{(1)}$	velocity of vessel in class i in waterway 1
$V_j^{(2)}$	velocity of vessel in class j in waterway 2
C_n	Number of coefficient
D_R	Relative draught
F_N	Ship collision frequency
I_L	Impact location
N_A	Number of ship collision candidates
P_C	Causation probability
R_L	Bow length
R_V	Bow base radius
R_b	Parabolic bow parameter
V_R	Relative velocity
V_{ij}	Crossing vessels relative velocity
$\mathrm{Vol}_{D,cr}$	Performance criteria
$\mathrm{Vol}_D(.)$	Damaged volume function

b_0, b_i, b_{ij}, b_{ii}	Regression coefficients	t	Thickness
		α	Sensitivity index
$\boldsymbol{x}_i$	Vector of a random variable	ε	Stochastic error
σ_u	Ultimate stress	θ	Collision angle
σ_y	Yield stress	β	Reliability index

Appendix

Table A1 Coefficients of the polynomial regression model

Ship	Container	Bulk Carrier	Passenger	General Cargo	Tanker	Other
b_0	−59.890 0	4.761 9	−21.059 0	−10.196 0	−1.554 2	2.084 5
$b(V_R)$	-0.803 8	−0.133 7	−0.304 3	0.084 1	−0.038 6	-0.029 4
$b(D_R)$	0.047 9	−0.067 4	−0.046 9	0.046 0	−0.009 7	0.006 3
$b(I_L)$	0.006 3	0.018 2	−0.022 3	−0.028 1	0.029 6	0.004 9
$b(\theta)$	−2.190 0	−9.995 1	−2.921 4	−1.061 3	−0.426 8	−0.177 0
$b(\sigma_0)$	0.252 0	0.014 1	0.093 3	0.040 6	0.006 9	−0.009 0
$b(t)$	170.510 0	165.730 0	295.930 0	59.449 0	28.485 0	55.184 0
$b(V_R D_R)$	0.002 5	−0.002 7	−0.000 9	−2.70E-05	0.000 4	−6.4E-06
$b(V_R I_L)$	0.000 3	−0.003 2	−0.002 4	−0.000 1	9.07E-05	−1.5E-05
$b(V_R \theta)$	−0.043 6	−0.048 1	−0.040 0	0.001 5	0.001 6	0.000 2
$b(V_R \sigma_0)$	−0.000 4	0.000 2	5.50E-05	−0.000 1	−7.9E-06	7.24E-05
$b(V_R t)$	−1.222 4	−1.001 0	−1.610 5	−0.182 5	0.403 8	0.022 1
$b(D_R I_L)$	0.000 4	−0.001 5	−0.001 2	−4.10E-05	4.58E-05	6.41E-06
$b(D_R \theta)$	−0.031 8	−0.013 0	−0.012 1	0.001 9	0.001 8	−0.000 2
$b(D_R \sigma_0)$	3.65E-06	1.78E-05	−0.000 2	−6.50E-05	1.17E-05	−2.1E-05
$b(D_R t)$	−1.175 4	−0.423 2	−0.611 5	−0.094 6	−0.076 1	−0.068 5
$b(I_L \theta)$	−0.006 6	0.022 7	−0.009 7	0.005 1	−0.008 4	0.003 2
$b(I_L \sigma_0)$	8.47E-06	−1.40E-05	2.46E-05	−1.80E-06	−1.9E-05	4.57E-06
$b(I_L t)$	2.546 0	7.105 4	12.996 0	1.791 5	−0.201 3	−0.118 8
$b(\theta \sigma_0)$	−0.000 2	0.014 4	−0.000 7	0.000 4	−8.1E-05	9.23E-07
$b(\theta t)$	−11.773 0	15.406 0	−8.182 9	−0.666 6	−3.331 1	−7.068 4
$b(\sigma_0 t)$	0.035 0	−0.093 8	−0.225 0	−0.135 0	−0.035 3	−0.088 1
$b(V_R^2)$	0.097 3	0.008 9	0.018 3	−0.000 9	0.001 3	−0.000 3
$b(D_R^2)$	0.000 2	0.002 4	0.004 0	−0.000 3	8.49E-05	0.000 1
$b(I_L^2)$	−0.003 7	−0.004 7	−0.007 0	−0.000 3	−0.000 4	−0.000 5
$b(\theta^2)$	1.098 6	0.876 0	1.227 4	0.295 8	0.217 2	0.053 3
$b(\sigma_0^2)$	−0.000 3	−4.00E-05	−8.70E-05	−3.80E-05	−6.3E-06	9.99E-06
$b(t^2)$	−3351.900	−4562.600 0	−6430.300	533.880 0	245.460 0	360.640 0

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