J. Marine Sci. Appl. (2015) 14: 156-162
DOI: 10.1007/s11804-015-1298-6

Scattering of Oblique Surface Waves by the Edge of Small Deformation on a Porous Ocean Bed

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Abstract: The scattering of oblique incident surface waves by the edge of a small cylindrical deformation on a porous bed in an ocean of finite depth, is investigated here within the framework of linearized water wave theory. Using perturbation analysis, the corresponding problem governed by modified Helmholtz equation is reduced to a boundary value problem for the first-order correction of the potential function. The first-order potential and, hence, the reflection and transmission coefficients are obtained by a method based on Green's integral theorem with the introduction of appropriate Green's function. Consideration of a patch of sinusoidal ripples shows that when the quotient of twice the component of the incident field wave number along x-direction and the ripple wave number approaches one, the theory predicts a resonant interaction between the bed and the free-surface, and the reflection coefficient becomes a multiple of the number of ripples. Again, for small angles of incidence, the reflected energy is more as compared to the other angles of incidence. It is also observed that the reflected energy is somewhat sensitive to the changes in the porosity of the ocean bed. From the derived results, the solutions for problems with impermeable ocean bed can be obtained as particular cases.

Keywords: oblique waves; bottom deformation; porous bed; Green's function; perturbation technique; reflection coefficient; transmission coefficient; scattering

Article ID: 1671-9433(2015)02-0156-07

1 Introduction

The propagation of surface waves over an obstacle or a geometrical disturbance at the bottom of an ocean is important for its possible applications in the area of coastal and marine engineering, and as such it is being studied with interest for a long time. A train of progressive waves traveling on the surface of an ocean, without any obstacle, experiences no reflection when the ocean is of uniform finite depth. If the bed of the ocean has a deformation, the wave train is partially reflected by it, and partially transmitted over it. However, the determination of reflection and transmission coefficients for a general type of bottom deformation is quite a difficult task for marine researchers. Scattering of surface waves by a small deformation of an impermeable ocean-bed with free-surface create interesting mathematical problems drawing attention of various types for obtaining their useful solutions.

Davies (1982) solved the reflection of normally incident surface water waves by a patch of sinusoidal undulations on the seabed in a finite region by using Fourier transform technique. Mei (1985) developed wave evolution and reflection theory at and near the Bragg resonance condition for shore-parallel sinusoidal bars. Miles (1981) and Davies and Heathershaw (1984) considered the problem of water-wave scattering by an undulating bottom topography in an ocean. Mandal and Basu (1990) generalized the problem of Miles (1981) to include the effect of surface tension at the free surface. Martha and Bora (2007a) worked on the propagation of obliquely incident surface waves over a small bottom undulation on the ocean bed. Employing a simplified perturbation analysis, they reduced the original boundary value problem (BVP) to another one up to first order and obtained the velocity potential, reflection and transmission coefficients up to the first order by using finite cosine transformation. For the problem of free surface flow over an undulating bed, the mild-slope equation, initially devised by Kirby (1986) and later on Chamberlain and Porter (1995) (modified mild slope equation) introduced approximate analytical techniques essentially involving depth-averaging under the assumption of the small variation of the bed. Staziker *et al*. (1996) considered the problem of two-dimensional wave scattering by a local bed elevation of any shape on an otherwise horizontal bed using linearized water wave theory. The behavior of water waves over periodic beds was considered by Porter and Porter (2003) in a two-dimensional context using linear water wave theory. They developed a transfer matrix method incorporating evanescent modes for the scattering problem, which reduced the computation to that required for a single period, without compromising full linear theory.

All the above works are focused only on the wave motion of the fluid region, where the effect of porosity of the ocean-bed was not taken into account. When the ocean bed is composed of porous material of a specific type, the hydrodynamic characteristics such as the wave energy dissipation, wave damping or decaying of wave height reaching towards the coast *etc*., are modified by the wave-induced pore pressure and soil

Received date: 2014-08-18.
Accepted date: 2014-11-17.
Foundation item: This work is partially supported by a research grant from Department of Science and Technology (DST), India (No. SB/FTP/MS-003/2013).
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displacements within the soil skeleton on the ocean bed. Therefore, due to many interesting applications in the theory of scattering of waves, porosity of the ocean bed becomes an extremely important aspect to be handled for marine researchers. Gu and Wang (1991) investigated the problem of water wave propagation within a porous ocean bed both theoretically and experimentally. Zhu (2001) studied the problem of water wave propagation within porous media on an undulating bed by employing a Galerkin eigenfunction expansion technique and obtained the reflection coefficient numerically. Silva *et al.* (2002) considered waterwave reflection and/or transmission problems where a porous medium was assumed to lie on an ocean-bed of varying quiescent depth. Jeng (2001) developed wave dispersion relation in a porous seabed by employing the complex wave number in the poro-elastic model of wave–seabed interaction. Martha and Bora (2007b) considered the problem of oblique wave scattering by a small patch of sinusoidal bottom undulation over a porous ocean bed and obtained the reflection and transmission coefficients by employing Fourier transform technique. Recently, Mohapatra (2014) investigated the problem of normally incident surface waves diffraction by small undulation on a porous bed in an ocean with ice-cover using Green's integral theorem with the introduction of appropriate Green's function.

In this article, we consider a three-dimensional problem involving an oblique incident progressive wave propagates in an ocean where the bottom is bounded by a porous surface which has a small deformation. The motion of the fluid below the porous bed of an ocean is not analyzed here and it is assumed that the fluid motions are such that the resulting boundary condition on the porous bed as considered here holds good and depends on a known parameter P, called porosity parameter, in this analysis. In this case, time-harmonic waves of a particular frequency can propagate with one wave number at the free-surface. Applying perturbation analysis involving a small parameter $\varepsilon(<<1)$, which measures the smallness of the deformation, the original problem governed by modified Helmholtz equation reduces to a simpler boundary value problem for the first-order correction of the potential function. The solution of this problem is then obtained by the use of a Green's integral theorem of the potential function describing the boundary value problem. The reflection and transmission coefficients are evaluated approximately up to the first-order of ε in terms of integrals involving the shape function when a train of oblique incident progressive waves propagates on the porous bed in an ocean having a small deformation. We present a special form of bottom deformation, that is, a patch of sinusoidal ripples and the first-order reflection coefficient is depicted graphically for various values of the different parameters. Investigations of such wave problems have gained reasonable importance due to various reasons. One of these is to understand the mechanism and effects of surface wave propagation through the porous bed of an ocean. Furthermore, another important reason for considering this kind of problems stems from the need to construct an effective reflector of the incident wave energy for protecting coastal areas from the rough ocean.

2 Mathematical formulation of the problem

The irrotational motion of an inviscid incompressible fluid flow of an ocean where the bottom is bounded by a porous surface which has a small cylindrical deformation is considered. A right-handed Cartesian coordinate system is used in which the xz-plane coincides with the undisturbed free-surface of the fluid. The y-axis points vertically downwards with y=0 as the mean position of the free-surface and y=h as the bottom surface. We further assume that the motion is time harmonic with angular frequency ω. Here, the bed has a porous type surface with a small deformation which is described by $y=h+\varepsilon c(x)$, where $c(x)$ is a bounded and continuous function describing the deformation of the ocean bed and $c(x)\to 0$ as $|x|\to\infty$ so that the ocean is of uniform finite depth h far to either side of the deformation. Under the assumptions of linear water wave theory, the velocity potential $\Phi(x,y,z,t)$ in the fluid of density ρ can be written as

$$\Phi(x,y,z,t)=\mathrm{Re}\left\{\phi(x,y)\mathrm{e}^{\mathrm{i}(\upsilon z-\omega t)}\right\}\qquad -\infty<x,z<\infty,\ 0\le y\le h+\varepsilon c(x) \tag{1}$$

where υ is the component of the incident field wave number along z-direction, Re stands for the real part and the potential $\phi(x,y)$ satisfies the modified Helmholtz equation:

$$\left(\nabla^2_{x,y}-\upsilon^2\right)\phi=0\qquad -\infty<x<\infty,\quad 0\le y\le h+\varepsilon c(x) \tag{2}$$

where $\nabla^2_{x,y}$ is the two-dimensional Laplacian operator. The linearized boundary conditions at the free-surface and at the bottom surface are:

$$\phi_y+K\phi=0,\ -\infty<x<\infty,\ y=0 \tag{3}$$

$$\phi_n-P\phi=0,\ -\infty<x<\infty,\ y=h+\varepsilon c(x) \tag{4}$$

where $K=\omega^2/g$, g the acceleration due to gravity, $\partial/\partial n$ the derivative normal to the bottom at a point (x,y) and P is the porous effect parameter on the ocean bed. The time dependence of $\mathrm{e}^{-\mathrm{i}\omega t}$ has been suppressed.

Within this framework in the fluid region, $-\infty<x<\infty$, $0\le y\le h$, a train of progressive surface waves takes the form (up to an arbitrary multiplicative constant)

$$\phi_0(x,y)=\frac{\cosh k(h-y)-(P/k)\sinh k(h-y)}{\cosh kh-(P/k)\sinh kh}\mathrm{e}^{\pm ix\sqrt{k^2-\upsilon^2}}\qquad -\infty<x<\infty,\ 0\le y\le h \tag{5}$$

is obliquely incident upon the bottom deformation from negative infinity, where $\upsilon=k\sin\theta\ (0\le\theta<\pi/2)$, θ is the angle of the oblique incident progressive waves (θ=0 corresponds to normal incidence), $\cosh kh-(P/k)\sinh kh\ne 0$ and k satisfies the following dispersion relation,

$$[k+(P/k)K]\tanh kh-(K+P)=0 \tag{6}$$

In the above dispersion equation, there is a positive real root $k=u$ (say), which indicates the propagating modes of the fluid at the free-surface and a countable infinity of purely imaginary roots $\mathrm{i}k_n \quad (n=1,2,...)$ that relate to a set of evanescent modes, where k_n's are real and positive satisfying the following equation in κ:

$$[\kappa-(P/\kappa)K]\tan\kappa h+K+P=0 \tag{7}$$

The negatives of all of these are also roots, being wave numbers of the waves traveling in the opposite direction. The intensity of the evanescent mode of waves decays exponentially with distance from the free-surface at which they are formed. Due to this evanescent mode of waves appearing in the fluid region, a part of the incident wave becomes trapped and leads to a standing wave pattern over the bottom irregularities, when the incident wave is scattered by the bottom undulation. Since the dispersion equation has exactly one nonzero positive real root u, so there exists one mode of wave propagating at the free-surface along the positive x-direction.

An oblique incident surface wave of mode u making an angle θ, $0\le\theta<\pi/2$ with the positive x-direction is of the form:

$$\phi_0(x,y)=\frac{\cosh u(h-y)-(P/u)\sinh u(h-y)}{\cosh uh-(P/u)\sinh uh}\mathrm{e}^{\mathrm{i}ux\cos\theta} \qquad -\infty<x<\infty,\ 0\le y\le h \tag{8}$$

Since the train of oblique incident progressive surface waves is partially reflected and partially transmitted over the bottom deformation, the far-field behavior of ϕ is given by

$$\phi(x,y)\sim\begin{cases}\phi_0(x,y)+R\phi_0(-x,y) & \text{as} \quad x\to-\infty\\ T\phi_0(x,y) & \text{as} \quad x\to\infty\end{cases} \tag{9}$$

where the unknown coefficients R and T are the reflection and transmission coefficients, respectively, and are to be determined. Here the perturbation method can be employed to obtain these coefficients up to first-order.

Assuming, for small bottom deformation, ε to be very small and neglecting the second order terms, the boundary condition (4) on the bottom surface $y=h+\varepsilon c(x)$ can be expressed in an appropriate form as

$$\begin{aligned}&\phi_y-\varepsilon\frac{\mathrm{d}}{\mathrm{d}x}\left[c(x)\phi_x(x,h)-c(x)\upsilon^2\phi(x,h)\right]-\\&P-[\phi+\varepsilon c(x)\phi_y]+O\left(\varepsilon^2\right)=0,\quad \text{on } y=h\end{aligned} \tag{10}$$

By using the perturbation technique, the entire fluid region $0\le y\le h+\varepsilon c(x),\ -\infty<x<\infty$ is reduced to the uniform finite strip $0\le y\le h,\ -\infty<x<\infty$ in the following mathematical analysis.

3 Method of solution

3.1 Perturbation technique

Suppose that a train of progressive waves of mode u to be obliquely incident at an angle θ, $0\le\theta<\pi/2$ on the bottom deformation on a porous surface in an ocean. If there is no bottom deformation, then the oblique incident wave train will propagate without any hindrance and there will be only transmission in this case. This, along with the appropriate form of the boundary condition (10), suggest that $\phi(x,y)$, R and T which were introduced in the last section, can be expressed in terms of the small parameter ε as:

$$\left.\begin{aligned}&\phi(x,y)=\phi_0(x,y)+\varepsilon\phi_1(x,y)+O\left(\varepsilon^2\right)\\&R=\varepsilon R_1+O\left(\varepsilon^2\right)\\&T=1+\varepsilon T_1+O\left(\varepsilon^2\right)\end{aligned}\right\} \tag{11}$$

where $\phi_0(x,y)$ is given by equation (8). It must be noted that such a perturbation expansion ceases to be valid at Bragg resonance when the reflection coefficient becomes much larger than the deformation parameter ε, as pointed out by Mei (1985). Also this theory is valid only for infinitesimal reflection and away from resonance. For large reflection, the perturbation series, as defined in Eq. (11), needs to be refined so that it can deal with the resonant case, which is reported in Mei (1985).

Using Eq. (11) in Eq. (2) and boundary conditions (3), (10), (9) and then comparing with the first-order terms of ε on both sides of the equations, we find a boundary value problem for the first-order potential $\phi_1(x,y)$ which satisfies the Eqs. (2) and (3) together with the following other conditions:

$$\phi_{1y}-P\phi_1=\frac{\left\{\mathrm{i}u\cos\theta\frac{\mathrm{d}}{\mathrm{d}x}\left[c(x)\mathrm{e}^{\mathrm{i}ux\cos\theta}\right]+\left(P^2-u^2\sin^2\theta\right)c(x)\mathrm{e}^{\mathrm{i}ux\cos\theta}\right\}}{\cosh uh-(P/u)\sinh uh}\equiv f(x) \qquad \text{on } y=h \tag{12}$$

$$\phi_1(x,y)\sim\begin{cases}R_1\phi_0(-x,y) & \text{as } x\to-\infty\\ T_1\phi_0(x,y) & \text{as } x\to\infty\end{cases} \tag{13}$$

To solve the above boundary value problem for ϕ_1, we need a two-dimensional source potential (in terms of Green's function) for the modified Helmholtz equation due to a source submerged in the fluid region. Then Green's integral theorem will be employed and the first-order coefficients R_1 and T_1 will be obtained in terms of integrals involving the shape function $c(x)$.

3.2 Introduction of Green's functions

In this section, Green's function method is introduced for solving the above boundary value problem. Then a two-dimensional source potential is obtained for the modified Helmholtz equation due to a source submerged in the fluid.

Suppose the source term (ξ,η) is submerged in the fluid region $-\infty<x<\infty,\quad 0<y<h$. Then for $0<\eta<h$, the source potential in terms of Green's function $G(x,y;\xi,\eta)$ satisfies the following boundary value problem:

$$\left(\nabla_{x,y}^{2}-\upsilon^{2}\right)G=0,\ -\infty<x<\infty,\ 0\le y\le h,\ \text{except at } (\xi,\eta) \tag{14}$$

$$G_y+KG=0 \quad \text{on } -\infty<x<\infty,\ \ y=0 \tag{15}$$

$$G_y-PG=0 \quad \text{on } -\infty<x<\infty,\ \ y=h \tag{16}$$

$$G\to \text{multiple of } \mathrm{e}^{\mathrm{i}\mu|x-\xi|} \quad \text{as } |x-\xi|\to\infty \tag{17}$$

$$G\sim K_0(\upsilon r) \ \text{ as } \ r=\sqrt{(x-\xi)^2+(y-\eta)^2}\to 0 \tag{18}$$

where $K_0(\bullet)$ denotes the modified Bessel function of the second kind. Now we try to solve the boundary value problem defined by Eqs. (14)–(18) in the form $G(x,y;\xi,\eta)$, where

$$G(x,y;\xi,\eta)=K_0(\upsilon r)-K_0(\upsilon r')+\int_{\upsilon}^{\infty}\frac{1}{\mu}\{A(k)\cosh k(h-y)+B(k)\sinh ky\}\cos\mu(x-\xi)\,\mathrm{d}k \tag{19}$$

where $\mu=\sqrt{k^2-\upsilon^2}$ and $r'=\sqrt{(x-\xi)^2+(y+\eta)^2}$.

With the help of the boundary conditions at the free-surface and at the bottom surface, we find $A(k)$ and $B(k)$ as

$$A(k)=\frac{2[k\cosh k(h-\eta)-P\sinh k(h-\eta)]}{\cosh kh\ \Delta(k)} \tag{20}$$

$$B(k)=\frac{2P[k\cosh k(h-\eta)-P\sinh k(h-\eta)]}{\cosh kh(k\cosh kh-P\sinh kh)\ \Delta(k)} \tag{21}$$

where

$$\Delta(k)=[k+(P/k)K]\sinh kh-(K+P)\cosh kh \tag{22}$$

It may be noted that $\Delta(k)$ has one simple non-zero positive root at $k=u$, from Eq. (6). Since $k=0$ and $\cosh kh-(P/k)\sinh kh=0$ will indicate that there is no wave in the fluid region, hence the terms k and $\cosh kh-(P/k)\sinh kh$ can never be zero. So the integrand in Eq. (19) has one simple pole at $k=u$ which will be from $\Delta(k)$ only. Since the source potential $G(x,y;\xi,\eta)$ behaves like outgoing waves as $|x-\xi|\to\infty$, so the path of integration is indented to pass beneath the simple pole at $k=u$. Solving Eq. (19) by using Eqs. (20) and (21), we obtain the solution $G(x,y;\xi,\eta)$ as $|x-\xi|\to\infty$, is given by

$$G(x,y;\xi,\eta)=2\pi\mathrm{i}\sec\theta\times\left[\frac{u\cosh u(h-\eta)-P\sinh u(h-\eta)}{u\Delta'(u)\cosh uh(u\cosh uh-P\sinh uh)}\right]\times[(u\cosh uh-P\sinh uh)\cosh u(h-y)+P\sinh uy]\times\mathrm{e}^{\mathrm{i}u\cos\theta|x-\xi|} \tag{23}$$

where Δ' denotes the derivative of Δ with respect to k. To calculate the value of $\phi_1(\xi,\eta)$, we apply the Green's integral theorem to $\phi_1(x,y)$ and $G(x,y;\xi,\eta)$ in the form

$$\int_C(\phi_1 G_n-G\phi_{1n})\mathrm{d}s=0 \tag{24}$$

where C is a closed contour in the xy-plane consisting of the lines $y=0$, $-X\le x\le X$; $y=h$, $-X\le x\le X$; $0\le y\le h$, $x=\pm X$ and a small circle of radius γ with center at (ξ,η) and ultimately let $X\to\infty$ and $\gamma\to 0$. Finally, the resultant integral Eq. (24) will give the determination of the solution ϕ_1 of the boundary value problem as given by

$$\phi_1(\xi,\eta)=\frac{1}{2\pi P}\int_{-\infty}^{\infty}G_y(x,h;\xi,\eta)f(x)\mathrm{d}x \tag{25}$$

The first-order reflection and transmission coefficients R_1 and T_1, respectively, are now obtained by letting $\xi\to\mp\infty$, in Eq. (25) and comparing with Eq. (13) by replacing (x,y) with (ξ,η). Thus we obtain the values of R_1 and T_1 as

$$R_1=\frac{\mathrm{i}\left(P^2+u^2\cos 2\theta\right)\sec\theta}{\Delta'(u)[\cosh uh-(P/u)\sinh uh]}\times\int_{-\infty}^{\infty}c(x)\,\mathrm{e}^{2\mathrm{i}ux\cos\theta}\mathrm{d}x \tag{26}$$

$$T_1=\frac{\mathrm{i}\left(P^2-u^2\right)\sec\theta}{\Delta'(u)[\cosh uh-(P/u)\sinh uh]}\int_{-\infty}^{\infty}c(x)\mathrm{d}x \tag{27}$$

Therefore, the first-order reflection and transmission coefficients due to oblique incident surface wave propagation over a small bottom deformation on the porous surface of an ocean bed are now can be evaluated from Eqs. (26) and (27), once the shape function $c(x)$ is known. Here, if $\theta=0$ is taken (i.e., the case of normal incidence), then the above results (26) and (27) coincide with the corresponding results as seen in Mohapatra (2014).

In the following section we proceed to examine the effects of reflection and transmission for a special sinusoidal form of the shape function $c(x)$.

4 A special form of the bottom surface

Here, we consider a special sinusoidal form of the shape function $c(x)$ for an uneven bottom to the porous surface in an ocean bed. As mentioned earlier, this functional form of the bottom disturbance closely resembles some naturally occurring obstacles formed at the bottom due to sedimentation and ripple growth of sands. Because of the importance of the bed topographies with sinusoidal ripples from the application point of view, significant emphasis is laid upon them and subsequent consideration of the following example is deemed appropriate.

The shape function $c(x)$ in the form of patch of sinusoidal bottom ripples on the bottom surface with amplitude a on an otherwise flat bottom has the form

$$c(x)=\begin{cases}a\sin lx, & L_1\le x\le L_2\\ 0 & \text{otherwise}\end{cases} \tag{28}$$

where $L_1=-n\pi/l$; $L_2=-m\pi/l$; m and n are positive integers, and l is the wave number of the patch in the region $L_1\le x\le L_2$. The patch of sinusoidal ripple bed consists of total $(m+n)/2$ number of ripples.

Substituting the value of $c(x)$ from the Eq. (28) into Eqs. (26) and (27), we obtain the reflection coefficient R_1 and transmission coefficient T_1, respectively, as follows:

$$R_1 = \left\{ \frac{\mathrm{i}al\left(P^2 + u^2\cos 2\theta\right)\sec\theta}{\Delta'(u)[\cosh uh - (P/u)\sinh uh]} \right\} \times \left[\frac{(-1)^n \mathrm{e}^{2\mathrm{i}(u\cos\theta)L_1} - (-1)^m \mathrm{e}^{2\mathrm{i}(u\cos\theta)L_2}}{l^2 - 4u^2\cos^2\theta} \right] \tag{29}$$

$$T_1 = \left\{ \frac{\mathrm{i}a\left(P^2 - u^2\right)\sec\theta}{\Delta'(u)[\cosh uh - (P/u)\sinh uh]} \right\} \times \left[\frac{(-1)^n - (-1)^m}{l} \right] \tag{30}$$

It may be noted from Eq. (29) that when the sinusoidal ripples wave number is approximately twice the component of the incident field wave number along x-direction (i.e., $2u\cos\theta \approx l$), the theory points at the possibility of a resonant interaction taking place between the bed and the free-surface waves. Hence, we find that near resonance, i.e., $2u\cos\theta \approx l$, the limiting value of the reflection coefficient assumes the value

$$R_1 \approx -\frac{a\left(P^2 + u^2\cos 2\theta\right)\sec^2\theta(m+n)\pi}{4u\Delta'(u)[\cosh uh - (P/u)\sinh uh]} \tag{31}$$

In this case, the reflection coefficient R_1 becomes a constant multiple of $(m+n)/2$, the total number of ripples in the patch. Hence, the reflection coefficient R_1 increases linearly with n and m. Although the theory breaks down when l=2$u$cosθ, a large amount of reflection of the incident wave energy by this special form of bed surface will be generated in the neighborhood of the singularity at l=2$u$cosθ.

Note that when $2u\cos\theta$ approaches l and the number of ripples in the patch of the deformation on the porous bed (m+n)/2 become large, the reflection coefficient becomes unbounded contrary to our assumption that R_1 is a small quantity, being the first-order correction of the infinitesimal reflection. Consequently, we consider only the cases excluding these two conditions in order to avoid the contradiction arising out of resonant cases.

Again, it is clear from Eq. (30) that when the number of ripples in the patch of the deformation on the porous bed is a positive integer (i.e., both m and n are even or odd), the first-order transmission coefficient vanishes identically.

5 Numerical results

In this section, the numerical computation and graphical presentation related to the special form of bottom surface mentioned in the previous section are shown for the first-order reflection and transmission coefficients. We consider the numerical computations for the non-dimensionalized first-order reflection coefficient $|R_1|$, which is calculated from Eq. (29), due to an oblique incident surface waves of wave number u making an angle θ to the positive x-direction propagating along the free-surface and a ripple bed with wave number l having (m+n)/2 number of ripples in the patch of the porous ocean bed. Again, in this case we consider the ratio of the amplitude of the ripples and the depth of the fluid (a/h) is taken as 0.1 .

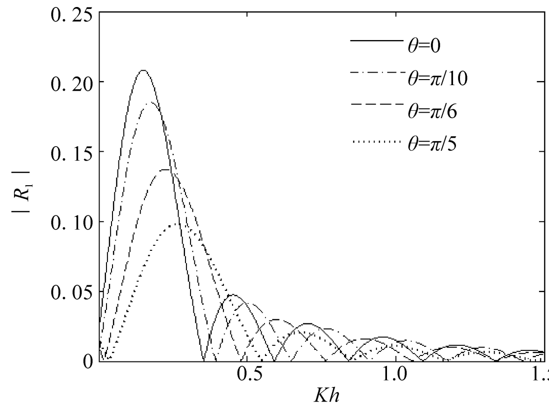

Fig. 1 Reflection coefficient $|R_1|$ plotted against *Kh* for *Ph*=0.1, *m*=2 and *n*=3

The different curves in Fig. 1 correspond to four different angles of incidence, $\theta = 0, \pi/10,\ \pi/6$ and $\pi/5$. For all these curves, m and n are fixed at $m = 2$ and $n = 3$, the porous parameter Ph as 0.1 and the ripple wave number lh as 1. It may be noted that for θ=0 (the case of normal incidence), the maximum value of $|R_1|$ is 0.208148 , attained at $uh = 0.506998$ (when Kh=0.151), that is, when the ripple wave number la of the bottom deformation on the porous ocean bed becomes approximately twice as large as the component of the incident field wave number uhcosθ along x-direction. The same can be observed when the angle of incidence θ is non-zero (the case of oblique incidence). Another common feature in Fig. 1 is the oscillating nature of the absolute value of the first-order coefficient as a function of the wave number Kh. As the angle of incidence θ increases, the peak value of $|R_1|$ decreases. When θ approaches to $\pi/4$, the reflection coefficient $|R_1|$ is much less as compared to the other angles of oblique incidence. In the case of normal incidence, the peak value of $|R_1|$ is the largest.

In Fig. 2, $|R_1|$ is plotted against Kh for different porous effect parameter Ph of the ocean bed, while we fixed the angle of incidence at θ=π/6, the ripple wave number lh as 1, m=2 and n=3. This is most evident in the curves that the peak value of $|R_1|$ increases as the porous effect parameter of the ocean bed increases. This shows that the first-order correction to the reflection coefficient is somewhat sensitive to the changes in the porous effect parameter of the ocean bed. The peak values of the first-order reflection coefficient corresponding to the porous effect parameters Ph=0, 0.01, 0.05 and 0.1 are attained at uh= 0.584073, 0.585435, 0.583963 and 0.581682, respectively. Here also it is observed that its peak value is attained when the ripple wave number lh of the bottom deformation on the porous ocean bed becomes approximately twice as large as the component of the incident field wave number $uh\cos\theta$ along x-direction.

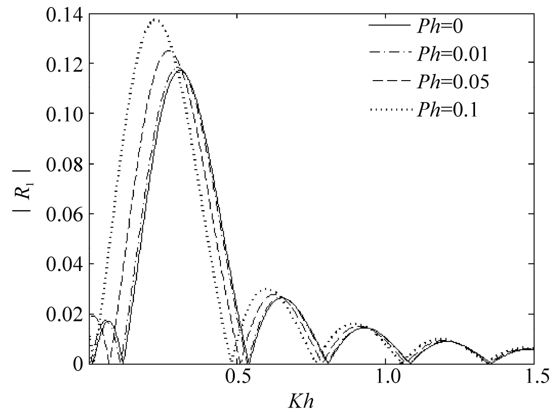

Fig. 2 Reflection coefficient $|R_1|$ plotted against Kh for θ=π/6, m=2 and n=3

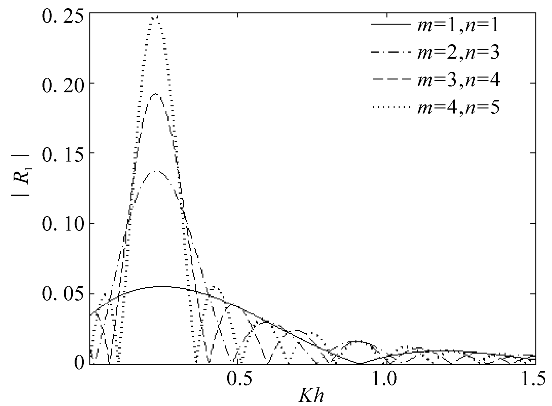

Fig. 3 Reflection coefficient $|R_1|$ plotted against Kh for Ph=0.1 and θ=π/6

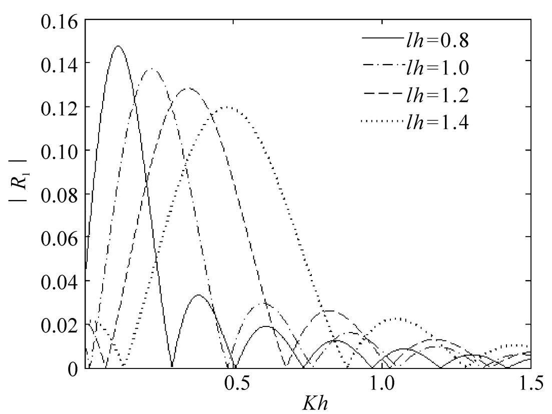

Fig. 4 Reflection coefficient $|R_1|$ plotted against Kh for θ=π/6, m=2 and n=3

In Fig. 3, different curves correspond to different number of ripples in the patch of the deformation on the porous bed. For all these curves, we consider the porous parameter Ph as 0.1, the angle of incidence θ as π/6 and the ripple wave number lh as 1. It is clear from this figure that as $(m+n)/2$, the number of ripples in the patch of the bottom deformation increases, the value of $uh\cos\theta$ converges to a number in the neighborhood of $lh/2$ and also the peak value of the reflection coefficient $|R_1|$ increases. But when the number of ripples, becomes very large, the reflection coefficient become unbounded. That means the perturbation expansion which is discussed in section 3.1, ceases to be valid when the reflection coefficient becomes much larger than the deformation parameter, as pointed out by Mei (1985). Its oscillatory nature against Kh is more noticeable with the number of zeros of $|R_1|$ increased but the general feature of $|R_1|$ remains the same.

In Fig. 4, different curves correspond to different ripple wave numbers lh=0.8, 1, 1.2 and 1.4 in the patch of the deformation on the porous ocean bed. In this figure, for all curves, we consider Ph=0.1, θ=π/6, m=2 and n=3. Here also, it has been clear that the peak values of the reflection coefficient are attained at different values of Kh. The reason is, the values of reflection coefficient $|R_1|$ (calculated from Eq. (29)) become maximum, only when $lh \approx 2uh\cos\theta$. It is also observed from this figure that as the ripple wave numbers increase the reflection coefficient $|R_1|$ becomes smaller than those for the bigger ripple wave numbers. That means when an oblique incident wave propagates over a porous bed in an ocean having a small ripple wave number in the patch of the deformation, a substantial amount of reflected energy can be produced.

From all the figures, it is also clear that the oscillating nature of the absolute values of the first-order coefficient as functions of the wave number Kh.

6 Conclusions

The work described in this article is the classical problem of scattering of oblique incident surface waves by a small bottom deformation on the porous surface in an ocean. In such a situation propagating waves can exist in only one wave number for any given frequency. A perturbation analysis has been deployed and thereby finding new expressions for the first-order corrections to the reflection and transmission coefficients for the problem by using a method based on Green's integral theorem with the introduction of appropriate Green's function. For the particular example of a patch of sinusoidal ripples, first-order approximations to the reflection and transmission coefficients are obtained in terms of computable integrals and the reflection coefficient depicted graphically through a number of figures. The main result that follows is that, the resonant interaction between the bed and the free-surface attains in the neighborhood of the singularity when the ripple wave numbers of the bottom deformation become twice the component of the incident field wave number along x-direction. This singularity point varies with the angles of oblique incident progressive waves, porous effect parameters of the ocean bed and the ripple wave numbers on the bottom surface. Another main advantage of this method, demonstrated through this example, is that a very few ripples may be needed to produce a substantial amount of reflected energy. It is also observed that for small angles of incidence, the reflected energy is more as compared to other angles of incidence (except at $\theta=\pi/4$). From the computational results it is observed that when the porosity of the ocean bed increases, the values of the reflected energy increase so that the amplitude of the generated wave increases. Again, from the derived results, the solutions for problems with impermeable ocean bed can be obtained as particular cases. Also the theory discussed in this article is valid only for infinitesimal reflection and away from resonance. The solution obtained here is expected to be qualitatively helpful for a wide class of surface waves scattering problems involving an uneven bottom on the porous surface in an ocean.

Acknowledgement

The author wishes to thank Prof. S.N. Bora, Department of Mathematics, Indian Institute of Technology Guwahati, India for his valuable discussions and suggestions to carry out the preparation of the manuscript. The author expresses his deep gratitude to the referees for their valuable comments and suggestions which enabled the author to carry out the desired revision of the manuscript.

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