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Wave Scattering by Porous Bottom Undulation in a Two Layered Channel

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Abstract: The scattering of plane surface waves by bottom undulations in channel flow consisting of two layers is investigated by assuming that the bed of the channel is composed of porous material. The upper surface of the fluid is bounded by a rigid lid and the channel is unbounded in the horizontal directions. There exists only one wave mode corresponding to an internal wave. For small undulations, a simplified perturbation analysis is used to obtain first order reflection and transmission coefficients in terms of integrals involving the shape function describing the bottom. For sinusoidal bottom undulations and exponentially decaying bottom topography, the first order coefficients are computed. In the case of sinusoidal bottom the first order transmission coefficient is found to vanish identically. The numerical results are depicted graphically in a number of figures.
Keywords: bottom undulations; two-layer fluid; porous bed; reflection and transmission coefficients; wave scattering

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1 Introduction

Wave scattering and generation problems in continuously stratified or multi-layered fluid have recently attracted a good deal of attention. Although, propagation of waves in a two-layer fluid was described by Stokes (1847) long back, till now the literature of two-layer fluid problems is rather limited. Linear wave motion in a two-layer fluid is described in the text books of Lamb (1932) and Landau and Lifshitz (1989). For normally incident waves, the corresponding problem of wave scattering by small bottom undulations in a two-layer fluid with the upper layer extending infinitely upwards and the lower layer having bottom undulations, was studied by Mandal and Basu (1993). The oblique interface wave scattering in such a two-layer fluid was also considered by Mandal and Basu (1994). A number of wave problems in such a two-layer fluid were studied by Dolai and Mandal (1994, 1995), Mandal and Chakrabarti (1995). A two-layer model of an ocean consisting of a layer of fresh water over a deep layer of saline water requires special attention as in this case there exist waves of two different wave numbers, one with lower wave number propagating along the free surface and the other with higher wave number propagating along the interface. Linton and McIver (1995) considered scattering of water waves by a horizontal cylinder in an infinitely deep two-layer fluid where in the upper layer has a free surface. Using linear theory, they examined the interaction of surface and interface waves with a horizontal circular cylinder. The motivation for their work came from a plan to build an underwater pipe-bridge across one of the Norweigian fjords, bodies of water which typically consist of a layer of fresh water on top of salt water. Cadby and Linton (2000) considered three-dimensional water wave scattering in such a two-layer fluid. They developed a general three-dimensional linear scattering theory and then illustrated it by solving problems involving submerged spheres.

Much work has been done on wave/structure interactions in such fluid region approximating the free surface by a rigid lid. With the free surface approximated by rigid lid Sturova (1994), for example, has studied the radiation of wave by an oscillating cylinder which is also moving uniformly in a direction perpendicular to its axis. Sturova (1999) considered the radiation and scattering problem in a cylinder in both a two and a three layer fluid bounded above and below by rigid horizontal walls. Gavrilov *et al*. (1999) also investigated the effects of a smooth pycnocline on wave scattering, again for horizontal circular cylinder where the fluid is bounded above and below by rigid walls.

Sherief *et al*. (2003) investigated the motion generated by a vertical wave-maker immersed in a two-layer fluid, the prescribed normal velocity on the wave-maker varying with depth and harmonically with time. The wave-maker was also assumed tobe porous. Sherief *et al*. (2004) also investigated a vertical cylindrical porouswave-maker immersed in a two layer fluid. Chamberlain and Porter (2005) considered two-layer fluid problem involving bottom variation while Ten and Kashiwagi (2004), Kashiwagi *et al*. (2006) studied hydrodynamics of a body floating in a two-layer fluid. Mase and Takeba (1994), Zhu (2001) and Silva *et al*. (2002) investigates the wave scattering problem involving porous bed. Martha *et al*. (2007) considered the problem of oblique water-wave scattering by small undulation on porous sea bed. They obtain the first order reflection and transmission coefficients. The problem of

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oblique wave propagation over a small deformation in a channel flow consisting of two layers was considered by Mahapatra and Bora (2012).

In the present paper scattering by porous bottom undulation in a two layered channel is investigated. Using linear theory, the problem is formulated as a coupled boundary value problem for the two potential functions describing the fluid motion in each of two layers. A simplified perturbation technique is employed to reduce the original boundary value problem coupled one upto first order. This problem is solved here by a method, based on the Fourier transform technique, to obtain the first order reflection and transmission coefficients in terms of integrals involving the shape function describing the bottom undulations. The first-order coefficients are depicted graphically against the wave number for two different shape functions. The effect of porosity is observed in the numerical results.

2 Mathematical formulations

We consider a two-layer invicide, incompressible fluid flowing through a channel with upper layer bounded by a rigid lid, while the lower layer has small cylindrical undulations at the porous bottom. Here a two dimensional co-ordinate system is chosen in such a way that $y=-h'$ denotes the position of the rigid plate and $y=0$ denotes the undisturbed interface while y-axis directed vertically downwards. The bottom of the lower layer can be represented by $y=h+\varepsilon c(x)$, where ε is a dimensionless small quantity, measures the smallness of the deformation and $c(x)$ is a bounded and continuously differentiable function describing the shape of the undulating bottom, such that $c(x)\to 0$ as $|x|\to\infty$. Thus the lower layer is of uniform finite depth h below the mean interface far away from the undulations on either side. As the fluid motion is irrotational, the time dependent harmonic velocity potentials of the upper and lower layer can be described by $\mathrm{Re}[\psi(x,y)\mathrm{e}^{-\mathrm{i}\sigma t}]$ and $\mathrm{Re}[\varphi(x,y)\mathrm{e}^{-\mathrm{i}\sigma t}]$, where σ is the angular frequency of the incoming wave. The density of the upper fluid is ρ_1 and the lower fluid is $\rho_2(>\rho_1)$. The functions $\psi(x,y)$ and $\varphi(x,y)$ satisfy

$$\nabla^2\psi=0 \quad \text{in the upper fluid} \tag{1}$$

$$\nabla^2\varphi=0 \quad \text{in the lower fluid} \tag{2}$$

The linearized boundary conditions at the interface and the two boundaries of the channel are given by

$$\frac{\partial\psi}{\partial y}=0 \quad \text{on} \quad y=-h' \tag{3}$$

$$K\varphi+\frac{\partial\varphi}{\partial y}=\rho(K\psi+\frac{\partial\psi}{\partial y}) \quad \text{on} \quad y=0 \tag{4}$$

$$\frac{\partial\varphi}{\partial y}=\frac{\partial\psi}{\partial y} \quad \text{on} \quad y=0 \tag{5}$$

and

$$\frac{\partial\varphi}{\partial n}-G\varphi=0 \quad \text{on} \quad y=h+\varepsilon c(x) \tag{6}$$

where $K=\dfrac{\sigma^2}{g}$, g being the acceleration due to gravity, $\rho=\dfrac{\rho_1}{\rho_2}(<1)$ and $\dfrac{\partial}{\partial n}$ denoting the normal derivative at a point (x,y) on the sea bed, G is the porous effect parameter on the porous bed. Though we have considered G as a real quantity, it is possible to find the dispersion relation for complex G too.

The velocity potentials of the progressive interface wave train coming from the negative infinity are $\psi_0(x,y)\mathrm{e}^{-\mathrm{i}\sigma t}$ and $\varphi_0(x,y)\mathrm{e}^{-\mathrm{i}\sigma t}$ in the lower and upper fluids, where $\psi_0(x,y)$ and $\varphi_0(x,y)$ has the following forms:

$$\psi_0(x,y)=\frac{\cosh k(h'+y)}{\sinh kh'}\mathrm{e}^{\mathrm{i}kx} \tag{7}$$

$$\varphi_0(x,y)=\frac{G\sinh k(h-y)-k\cosh k(h-y)}{k\sinh kh-G\cosh kh}\mathrm{e}^{\mathrm{i}kx} \tag{8}$$

where k is real, positive and satisfies the dispersion relation

$$\Delta(k)=0 \tag{9}$$

where

$$\Delta(k)=(1-\rho)k-K(\coth kh+\rho\coth kh')-\frac{KG\operatorname{csch}^2 kh}{(k-G\coth kh)} \tag{10}$$

The dispersion equation has exactly one positive real root, m $(m>0)$, say; describing mode of the wave propagating through the interface. Since upper layer is bounded by a rigid lid, there is only one wave mode, corresponding to an internal wave. The explanation of roots of the dispersion equation has been given in the Appendix.

The progressive waves of mode m in the upper and lower fluid are given by the velocity potentials

$$\psi^{(m)}(x,y)=\frac{\cosh m(h'+y)}{\sinh mh'}\mathrm{e}^{\mathrm{i}mx} \tag{11}$$

and

$$\varphi^{(m)}(x,y)=\frac{G\sinh m(h-y)-m\cosh m(h-y)}{m\sinh mh-G\cosh mh}\mathrm{e}^{\mathrm{i}mx} \tag{12}$$

When a train of progressive wave with mode m incident upon the undulating porous sea bottom, it produces reflected wave train in the negative x direction for $x\to-\infty$ and transmitted wave train in the positive x direction for $x\to+\infty$ mode m. These conditions can be mathematically expressed as far field conditions for ψ and φ by

$$\psi(x,y)\to\begin{cases}\psi^{(m)}(x,y)+r^m\psi^{(m)}(-x,y), & \text{as} \quad x\to-\infty\\ t^m\psi^{(m)}(x,y), & \text{as} \quad x\to\infty\end{cases} \tag{13}$$

$$\varphi(x,y)\to\begin{cases}\varphi^{(m)}(x,y)+r^m\varphi^{(m)}(-x,y), & \text{as} \quad x\to-\infty\\ t^m\varphi^{(m)}(x,y), & \text{as} \quad x\to\infty\end{cases} \tag{14}$$

where r^m and t^m denotes the reflection and transmission coefficients corresponding to the reflected wave and transmitted wave of mode m respectively.

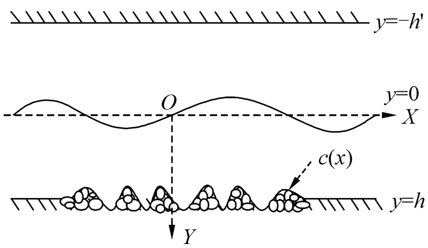

Fig. 1 Sketch of the problem

3 Method of solution

The bottom condition (6) can be expressed approximately as

$$\frac{\partial\varphi}{\partial y}-\varepsilon\frac{\mathrm{d}}{\mathrm{d}x}\left\{c(x)\frac{\partial\varphi}{\partial x}\right\}-G\left[\varphi+\varepsilon c(x)\frac{\partial\varphi}{\partial y}\right]\text{on } y=h \tag{15}$$

The form of the approximate bottom condition (15) suggests that ψ , φ , r^m , t^m have the following perturbational expansions, in terms of the small parameter ε as

$$\begin{aligned}
&\psi(x,y)=\psi^{(m)}(x,y)+\varepsilon\psi_1(x,y)+O(\varepsilon^2)\\
&\varphi(x,y)=\varphi^{(m)}(x,y)+\varepsilon\varphi_1(x,y)+O(\varepsilon^2)\\
&t^m(x,y)=1+\varepsilon t_1^{\,m}(x,y)+O(\varepsilon^2)\\
&r^m(x,y)=\varepsilon r_1^{m}(x,y)+O(\varepsilon^2)
\end{aligned}\tag{16}$$

where $\psi^{(m)}$, $\varphi^{(m)}$ are given by the expressions (11) and (12).

On substituting (16) in Eqs. (3)–(5), (15) and on equating the coefficients of ε from both sides, results the following coupled BVP for first order potential functions as:

$$\begin{aligned}
&\nabla^2\psi_1=0 \text{ in } -h'<y<0, -\infty<x<\infty\\
&\nabla^2\varphi_1=0 \text{ in } 0<y<h, -\infty<x<\infty\\
&\frac{\partial\psi_1}{\partial y}=0 \text{ on } y=-h'\\
&K\varphi_1+\frac{\partial\varphi_1}{\partial y}=\rho(K\psi_1+\frac{\partial\psi_1}{\partial y}) \text{ on } y=0\\
&\frac{\partial\varphi_1}{\partial y}=\frac{\partial\psi_1}{\partial y} \text{ on } y=0\\
&\frac{\partial\varphi_1}{\partial y}-G\varphi_1\equiv V(x) \text{ on } y=h
\end{aligned}\tag{17}$$

where,

$$V(x)=\mathrm{i}\,m\,A\frac{\mathrm{d}}{\mathrm{d}x}\left\{c(x)\mathrm{e}^{\mathrm{i}mx}\right\}+AG^2c(x)\mathrm{e}^{\mathrm{i}mx} \tag{18}$$

and

$$A=\frac{m}{G\cosh mh-m\sinh mh} \tag{19}$$

The infinite requirements (13) and (14) give

$$\psi_1(x,y)\to\begin{cases} r_1^{\,m}\psi^{(m)}(-x,y), \text{as } x\to-\infty\\ t_1^{\,m}\psi^{(m)}(x,y), \ \text{as } x\to\infty\end{cases} \tag{20}$$

$$\varphi_1(x,y)\to\begin{cases} r_1^{\,m}\varphi^{(m)}(-x,y), \ \text{as } x\to-\infty\\ t_1^{\,m}\varphi^{(m)}(x,y), \ \text{as } x\to\infty\end{cases} \tag{21}$$

4 First order reflection and transmission coefficients

Solutions of the problem described by Eq. (17) for the potentials $\psi_1(x,y)$ and $\varphi_1(x,y)$ are obtained by using Fourier transform technique.

Let us define the Fourier transform of $\psi_1(x,y)$ and $\varphi_1(x,y)$ by

$$\psi_1(k,y)=\int_{-\infty}^{\infty}\psi_1(x,y)\mathrm{e}^{-\mathrm{i}kx}\,\mathrm{d}x \tag{22}$$

$$\varphi_1(k,y)=\int_{-\infty}^{\infty}\varphi_1(x,y)\mathrm{e}^{-\mathrm{i}kx}\,\mathrm{d}x \tag{23}$$

The above transformation exists when $\psi_1(x,y)$ and $\varphi_1(x,y)$ decreases exponentially as $|x|\to\infty$ and it is possible if we assume that, k has a small positive imaginary part, i.e, we are replacing k by $k+\mathrm{i}k_1$.

To decouple the BVP (17) we write

$$\frac{\partial\varphi_1}{\partial y}=p(x), \text{ on } y=0 \tag{24}$$

so that

$$\frac{\partial\psi_1}{\partial y}=p(x), \text{ on } y=0 \tag{25}$$

where $p(x)$ is an unknown function.

We get the following boundary value problems

$$\begin{aligned}
&\frac{\partial^2\Psi_1}{\partial y^2}-k^2\Psi_1=0, \text{ in } -h'<y<0\\
&\frac{\partial\Psi_1}{\partial y}=\overline{p}(k), \text{ on } y=0\\
&\frac{\partial\Psi_1}{\partial y}=0, \text{ on } y=-h'
\end{aligned}\tag{26}$$

and

$$\begin{aligned}
&\frac{\partial^2\Phi_1}{\partial y^2}-k^2\Phi_1=0, \text{ in } 0<y<h\\
&\frac{\partial\Phi_1}{\partial y}=\overline{p}(k), \text{ on } y=0\\
&\frac{\partial\Phi_1}{\partial y}-G\Phi_1=\overline{V}(k), \text{ on } y=h
\end{aligned}\tag{27}$$

where, $\overline{p}(k)$ and $\overline{V}(k)$ respectively, given by

$$\overline{p}(k)=\int_{-\infty}^{\infty}p(x)\mathrm{e}^{-\mathrm{i}kx}\,\mathrm{d}x \tag{28}$$

$$\overline{V}(k)=\int_{-\infty}^{\infty}V(x)\mathrm{e}^{-\mathrm{i}kx}\,\mathrm{d}x \tag{29}$$

The solution of the BVP (26)–(27) can be expressed as

$$\psi_1(k,y)=\frac{\overline{p}(k)}{k}\frac{\cosh k(h'+y)}{\sinh kh'},\ \text{in}-h'<y<0 \tag{30}$$

$$\Phi_1(k,y)=\frac{\overline{p}(k)}{k}\left[\frac{G\sinh k(h-y)-k\cosh k(h-y)}{k\sinh kh-G\cosh kh}\right]+\frac{\overline{V}(k)\cosh ky}{k\sinh kh-G\cosh kh},\ \text{in}\,0<y<h \tag{31}$$

where $\overline{p}(k)$ is given by

$$\overline{p}(k)=\frac{Kk\overline{V}(k)}{\sinh kh\,\Delta(k)} \tag{32}$$

Using (30) and (31), we get

$$\psi_1(k,y)=\frac{K\overline{V}(k)}{\sinh kh\,\Delta(k)}\frac{\cosh k(h'+y)}{\sinh kh'},\ \text{in}-h'<y<0 \tag{33}$$

$$\Phi_1(k,y)=\frac{K\overline{V}(k)}{\sinh kh\,\Delta(k)}\left[\frac{G\sinh k(h-y)-k\cosh k(h-y)}{k\sinh kh-G\cosh kh}\right]+\frac{\overline{V}(k)\cosh ky}{k\sinh kh-G\cosh k},\ \text{in}\,0<y<h \tag{34}$$

By inverse Fourier transform, Eqs. (33) and (34) gives

$$\psi_1(x,y)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\Psi_1(k,y)e^{ikx}\,dk,\ \text{in}-h'<y<0 \tag{35}$$

$$\varphi_1(x,y)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\Phi_1(k,y)e^{ikx}\,dk,\ \text{in}\,0<y<h \tag{36}$$

Therefore,

$$\psi_1(x,y)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{K\overline{V}(k)}{\sinh kh\,\Delta(k)}\frac{\cosh k(h'+y)}{\sinh kh'}e^{ikx}\,dk,\quad \text{in}-h'<y<0 \tag{37}$$

$$\varphi_1(x,y)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{K\overline{V}(k)}{\sinh kh\,\Delta(k)}\left[\frac{G\sinh k(h-y)-k\cosh k(h-y)}{k\sinh kh-G\cosh kh}+\frac{\overline{V}(k)\cosh ky}{k\sinh kh-G\cosh kh}\right]e^{ikx}\,dk,\ \text{in}\,0<y<h \tag{38}$$

Now, since $\Delta(k)=\Delta(-k)$, we can write

$$\psi_1(x,y)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{K}{\sinh kh\,\Delta(k)}\frac{\cosh k(h'+y)}{\sinh kh'}\times\left[\overline{V}(k)e^{ikx}+\overline{V}(-k)e^{-ikx}\right]dk,\ \text{in}-h'<y<0 \tag{39}$$

$$\varphi_1(x,y)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{K}{\sinh kh\,\Delta(k)}\left[\frac{G\sinh k(h-y)-k\cosh k(h-y)}{k\sinh kh-G\cosh kh}+\frac{\cosh ky}{k\sinh kh-G\cosh kh}\right]\left[\overline{V}(k)e^{ikx}+\overline{V}(-k)e^{-ikx}\right]dk,\ \text{in}\,0<y<h \tag{40}$$

The integrands on the right hand side of (39) and (40) has singularities at the zeros of $\Delta(k)$. As $k_1\to 0$ the dispersion equation has only two real values of k and an infinite no of values $\pm ik_n$, $n=0,1,2,...$; satisfying the equation

$$(1-\rho)k_n+K(\cot k_nh+\rho\cot k_nh')-\frac{KG\csc^2 k_nh}{(k_n+G\cot k_nh)}=0 \tag{41}$$

The first order reflection and transmission coefficients r_1^m and t_1^m are obtained by making $x\to\mp\infty$ in (39) and (40) and comparing with the infinite requirements (20) and (21) respectively.

As $x\to\infty$ the transmission coefficient is obtained by rotating the contour of integration involving e^{ikx} into a contour in the first quadrant by an angle $\theta\left(0<\theta<\frac{\pi}{2}\right)$ so that the pole at $k=m$ lies inside it and the contour of the integration involving e^{-ikx} into the fourth quadrant by the same angle θ. Then the integrand involving e^{-ikx} has no effect in calculating $\psi_1(x,y)$ and $\varphi_1(x,y)$. By complex integration technique we get

$$\psi_1(x,y)\to\frac{iK}{\sinh mh\,\Delta'(k)}\frac{\cosh m(h'+y)}{\sinh mh'}\overline{V}(m)e^{imx}\ \text{as}\,x\to\infty \tag{42}$$

and

$$\varphi_1(x,y)\to\frac{iK}{\sinh mh\,\Delta'(k)}\left[\frac{G\sinh m(h-y)-m\cosh m(h-y)}{m\sinh mh-G\cosh mh}\right]\times\overline{V}(m)e^{imx},\ \text{as}\,x\to\infty \tag{43}$$

where,

$$\overline{V}(m)=A(G^2-m^2)\int_{-\infty}^{\infty}c(x)dx \tag{44}$$

Similarly, as $|x|\to-\infty$ the first order reflection coefficient is obtained from the analysis of the behavior of $\psi_1(x,y)$ and $\varphi_1(x,y)$ in Eqs. (39) and (40) by rotating the path of the second integrals into a contour in the first quadrant, so that we must include the residue term at $k=m$. In this case the term involving e^{ikx} has no contribution in the expressions of ψ_1 and φ_1 and these are:

$$\psi_1(x,y)\to\frac{iK}{\sinh mh\,\Delta'(k)}\frac{\cosh m(h'+y)}{\sinh mh'}\overline{V}(-m)e^{imx}\quad \text{as}\,x\to-\infty \tag{45}$$

$$\varphi_1(x,y)=\frac{iK}{\sinh mh\,\Delta'(k)}\left[\frac{G\sinh m(h-y)-m\cosh m(h-y)}{m\sinh mh-G\cosh mh}\right]\times\overline{V}(-m)e^{imx}\,\text{as}\,x\to-\infty \tag{46}$$

where,

$$\overline{V}(-m)=A(G^2+m^2)\int_{-\infty}^{\infty}c(x)e^{2imx}\,dx \tag{47}$$

Comparing (42) and (43), (45) and (46) with (20) and (21), we get the first order coefficients as:

$$r_1^m=\frac{iKA(G^2+m^2)}{\sinh mh\,\Delta'(m)}\int_{-\infty}^{\infty}c(x)e^{2imx}\,dx \tag{48}$$

$$t_1^m=\frac{iKA(G^2-m^2)}{\sinh mh\,\Delta'(m)}\int_{-\infty}^{\infty}c(x)dx \tag{49}$$

where $\Delta(m)$ and A are given by equations (10) and (19) respectively.

5 Numerical results

Example1: For sinusoidal undulations at the bottom of the two-layer fluid, the shape function $c(x)$ as

$$c(x)=\begin{cases} a\sin\mu x, \text{ for} -\dfrac{n\pi}{\mu}\le x\le\dfrac{n\pi}{\mu} \\ 0, \text{ otherwise}\end{cases} \tag{50}$$

where n is a positive integer. a and μ are the amplitude of the sinusoidal ripple on the bottom surface and the ripple wave number respectively.

Substitute (50) in the expressions (48)–(49), the first order reflection coefficients and the transmission coefficients are given as follows:

$$r_1^m=\frac{(-1)^{n+1}2KaA(G^2+m^2)\mu}{\sinh mh(4m^2-\mu^2)\Delta'(m)}\sin\frac{2n\pi m}{\mu} \tag{51}$$

$$t_1^m=0 \tag{52}$$

where $\Delta(m)$ and A are given as in expressions (10) and (19) respectively.

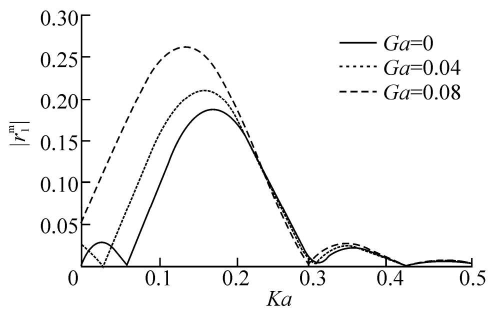

Fig. 2 Sinusoidal undulations $|r_1^m|$ is plotted against *Ka* for different *Ga*

The Fig. 2 depicted here is the first order reflection coefficient against *Ka*. The graph plotted for ρ=0.05, $h=5a$, $h'=5a$, $\mu a=0.47$, $n=2$ and three different values of *Ga*, viz. 0.0, 0.04, 0.08. The results obtained here agree with the known results when the bed has no porous effect (*Ga*=0.0). We have considered a very small density ratio for sake of batter interfacial effect, but it can take any value less than one. In the expression (51), $|r_1^m|$ is unbounded when μ~$2m$, i.e.. The graph showing that, the energy reflection increases with the porous effect.

Fig. 3 showing the reflection coefficients for different values of the bottom ripple number (n=1, 3, 5) and for $\rho=0.05$, $h=5a$, $h'=5a$, $\mu a=0.47$ when $Ga=0.05$ and it is clearly seen that the peak values of the coefficients increases. Which shows that, if the ripple number increases indefinitely, the first order coefficients become unbounded for certain value of Ka .

In Fig. 4, $|r_1^m|$ are plotted for three different density ratios of the fluid layers and for $Ga=0.04$, $h=5a$, $h'=5a$, $\mu a=0.47$, $n=5$ found that for a particular porous effect the wave energy decreases with the density ratio.

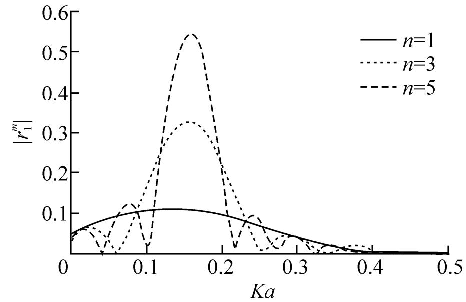

Fig. 3 Sinusoidal undulations $|r_1^m|$ is plotted against *Ka* for different *n*

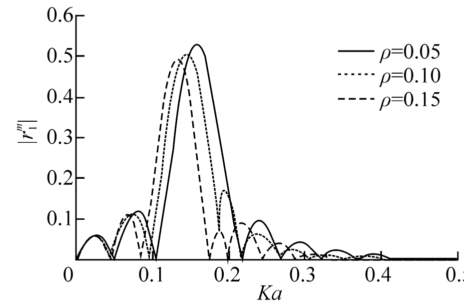

Fig. 4 Sinusoidal undulations $|r_1^m|$ is plotted against *Ka* for different ρ

In expression (51), $|r_1^m|$ is a periodic function of $\dfrac{m}{\mu}$ with period $\dfrac{1}{n}$. So, the number of oscillation increases as the ripple number. Also when $\dfrac{m}{\mu}$ is exactly half a unit, i.e. the ripple wave number is approximately equal to twice of the surface wave number the reflection coefficient becomes unbounded and in that case Bragg resonance occurred. At Bragg resonance the reflection coefficient is given by

$$\frac{KaA(G^2+m^2)}{\sinh mh\Delta'(k)}\frac{n\pi}{\mu} \tag{53}$$

Example 2: We consider the shape function $c(x)$ in the form of an exponentially decaying bottom as

$$c(x)=a\,\mathrm{e}^{-\mu|x|},\ \text{for } -\infty<x\le\infty \text{ and } \mu>0 \tag{54}$$

where a and μ are the amplitude of the ripple on the bottom surface and the ripple wave number respectively. On substitution (54) in the expressions (48)–(49), the first order reflection and transmission coefficients are given as follows:

$$r_1^m=\frac{2\mathrm{i}KaA(G^2+m^2)\mu}{\sinh mh(4m^2+\mu^2)\Delta'(m)} \tag{55}$$

$$t_1^m=\frac{2\mathrm{i}KaA(G^2-m^2)}{\mu\sinh mh\Delta'(m)} \tag{56}$$

where $\Delta(m)$ and A are given by the expressions (10) and (19).

The graphs depicts in Figs. 5 and 6 are $|r_1^m|$ and $|t_1^m|$ as a function of *Ka* for three different values of *Ga* (viz. 0.0, 0.04, 0.08) and $\rho=0.05$, $h=5a$, $h'=5a$, μa =0.47. In each of these figures the peak value increases as with porous effect parameter.

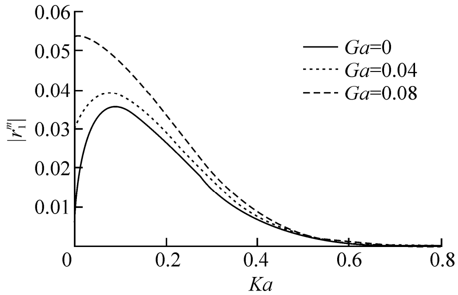

Fig. 5 Exponentially decaying bottom $|r_1^m|$ is plotted against *Ka* for different *Ga*

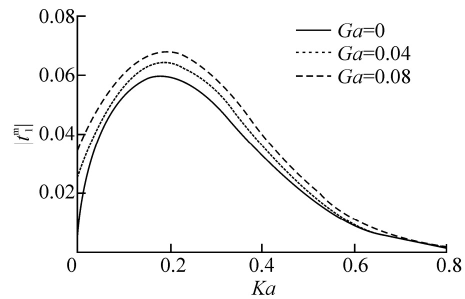

Fig. 6 Exponentially decaying bottom $|t_1^m|$ is plotted against *Ka* for different *Ga*

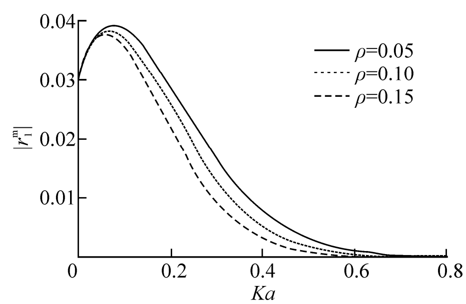

Fig. 7 Exponentially decaying bottom $|r_1^m|$ is plotted against *Ka* for different *ρ*

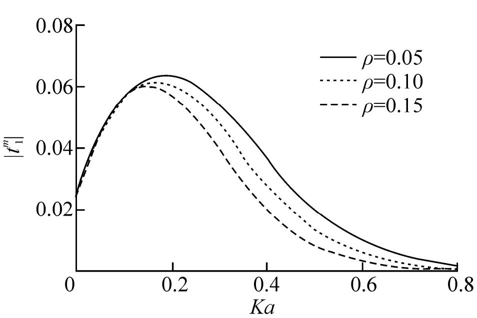

Fig. 8 Exponentially decaying bottom $|t_1^m|$ is plotted against *Ka* for different *ρ*

In Figs. 7 and 8, reflection and transmission coefficients $|r_1^m|$ and $|t_1^m|$ are depicted against Ka for different values of the density ratio ρ for Ga=0.04, h=5a, $h'=5a$, μa =0.47. In each of these two figures it is observed that as ρ increases the peak value of $|r_1^m|$ and $|t_1^m|$ decreases. Thus the first order coefficients are quite sensitive to the density ratio.

6 Conclusions

Scattering of surface waves by porous bottom undulation in a two layered channel is investigated. Using a simplified perturbation analysis, the problem is reduced upto first order to a coupled boundary value problem. The boundary value problem is solved by Fourier transform technique. First order reflection and transmission coefficients are obtained in terms of integrals involving the shape function representing the bottom undulations. The bottom undulations are described by sinusoidal ripples on an otherwise flat bed and also by an exponentially decaying profile. For the important case of sinusoidal bottom undulations, the first-order correction for the reflection coefficient is depicted graphically against the wave number, while the same for transmission coefficient vanishes identically. For the case of exponentially decaying bottom topography the first order corrections to reflection and transmission coefficients are also depicted graphically. It is observed that the reflection coefficient increases with increasing porous effect. Also for the sinusoidal bottom the wave reflection increases as the ripple number increases.

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Appendix: roots of the dispersion equations

Here we find the roots of the dispersion equations in a fluid of finite depth with porous bottom when the upper surface of the fluid is bounded by a rigid lid.

The dispersion equation given by (9) is

$$(1-\rho)k - K(\coth kh + \rho \coth kh') - \frac{KG \operatorname{csch}^2 kh}{(k - G\coth kh)} = 0 \quad \text{(A1)}$$

where $K = \dfrac{\omega^2}{g}$.

The plot (Fig. A1) of the functions $Kh(\coth kh + \rho\coth kh') - \dfrac{Kh \cdot Gh \operatorname{csch}^2 kh}{(kh - Gh\coth kh)}$ and $(1-\rho)kh$ intersect exactly at one point for $Kh > 0$ for $Kh = 0.2$, $Gh = 0.05$, $\mu = \dfrac{h'}{h} = 1$, $\rho = 0.05$. Similarly when we plot for $\mu <\text{or}> 1$, the result will be the same. Since each of these functions is odd in kh, there are always exactly two real roots occurring as plus and minus of some positive quantity which we denote as m.

When k is purely imaginary, $k = \mathrm{i}\kappa$ for some real κ, the Eq. (A1) becomes

$$(1-\rho)\kappa + K(\cot\kappa h + \rho\cot\kappa h') - \frac{KG \csc^2 \kappa h}{(\kappa + G\cot\kappa h)} = 0 \quad \text{(A2)}$$

Thus the purely imaginary roots of the Eq. (A2) are obtained from the plots (Fig. A2) of $-Kh(\cot\kappa h + \rho\cot\kappa h') - \dfrac{Kh \cdot Gh \csc^2 \kappa h}{(\kappa h + Gh\cot\kappa h)}$ and $(1-\rho)\kappa h$ against κh.

It is obvious that there exists an infinite no of purely imaginary roots of the Eq. (A2) given by $\pm\mathrm{i}\kappa_n$, n=1,2,3…(say).

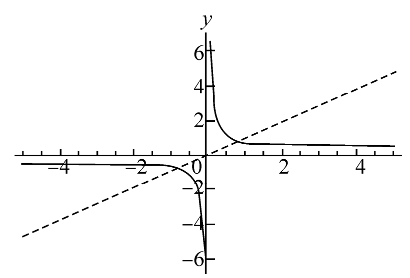

Fig. A1 $y = Kh(\coth kh + \rho\coth kh') - \dfrac{Kh \cdot Gh\operatorname{csch}^2 kh}{(kh - Gh\coth kh)}$, $y = (1-\rho)kh$

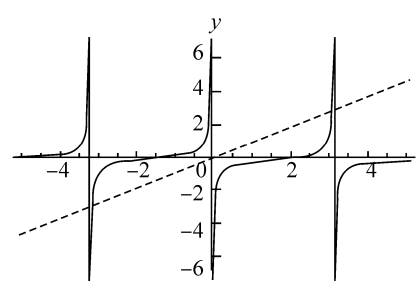

Fig. A2 $y = -Kh(\cot\kappa h + \rho\cot\kappa h') - \dfrac{Kh \cdot Gh \csc^2 \kappa h}{(\kappa h + Gh\cot\kappa h)}$, $y = (1-\rho)\kappa h$

By Rauche's theorem of complex variable theory, we can show that the dispersion Eq. (A1) has two real roots, infinite no of purely imaginary roots and there is no other roots (Das and Mandal, 2005).