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Fully Nonlinear Simulation for Fluid/Structure Impact: A Review

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Abstract: This paper presents a review of the work on fluid/structure impact based on inviscid and imcompressible liquid and irrotational flow. The focus is on the velocity potential theory together with boundary element method (BEM). Fully nonlinear boundary conditions are imposed on the unknown free surface and the wetted surface of the moving body. The review includes (1) vertical and oblique water entry of a body at constant or a prescribed varying speed, as well as free fall motion, (2) liquid droplets or column impact as well as wave impact on a body, (3) similarity solution of an expanding body. It covers two dimensional (2D), axisymmetric and three dimensional (3D) cases. Key techniques used in the numerical simulation are outlined, including mesh generation on the multivalued free surface, the stretched coordinate system for expanding domain, the auxiliary function method for decoupling the mutual dependence of the pressure and the body motion, and treatment for the jet or the thin liquid film developed during impact.

Keywords: fluid/structure impact; boundary element method; 3D surface mesh generation; water entry; wave impact; similarity solution; fully nonlinear simulation

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1 Introduction

Fluid/structure impact occurs when fluid and solid approach each other with high relative speed. It has a wide range of practical applications. The physical process usually involves air cushion effect (Smith *et al*., 2003), trapped air cavity or bubbles (Hattori *et al*., 1994; Kiger and Duncan, 2012), cavitation inception (Arndt, 2002), high speed jet or thin liquid film (Wu *et al*., 2004), extremely large impulsive pressure and acceleration (Peregrine, 2003), structural deformation (Lu *et al*., 2000; Korobkin *et al*. 2006), liquid compressibility (Lesser and Field, 1983; Korobkin and Pukhnachov, 1988; Korobkin *et al*., 2008), *etc*. Many of these physical parameters change rapidly in the space and with time. This makes experimental and numerical studies of this problem extremely challenging. The present work does not intend to cover all these aspects. The air cushion effect will not be considered based on the assumption that the impact starts with a small contact area and air can escape before collision. Air trapping will be ignored, assuming the liquid and solid surfaces will not form a closed volume at the moment of impact. The relative speed of impact may be large, but it will still be much smaller than the speed of the sound in the liquid, and therefore its compressibility can be ignored. The viscosity may also be neglected if the period of impact is short as its effect takes time to develop (Batchelor, 1967). Therefore we will focus on the inviscid and incompressible liquid with irrotational flow. However, we shall follow the deformation of the liquid surface with the jet and thin liquid film, on which the fully nonlinear boundary conditions will be imposed.

Mathematically, the governing equation for the irrotational flow of an ideal and incompressible liquid is the Laplace equation. The pioneering work was started by Von Karman (1929) through a water entry problem. When a two dimensional body entered water, he used an equivalent plate with its width equal to distance between intersections of the body with the calm water surface. The analytical solution for a plate on the calm water surface was used as an approximation at each time step. When the body continues entering the water, the width of the plate changes. While Von Karman's work did not consider the water surface deformation, Wagner (1932) introduced a correction. From Von Karman's theory, the vertical velocity on the free surface could be obtained. Using that, the free surface elevation was obtained. The intersection of the body with the elevated free surface was then used as the width of the equivalent plate in the Wagner's theory. This theory has been widely used in the fluid/structure impact problem ever since. Typical work based on analytical or semi-analytical solution includes those by Armand and Cointe (1987), Howison *et al*. (1991), Scolan and Korobkin (2001, 2012), Korobkin and Scolan (2006) and Moore *et al.* (2012).

In addition to the Wagner's approximation, there has been extensive work using the analytical method for the fully nonlinear problem. Cumberbatch (1960) obtained the mathematical solution for vertical impact of a symmetrical liquid wedge on a horizontal flat surface. Dobrovol'skaya (1969) converted the problem into an integro-differential

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equation for $f(t)$ between $0 \le t \le 1$. Shu (2004) solved the oblique impact of a water wedge on a flat wall at initial stage by using the Taylor expansion in terms of time, similar to that used by Korokin and Wu (2000) for the impact caused by the impulsive motion of a floating semi-circular cylinder. Semenov and Iafrati (2006) used the integral hodograph method for vertical entry of an asymmetric wedge into calm water. The same method was used by Semenov and Wu (2013) for the steady flow problem of a body gliding along the free surface and by Semenov *et al*. (2013) for the problem of collision between two liquid wedges. Christodoulides and Dias (2009) used conformal mapping method for the steady flow problem of a rising stream hitting a plate of finite extent.

Here we shall give an overview for the extensive work based on the boundary element method (BEM). This method has particular advantage for this kind of impact problem. Although physical parameters change rapidly during impact, the affected area is usually confined compared with body/water wave interaction problem which is usually over a much larger domain. In such a case, the BEM is usually computationally more efficient compared with the volume mesh based method, such as the finite element method (Wu and Eatock Taylor, 1995; Wu and Eatock Taylor, 2003). The jet or liquid film also makes the mesh generation in the latter method more difficult. In the section below, we will give an overview for the following fluid/structure impact problems (1) water entry, (2) water droplets and water block, wave impact on a wall and (3) similarity solution for water entry of an expanding body with curvature.

2 The fully nonlinear boundary element method

The fluid is assumed to be inviscid and incompressible, and flow to be irrotational. Thus a velocity potential ϕ with its gradient equal to the velocity can be introduced and it satisfies Laplace equation

$$\nabla^2\phi=0 \tag{1}$$

From Green's identity, this equation can be transformed into an integral form

$$A(p)\phi(p)=\int\left[G\frac{\partial\phi(q)}{\partial n_q}-\phi(q)\frac{\partial G}{\partial n_q}\right]\mathrm{d}s_q \tag{2}$$

where integration is performed with respect to point q over the boundary of the fluid domain, $A(p)$ is the solid angle at point p and G is the Green function. For 2D problems $G=\ln R_{pq}$, and for 3D problems $G=-1/R_{pq}$ where $R_{pq}=\left|\boldsymbol{r}_q-\boldsymbol{r}_q\right|$ is the distance between the field point p and source point q, $\boldsymbol{r}=x\boldsymbol{i}+y\boldsymbol{j}+z\boldsymbol{k}$ denotes position vector from the origin of the Cartesian system $O-xyz$, and $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ are unit vectors in the x, y, z directions respectively. In the 2D case, Eq. (2) can also be written based on the complex potential through the Cauchy theorem, and thus the BEM can be written in a different form involving the velocity potential and the stream function, rather than the potential and its normal derivative.

The potential satisfies the impermeable boundary condition on the wetted body surface

$$\frac{\partial\phi}{\partial n}=\boldsymbol{v}\cdot\boldsymbol{n}+\boldsymbol{\Omega}\times\boldsymbol{r}\cdot\boldsymbol{n} \tag{3}$$

where $\boldsymbol{v}=U\boldsymbol{i}+V\boldsymbol{j}+W\boldsymbol{k}$ and $\boldsymbol{\Omega}=\Omega_x\boldsymbol{i}+\Omega_y\boldsymbol{j}+\Omega_z\boldsymbol{k}$ are respectively the translational and rotational velocities of the body and $\boldsymbol{n}$ is the normal pointing out of the fluid domain. In the Lagrangian framework, the kinematic and dynamic boundary conditions on the free surface can be respectively written as

$$\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{\partial\phi}{\partial x},\quad \frac{\mathrm{d}y}{\mathrm{d}t}=\frac{\partial\phi}{\partial y},\quad \frac{\mathrm{d}z}{\mathrm{d}t}=\frac{\partial\phi}{\partial z} \tag{4}$$

$$\frac{\mathrm{d}\phi}{\mathrm{d}t}=\frac{1}{2}\nabla\phi\cdot\nabla\phi \tag{5}$$

where the effect of gravity has been ignored in Eq. (5) on the basis that the impact time is much smaller than the ratio of the relative impact speed and the acceleration due to gravity. Eqs. (4) and (5) can also be written based on the Eulerian framework through wave elevation ζ, or

$$\frac{\partial\zeta}{\partial t}=\phi_z-\phi_x\varsigma_x-\phi_y\varsigma_y \tag{6}$$

$$\phi_t+\frac{1}{2}\nabla\phi\cdot\nabla\phi=0 \tag{7}$$

Far away from the impact, the fluid is assumed to be undisturbed by the impact. The potential there is assumed to be zero or the incident potential due to incoming flow.

3 The stretched coordinate system

Many problems of fluid structure impact starts with a contact point. Earlier work let the contact zone be initially finite, or a small part of the body be put into the water (Lu *et al*., 2000; Zhao and Faltinsen, 1993, 1999; Battistin and Iafrati, 2003) and assumed that the water surface was flat and the potential on it was zero. It was also noticed by Wu *et al*. (2004) that this practice became problematic for the free fall motion, or other problems when the body motion/deformation was nonlinearly coupled with the flow. They proposed to solve the problem using the stretched coordinate system $\mathrm{O}-\alpha\beta\gamma$ defined as

$$\alpha=x/s,\quad \beta=y/s,\quad \gamma=z/s \tag{8}$$

Here, the length scale s can be appropriately chosen. In the water entry problem of a wedge (Wu *et al*., 2004), it was chosen as the distance that the wedge has travelled into the water. For a 2D water column or liquid droplet of curvature it was found that the local varying width of the liquid is a more rational choice for s (Wu, 2007a, 2007b). The use of the stretched coordinate system in Eq. (8) allows us to adopt the similar element size and computational domain in the space as the time step is marching forward, while in the physical system the element size is very small initially and then increases with the time step. Furthermore, the initial $s = s_0$ in Eq. (8) can be chosen as small as possible, for example 10^{-6} or smaller as in Wu *et al*. (2004). This can reduce or remove the initial transient effect on the results of interest at later stage. It fact, detailed analysis by Sun and Wu (2013a) for the case of water entry of a cone has shown that when $s / s_0 = 20 - 30$, the initial transient effect is hardly noticeable. Because of its advantages, this stretched coordinate system method has been used in the wide range of problems (Wu *et al*. 2004; Wu, 2007a, 2007b; Wu, 2006, 2012; Duan *et al*. 2009; Xu *et al*. 2008, 2010, 2011a; Xu *et al*. 2011b; Sun and Wu, 2013a, 2013b, 2014; Wu and Sun, 2014).

4 Jet and intersection point treatments

During fluid/structure impact, it is common that, a thin liquid layer moving along the body surface with very high speed will be formed. To ensure numerical accuracy, the elements on the two sides of the thin fluid layer must have the size smaller than the thickness of the layer. This will lead to an extremely large number of elements, which can easily beyond the capacity of computers. To resolve this problem, Zhao and Faltinsen (1993) and Lu *et al*. (2000) cut the thin liquid layer and used a line element to connect the body surface and the free surface. This may cause numerical fluctuation locally, but it does not affect the global result when the fluctuation is not fed back into the flow. Wu *et al*. (2004) treated the thin jet on the wedge surface by assuming that the potential across the layer was a linear function. Because of the Laplace equation, the potential is then also a linear function in the other direction. As a result the potential and velocity become known on the both sides of the thin layer and they do not have to be solved from the boundary integral equation. The advantage of this is therefore that the inclusion of the jet does not increase the number of unknowns and the CPU and memory requirement still depends on the number of elements in the main flow region. This method was refined by Wu (2007b) for more general 2D cases and was extended by Sun and Wu (2013a) for 3D problems.

5 Pressure and force calculation

The pressure on the body surface can be written based on the Bernoulli's equation

$$p = -\rho(\phi_t + \frac{1}{2}\nabla\phi \cdot \nabla\phi) \tag{9}$$

We notice that the meaning of ϕ_t is the partial temporal derivative for a point fixed in the space. This could be problematic on the body surface. The numerical calculation of ϕ_t has to take the value of ϕ from the previous time step (explicit scheme) or the next time step (implicit scheme). However due to the body motion, the point fixed in the space may not be in the fluid domain at the previous time or the next time step. To overcome that, Eq. (9) can be written in the following from

$$\begin{aligned} p = -\rho[\frac{\mathrm{d}\phi}{\mathrm{d}t} - (U - \Omega_y z + \Omega_z y)\phi_x - \\ (V - \Omega_z x + \Omega_x z)\phi_y - \\ (W - \Omega_x y + \Omega_y x)\phi_z + \frac{1}{2}\nabla\phi \cdot \nabla\phi] \end{aligned} \tag{10}$$

Here $\frac{\mathrm{d}\phi}{\mathrm{d}t}$ is taken for a point fixed on the body surface. This avoids the problem associated with ϕ_t. However for a body at the free surface, finite difference calculation of $\frac{\mathrm{d}\phi}{\mathrm{d}t}$ usually causes very sharp spikes in the pressure curve.

Alternative to calculation of ϕ_t through finite difference for $\frac{\mathrm{d}\phi}{\mathrm{d}t}$ is to treat ϕ_t as another potential problem and find it through the boundary integral equation similar to Eq. (2). The boundary condition of ϕ_t on the free surface can be obtained easily from the zero pressure condition. Its rigid body surface boundary condition can be obtained from the equation derived by Wu (1998). However, the latter involves the body acceleration which depends on the fluid force. The fluid force then depends on the pressure in Eq. (9) which further depends on the acceleration. To avoid this nonlinear mutual dependence, Wu and Eatock Taylor (2003) introduced some auxiliary functions χ_k, $k = 1,\dots 6$. They satisfy the Laplace equation, $\chi_k = 0$ on the instantaneous free surface and $\frac{\partial\chi_k}{\partial n} = n_k$, where $\boldsymbol{n} = (n_1, n_2, n_3)$ and $\boldsymbol{r} \times \boldsymbol{n} = (n_4, n_5, n_6)$. Through the use of these auxiliary functions, the body acceleration can be obtained without the knowledge of the pressure distribution. This method was used by Wu *et al*. (2004) and Xu *et al*. (2010, 2011a). When the body has the constant velocity or the known acceleration, ϕ_t can be solved directly without the need of the auxiliary function (Wu, 2007a, 2007b; Sun and Wu, 2013a, 2013b; Wu, 2006, 2012; Duan *et al*. 2009; Xu *et al*. 2008).

6 Case studies and discussions

6.1 Water entry

A classic fully nonlinear solution for water entry of a wedge at constant speed was obtained by Dobrovol'skaya (1969) using the complex potential to convert problem into an integral equation along a straight line. This integral equation was resolved by Zhao and Faltinsen (1993) with a much higher degree of accuracy. In the same paper, they solved the problem using the BEM described in Eq. (2). They provided details on how the free surface was updated in the time marching method and how the BEM elements were regularly generated, which laid a good foundation for much of the work followed. They also carefully cut the jet and showed that although that caused some local fluctuation of the pressure, this was very much confined to a small area where the jet was cut. Battistin and Iafrati (2003) considered water entry of a 2D cylinder and an axisymmetric body of curvature. Jet cutting was also applied. The water entry of two wedges at constant speed was solved by Wu (2006). At initial stage, the ratio of the distance between the two wedges to the distance which the body has travelled into the water is virtually infinite. Thus two wedges can be considered as fully independent. The initial solution for each wedge can be obtained from the self similar flow. As time progresses, the interaction between the two wedges becomes important. The flow is no longer self similar and the problem of the combined two wedges has to be solved in the time domain. Xu *et al*. (2008) considered the problem of oblique entry of an asymmetric wedge at constant speed. When the horizontal velocity is zero, or the body enters water vertically, they compared their results with those obtained by Semenov and Iafrati (2006) using the integral hodograph method and very good agreement was found. While all the work above are on 2D problems, Sun and Wu (2013a) solved a 3D problem of oblique water entry of a cone. One of the main challenges to BEM in this kind of problem is regular generation of 3D meshes in the time domain. This becomes particularly complex when the free surface elevation is non single valued, or a vertical line will intersect the free surface more than once. To resolve that, Sun and Wu (2013a) first generated line elements in each given azimuth of the cylindrical coordinate system. The line element nodes were then linked in the circumferential direction to form surface elements. The free surface was updated in a modified Eulerian method. In particular, instead of in the vertical direction, the free surface elevation was updated in the direction parallel to the body surface in each given azimuth. This worked well apart from in the liquid film attached the body, where the slope of the free surface elevation in the direction parallel to the local body surface becomes extremely large. Sun and Wu (2013b) overcome the difficulty by replacing this direction with the normal direction of the free surface or the direction of the relative velocity between the local flow and the body surface. This has laid a good foundation for successful simulations for 3D problems.

For non-constant speed, Zhao *et al*. (1997) solved water entry problem of arbitrary two-dimensional sections with prescribed speed variation. Wu *et al*. (2004) undertook the numerical simulation and experimental study of vertical water entry of a wedge in free fall motion. As the body speed is no longer prescribed and has to be updated from its acceleration obtained from the hydrodynamic force, accuracy of the solution at each time step becomes more important. The pressure fluctuation due to jet cutting could be fed back into the fluid flow through the body motion. Great care was therefore taken by Wu *et al*. (2004). When the wedge touched the water surface, they used the self similar flow as the initial solution to allow a smooth start with the pressure distribution and the acceleration. The nonlinear mutual dependence of the body motion and the fluid flow was decoupled by the use of the auxiliary function described in Section 4. They also performed the model test and found the numerical results and experimental data were in good agreement. This work was extended by Xu *et al*. (2010). In addition to the vertical motion, they included the horizontal and rotational motions when the wedge was in free fall. Despite the fact that the similarity solution was no longer possible due to rotation, they found that use of the self similar flow with equivalent speed as the initial solution could greatly improve the accuracy of the solution at the later stage. Xu *et al*. (2011a) further consider an axisymmetric problem of vertical water entry of a cone in free fall motion. The auxiliary function method was once again used. The calculated results were found in good agreement with the experimental data by Baldwin (1971).

It was known that solution for water entry of a wedge at constant speed would be self similar, which led to the pioneering work of Dobrovol'skaya (1969). Wu (2012) demonstrated that self similar solution would be also possible for a wedge with a varying speed, proved its travelled distance s into water and time t follow the relationship $s = Dt^{\lambda}$ where D and λ are constants. When $\lambda = 1$, this becomes the case of constant speed. When $\lambda \neq 1$, Wu (2012) showed that the line linking the origin on the undisturbed water surface and the interaction of the wedge surface and the free surface is perpendicular to the free surface. Sun and Wu (2013b) extended this to 3D. They showed that even when the body was not axisymmetric, self similar solution would still possible under the same condition of $s = Dt^{\lambda}$, provided the plan cutting through the body axis at each azimuth formed a triangle.

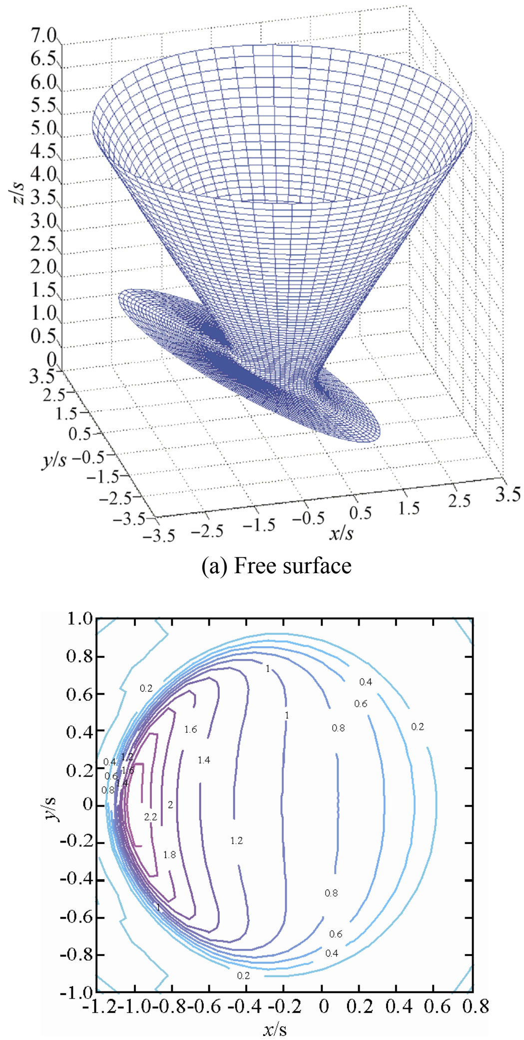
For non-rigid bodies, Lu *et al*. (2000) conducted the coupled hydrodynamic and structural analysis for water entry of an elastic wedge at constant speed. The nonlinear velocity product term in the Bernoulli equation was calculated using the velocity at the current time step (unknown to be found) and the velocity at the previous step (known). This allows the differential equation for the body deformation to be solved based on Newmark integral method (Bathe and Wilson, 1976). Lu *et al*. (2000) also used velocity at current step for the product term in the Bernoulli equation and used iteration to obtain the body deformation. They found that the results from the two methods were in good agreement.

6.2 Water droplets and water block, wave impact on a wall

We focus on our discussions in the context of fluid/structure impact. More general discussions on droplets and bubbles can be found in Yarin (2006) and Thoroddsen *et al*. (2008). Discussion on wave impact in a wider context can be found in Peregrine (2003).

A typical earlier work on wave impact on a wall was that by Cooker and Peregrine (1991). One very interesting thing which they noted is flip through. In such a case, the incoming wave will not hit the wall directly. Instead the free surface at the wall will rise rapidly in the form of a jet. The local acceleration could be thousands of times larger than the acceleration due to gravity. Zhang *et al*. (1996) obtained the self-similar solution of a water wedge impact on a wall and used this solution as the initial solution for plunging wave impact. They approximated the free surface shape using the exponential function. Duan *et al*. (2009) solved the problem of a water wedge impact on a wall without such an approximation and obtained far more accurate solution, which was confirmed by Semenov and Wu (2012) using the integral hodograph method. In the same paper, Duan *et al*. (2009) considered the oblique impact of a liquid wedge on a solid wedge, extended from the work of Wu (2007b) on perpendicular impact. In 3D, Sun and Wu (2014) considered the oblique impact of a water cone on a wall. The 3D mesh was generated and then regularly regenerated following the deformation of the free surface, based on the technique developed in Sun and Wu (2013a, 2013b). A snapshot of the mesh of quadrilateral elements on the instantaneous free surface is shown in Fig.1(a) together with the obtained pressure distribution on the wall shown in Fig.1(b), taken from Sun and Wu (2014). In the figure γ_1 and γ_2 are the angles from the cone surface and the wall surface to the plane perpendicular to the axis of the water cone.

The surface of a liquid wedge or a cone has no curvature (in a given azimuth for the case of cone). Wu (2007a) considered a case of a 2D water column described by $y = f(x)$ hitting on a wall with constant speed *W*. He found that if $s = Wt$ was used, the initial wetted surface tended to infinity, which became problematic in numerical simulation. Thus it is crucial to choose s properly. In that case, when $s = f(Wt)$, the above difficulty could be resolved. In the same paper, it was shown when a liquid column with $f = a^{1-\lambda}x^{\lambda}$ hitting on a wall with speed *W*, where $0 < \lambda < 1$, the pressure distribution depends only on Wt/a and it does not depend on *W* and a individually, which is consistent with the PI theorem. Wu (2007b) further considered liquid column hitting on a solid wedge. In the same paper, he also considered a liquid droplet hitting on the solid wedge. The simulation started from the solid wedge cutting into the droplet on one side in the stretched coordinate system. The simulation returned to the physical domain when the width of droplet began to decrease. A numerical scheme was introduced to allow the wedge edge to come out from the other side of the droplet, or to allow the wedge to bisect the droplet. The single droplet was then split into two parts sliding on each side of the solid wedge. In the paper, it was also found that when a 2D body has no curvature, the normal derivative of the pressure on the body surface is zero. Xu *et al*. (2011b) solved the problem an axisymmetric problem of a liquid block hitting on a solid cone. The simulation, however, was not sufficiently long to let the tip of the cone to pierce through the other side of the liquid block. Water droplet impact is important in a much wider range of problems. Further details can be found in Yarin (2006).



(a) Free surface

(b) Pressure distribution

Fig. 1 Oblique impact of a water cone on an inclined wall, the deadrise angle of the water column is $\gamma_1 = \pi/3$ and the inclined angle of the wall $\gamma_2 = \pi/12$. (Sun and Wu, 2014)

6.3 Similarity solution for water entry of an expanding body with curvature

In general, the spatial and temporal dependence of the fluid/structure impact problem is fully separate. However there are many cases in which the spatial and temporal variables can be combined to form new variables and the flows become self similar in these cases. Importantly even when the similarity solution is for special cases, it can play some crucial role in fluid/structure impact problems. It can help to resolve the local singularity in the solution (Zeff *et al*., 2000; Iafrati and Korobkin, 2004). The use of the self-similar solution as the initial solution for general transient problem can greatly improve the numerical results at later stages (Wu *et al*., 2004; Xu *et al*., 2010, 2011a). In some cases, the limit of a similarity solution tends to a steady solution (Semenov and Wu, 2012).

As discussed by Wu and Sun (2014), among many others, well known examples of similarity solutions include those by Cumberbatch (1960) for liquid wedge impacting on a flat wall, by Dobrovol'skaya (1969) and Zhao and Faltinsen (1993) for a solid wedge entering a calm water surface, by Semenov and Iafrati (2006) and Xu *et al*. (2008) for an asymmetric wedge entering water vertically and obliquely, respectively, by Wu (2007b) and Duan *et al*. (2009) for impact of a liquid wedge and solid wedge. Similarity solution can also be found in impact of two liquid wedges of same density (Semenov *et al*. 2013) and impact of two liquid wedges of different densities (Semenov *et al*., 2014).

It may be noticed that the self-similar solution in water entry of a rigid body was obtained only for a wedge in 2D or a cone in 3D. This is expected as the body continues moving into the water, the shape below the water surface must be geometrically similar even though its size increases. Thus it seems to rule out a body with curvature in the vertical plane. Wu and Sun (2014) realized that this might be true when a body was rigid. However if the body was allowed to deform during water entry to maintain the geometric similarity, self similar solution might be still possible. They used the paraboloid as an example and discovered when the body expanded in a particularly prescribed way, the self similar flow was indeed possible. As discussed by Wu and Sun (2014), such consideration is not purely for mathematical purpose and it has important physical significance and practical applications. Its importance could be partly reflected by the role of the Dirichlet's ellipsoid (Lamb, 1932) in the free surface flow problem. Similar examples could also be found in the work of Longuet-Higgins (1976) on a family of self similar problem for an expanding Dirichlet ellipse, hyperbola and other shapes. These mathematical solutions were later found to give significant insight into wave breaking (Longuet-Higgins and Cokelet, 1976; Longuet-Higgins, 1980, 1983a) and bubble busting through the free surface (Longuet-Higgins, 1983b). Wu and Sun (2014) commented that a potential application of the water entry of an expanding body could be in the 2D+*t* theory for a ship (Faltinsen *et al*., 1991). When calculation starts from a 2D section at the bow and then continues along the ship length, it is equivalent to the problem that the body is expanding (Tassin *et al*., 2013).

While similarity solution can be possible for a variety of bodies with varying shape, a specific example considered by Wu and Sun (2014) is the following paraboloid

$$f(x,y,z,t) = x^2 + y^2 - \lambda s \cdot (z+s) = 0 \qquad (11)$$

where $s=Wt$ and λ is a constant. Thus the body enters water with speed W and the radius of its horizontal cross section increases at the rate of $s^{1/2}$. In this case, the problem will no longer depend on the time explicitly in the stretched coordinate system. However, the free surface boundary conditions are fully nonlinear and its shape is unknown. Wu and Sun (2014) converted the differential equations for the free surface boundary conditions into integral equations along a line at each given azimuth plane, similar to that used in Wu *et al*. (2004). These conditions were then satisfied through iterations.

7 Concluding remarks and further research

Fluid/structure impact is of vital importance in many engineering applications. It is also one of the most challenging problems in experiment and numerical simulations. Tremendous progress in both solution techniques and understanding of the physical nature has been achieved since the time of Von Karman (1929), as the present review has shown. However, the success in many aspects is still limited. Further research is required in a wide range of problems.

(1) Kutta condition at shape edge.

During the oblique entry of a wedge (Xu *et al*., 2008) or a cone (Sun and Wu, 2013a), pressure is found to be discontinuous or even singular. This is of course not unique for the impact problem. A well known example of this is a hydrofoil. An effective solution for the foil within the framework of the potential theory is to use Kutta condition. Through allowing the vortices being shed from the trailing edge, the pressure there becomes continuous. Numerical implementation of the Kutta condition has been made for a foil with large amplitude motion and moving vortex sheet shed from the trailing edge (Xu and Wu, 2013). This scheme can be incorporated into the water entry of a wedge.

(2) Water entry into waves with gravity effect.

When a body enters a wave instead of calm water, the fluid motion due to the incident wave changes the relative impact velocity between body and fluid. The slope of the wave will also change the effective deadrise angle of the body. The total potential will involve both incident potential and the disturbed potential caused by the body. The method based on the stretched coordinate system described in Section 2 will have to be modified.

(3) Impact with compressible liquid.

In some cases, even though the speed of the body is much smaller than the speed of the sound, the magnitude of the local fluid velocity can be much larger. Also when a liquid contains bubbles, the speed of sound can be significantly reduced. In such cases, the compressibility of the liquid is no longer negligible. The governing equation is no longer Laplace equation but wave equation. When the latter is converted into an integral equation, it involves not only the integration over the boundary but also an integration over the time (Zhang *et al*., 2013), which is in fact the delayed effect as a disturbance travels at the speed of sound. The solution procedure for such an integral equation becomes much more complicated.

Other effects in more general cases to be considered in further research include (1) large structural deformation, (2) trapped air and bubble, (3) cavitation inception, (4) air cushion, (5) flow detachment from the body (6) viscousity, (7) surface tension, *etc*.

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References

Armand JL, Cointe R (1987). Hydrodynamic impact analysis of a cylinder. *Journal of Offshore Mechanics and Arctic Engineering*, **109**(3), 237-243.

Arndt EA (2002). Cavitation in vortical flows. *Annual Review of Fluid Mechanics*, **34**, 143-175.

Baldwin JL (1971). *Vertical water entry of cones*. Naval Ordnance Laboratory, White OAK, Silver Spring, Maryland, USA.

Batchelor GK (1967). *An introduction to fluid dynamics*. Cambridge University Press, Cambridge, UK, 471.

Bathe KJ, Wilson EL (1976). *Numerical Methods in Finite Element Analysis*. Prentice-Hall, Englewood Cliffs, NJ, USA.

Battistin D, Iafrati A (2003). Hydrodynamic loads during water entry of two-dimensional and axisymmetric bodies. *Journal of Fluids and Structures*, **17**(5), 643-664.

Christodoulides P, Dias F (2009). Impact of a rising stream on a horizontal plate of finite extent. *Journal of Fluid Mechanics*, **621**, 243-258.

Cooker MJ, Peregrine DH (1991). Violent motion as near breaking waves meet a vertical wall. *International Union of Theoretical and Applied Mechanics*, Sydney, Australia, 291-297.

Cumberbatch E (1960). The impact of a water wedge on a wall. *Journal of Fluid Mechanics*, **7**(3), 353-374.

Dobrovol'Skaya ZN (1969). On some problems of similarity flow of fluid with a free surface. *Journal of Fluid Mechanics*, **36**(4), 805–829.

Duan WY, Xu GD, Wu GX (2009). Similarity solution of oblique impact of wedge-shaped water column on wedged coastal structures. *Coastal Engineering*, **56**(4), 400-407.

Faltinsen O, Zhao R, Umeda N(1991). Numerical predictions of ship motions at high forward speed. *Philosophical Transactions of the Royal Society A*, **334**(1634), 241-252.

Hattori M, Arami A, Yui T (1994). Wave impact pressure on vertical walls under breaking waves of various types. *Coastal Engineering*, **22**(1-2), 79-114.

Howison SD, Ockendon JR, Wilson SK (1991). Incompressible water-entry problems at small deadrise angles. *Journal of Fluid Mechanics*, **222**, 215–230.

Iafrati A. and Korobkin AA (2004). Initial stage of flat plate impact onto liquid free surface. *Physics of Fluids*, **16**(7), 2214-2227.

Kiger KT, Duncan JH (2012) Air-entrainment mechanisms in plunging jets and breaking waves. *Annual Review of Fluid Mechanics*, **44**, 563-596.

Korobkin AA, Gueret R, Malenica S (2006). Hydroelastic coupling of beam finite element model with Wagner theory of water impact. *Journal Fluids and Structures*, **22**(4), 493-504.

Korobkin AA., Khabakhpasheva TI, WU GX (2008). Coupled hydrodynamic and structural analysis of compressible jet impact onto elastic panels. *Journal of Fluids and Structures*, **24**(7), 1021-1041.

Korobkin AA, Pukhnachov VV (1988). Initial stage of water impact. *Annual Review of Fluid Mechanics*, **20**, 159-185.

Korobkin AA, Scolan YM (2006). Three-dimensional theory of water impact. Part 2. linearized Wagner problem. *Journal of Fluid Mechanics*, **549**, 343-373.

Korobkin AA, Wu GX (2000). Impact on a floating circular cylinder. *Proceedings of The Royal Society A*, **456**(2002), 2489-2514.

Lesser MB, Field JE (1983). The impact of compressible liquids. *Annual Review of Fluid Mechanics*, **15**, 97-122.

Lamb H (1932). *Hydrodynamics,* 6th edition. Cambridge University Press, Cambridge, UK, 382.

Longuet-Higgins MS (1976). Self-similar, time-dependent flows with a free surface. *Journal of Fluid Mechanics*, **73**(4), 603-620.

Longuet-Higgins MS (1980). On the forming of sharp corners at a free surface. *Proceedings of the Royal Society A*, **371**(1747), 453-478.

Longuet-Higgins MS (1983a). Rotating hyperbolic flow : particle trajectories and parametric representation. *Quarterly Journal of Mechanics and Applied Mathematics*, **36**(2), 247-270.

Longuet-Higgins MS (1983b). Bubbles, breaking waves and hyperbolic jets at a free surface. *Journal of Fluid Mechanics*, **127**, 103-121.

Longuet-Higgins MS, Cokelet ED (1976). The deformation of steep surface waves on water. I. A numerical method of computation. *Proceedings of the Royal Society A*, **350**(1660), 1-26.

Lu CH, He YS, Wu GX (2000). Coupled analysis of nonlinear interaction between fluid and structure during impact. *Journal of Fluids and Structures*, **14**(1), 127-146.

Moore MR, Howison SD, Ockendon JR, Oliver JM (2012). Three-dimensional oblique water-entry problems at small deadrise angle. *Journal of Fluid Mechanics*, **711**, 259-280.

Peregrine DH (2003). Water-wave impact on walls. *Annual Review of Fluid Mechanics*, **35**, 23-43.

Scolan YM, Korobkin AA (2001). Three-dimensional theory of water impact, Part 1: Inverse Wagner problem. *Journal of Fluid Mechanics*, **440**, 293-326.

Scolan YM, Korobkin AA (2012). Hydrodynamic impact (Wagner) problem and Galin's theorem. *27th International Workshop on Water Waves and Floating Bodies*, Copenhagen, Denmark, 22-25.

Semenov YA, Iafrati A (2006). On the nonlinear water entry problem of asymmetric wedges. *Journal of Fluid Mechanics*, **547**, 231-256.

Semenov YA, Wu GX (2012). Asymmetric impact between liquid and solid wedges. *Proceedings of the Royal Society A*, **469**(2150), 1-20.

Semenov YA, Wu GX (2013). The nonlinear problem of a gliding body with gravity. *Journal of Fluid Mechanics*, **727**, 132-160.

Semenov YA, Wu GX, Korobkin AA (2014). Impact of liquids of different densities. *The 29th IWWWFB*, Osaka, Japan, 1-4.

Semenov YA, Wu GX, Oliver JM (2013). Splash jet generated by collision of two liquid wedges. *Journal of Fluid Mechanics*, **737**, 132-145.

Shu JJ (2004). Impact of an oblique breaking wave on a wall. *Physics of Fluids*, **16**(3), 610-614.

Smith FT, Li T, Wu GX (2003). Air cushioning with a lubricating/inviscid balance. *Journal of Fluid Mechanics*, **482**, 291-318.

Sun SL, Wu GX (2013a). Oblique water entry of a cone by a fully three dimensional nonlinear method. *Journal of Fluids and Structures*, **42**, 313-332.

Sun SL, Wu GX (2013b). Oblique water entry of non-axisymmetric bodies at varying speed by a fully nonlinear method. *Quarterly Journal of Mechanics and Applied Mathematics*, **66**(3), 366-393.

Sun SL,Wu GX (2014). Oblique impact of a water cone on a solid wall. *European Journal of Mechanics B-Fluid*, **43**, 120-130.

Tassin A, Piro DJ, Korobkin AA, Maki KJ, Cooker MJ (2013). Two-dimensional water entry and exit of a body whose shape varies in time. *Journal of Fluids and Structures*, **40**, 317-336.

Thoroddsen ST, Etoh TG, Takehara K (2008). High-speed imaging of drops and bubbles. *Annual Review of Fluid Mechanics*, **40**, 257-285.

Von Karman T (1929). The impact of seaplane floats during landing. Technical Report, 321, NACA.

Wagner H (1932). The phenomena of impact and planning on water. Technical Report, 1366, NACA.

Wu GX (1998). Hydrodynamic force on a rigid body during impact with liquid. *Journal of Fluids and Structures*, **12**(5), 549-559.

Wu GX (2006). Numerical simulation of water entry of twin wedges. *Journal of Fluids and Structures*, **22**(1), 99-108.

Wu GX (2007a). Fluid impact on a solid boundary. *Journal of Fluids and Structures*, **23**(5), 755-765.

Wu GX (2007b). Two dimensional liquid column and liquid droplet impact on a solid wedge. *Quarterly Journal of Mechanics and Applied Mathematics*, **60**(4), 497-511.

Wu GX (2012). Numerical simulation for water entry of a wedge at varying speed by a high order boundary element method. *Journal of Marine Science and Application*, **11**(2), 143-149.

Wu GX, Eatock Taylor R (1995). Time stepping solutions of the two dimensional non-linear wave radiation problem. *Ocean Engineering*, **22**(8), 785-798.

Wu GX, Eatock Taylor R (2003). The coupled finite element and boundary element analysis of nonlinear interactions between waves and bodies. *Ocean Engineering*, **30**(3), 387-400.

Wu GX, Sun H, He YS (2004). Numerical simulation and experimental study of water entry of a wedge in free fall motion. *Journal of Fluids and Structures*, **19**(3), 277-289.

Wu GX, Sun SL (2014). Similarity solution for oblique water entry of an expanding paraboloid. *Journal of Fluid Mechanics*, **745**, 398-408.

Xu GD, Duan WY, Wu GX (2008). Numerical simulation of oblique water entry of an asymmetrical wedge. *Ocean Engineering*, **35**(16), 1597-1603.

Xu GD, Duan WY, Wu GX (2010). Simulation of water entry of a wedge through free fall in three degrees of freedom. *Proceedings of the Royal Society A*, **466**, 2219-2239.

Xu GD, Duan WY, Wu GX (2011a). Numerical simulation of water entry of a cone in free fall motion. *Quarterly Journal of Mechanics and Applied Mathematics*, **64**(3), 265-285.

Xu GD, Wu GX, Duan WY (2011b). Axisymmetric liquid block impact on a solid surface. *Applied Ocean Research*, **33**(4), 366-374.

Yarin AL (2006). Drop impact dynamics, splashing, spreading, receding, bouncing. *Annual Review of Fluid Mechanics*, **38**, 159 -192.

Zeff BW, Kleber B, Fineberg J, Lathrop DP (2000). Singularity dynamics in curvature collapse and jet eruption on a fluid surface. *Nature*, **403**, 401-404.

Zhang AM, Wang SP, Wu GX (2013). Simulation of bubble motion in a compressible liquid based on the three dimensional wave equation. *Engineering Analysis with Boundary Elements*, **37**(9), 1179-1188.

Zhang S, Yue DKP, Tanizawa K (1996). Simulation of plunging wave impact on a vertical wall. *Journal of Fluid Mechanics*, **327**, 221-254.

Zhao R, Faltinsen O (1993). Water entry of two-dimensional bodies. *Journal of Fluid Mechanics*, **246**, 593–612.

Zhao R, Faltinsen O (1999). Water entry of arbitrary axisymmetric bodies with and without flow separation. *Twenty-Second Symposium on Naval Hydrodynamics*, Washington D. C., USA, 652-664.

Zhao R, Faltinsen O, Aarsnes J (1997). Water entry of arbitrary two dimensional sections with and without flow separation. *Twenty-First Symposium on Naval Hydrodynamics*, Trondheim, Norway, 408-425.

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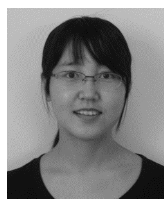

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