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3D Numerical Modeling of Wave Forces on Tandem Fixed Cylinders Using the BEM

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Abstract: In this paper a 3D numerical model was developed to study the complicated interaction between waves and a set of tandem fixed cylinders. The fluid was considered to be inviscid and irrotational. Therefore, the Helmholtz equation was used as a governing equation. The boundary element method (BEM) was adopted to discretize the relevant equations. Open boundaries were used in far fields of the study domain. Linear waves were generated and propagated towards tandem fixed cylinders to estimate the forces applied on them. Special attention was paid to consideration of the effect on varying non-dimensional cylinder radius and distance between cylinders, *ka* and *kd* on forces and trapped modes. The middle cylinder wave forces and trapped modes in a set of nine tandem cylinders were validated utilizing analytical data. The comparisons confirm the accuracy of the model. The results of the inline wave force estimation on *n* tandem cylinders show that the critical cylinder in the row is the middle one for odd numbers of cylinders. Furthermore the results show that the critical trapped mode effect occurs for normalized cylinder radiuses close to 0.5 and 1.0. Finally the force estimation for *n* tandem cylinders confirms that force amplitude of the middle cylinder versus normalized separation distance fluctuates about that of a single cylinder.

Keywords: tandem cylinders; boundary element method (BEM); wave force; diffraction; trapped mode; Henkel function

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1 Introduction

Estimation of wave forces on large structures, such as floating airports, fixed and floating bridges, artificial islands and offshore wind turbines is a very important subject for marine engineers. Two main ideas were applied for solving this type of problem:

1) A floating plane which is directly encountered with free surface.

2) A plane which is supported by a series of piercing free surface floating or fixed cylinders.

The second form needs special considerations in hydrodynamics analyzing for wave-structure interaction associated with multi-scattering effects of wave from cylinders. Therefore, the trapped mode problem and resonance phenomenon in special frequencies (trapped frequencies) possibly will occur. This fluid-structure behavior depends on the number of repeated tandem units, the distance between them and the frequency bound of encounter waves. The main purpose of this article is to study the diffracted wave force due to fluid interaction with tandem cylinders, especially in resonance mode of trapped frequency. In general, wave-structure interaction is a 3D phenomenon leading to a full nonlinear problem. If the body dimension is large enough with respect to the wave length and amplitude, the separation effect of fluid due to viscosity can be neglected and diffraction effect is dominated. In this study linear wave theory was used, and it was assumed that the fluid was incompressible and irrotational and surface tension was neglected. Therefore, scalar velocity potential which satisfies Laplace equation was applied.

Havelock (1940) developed an analytical solution for regular wave diffraction by single cylinder in infinite water. McCamy and Fuchs (1954) studied this phenomenon for finite water depth. When waves encounter tandem bodies, the effect of one body on encountering a wave, generates a scattered wave, which become scattered again by adjacent bodies. In order to obtain velocity potential, consideration must be given to the scattered encounter wave by each body and the multi-scattered waves by other bodies. An analytical solution of this problem was prepared for tandem piles and double hull ships. It is generally restricted to a first order force evaluation (Kagemoto and Yue, 1986).

Another possible solution to this problem was consideration of the tandem bodies as a unit body. The more the number of tandem bodies, the more computational difficulties. Twersky (1952) presented a successful iterative method for reflecting waves between cylinders. This method cannot be applied for a large number of cylinders. Another applicable method based on multi-scattering is the direct matrix method presented by Spring and Monkmever (1974). Approximating the scattering waves from each body as plan waves for other bodies, Simon (1982) presented a direct matrix solution to asymmetrical bodies with the same distance from each other. This assumption can be used only for large spacing's and is known as plan-wave or space-width approximation.

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Kim and Cao (2008) calculated the wave loads on fixed tandem cylinders located in a unit axis using panel-Galerkin method. They noticed that as the number of cylinders become large the finite and critical spacing between them become a portion of wave length, and quasi-resonance modes associated with canal trapped waves between adjacent cylinders occur. Evans and Porter (1997), Han and Ohkusu (1995), and recently Mainer and Newman (1997), studied these trapped modes. Their results showed that forces on 4, 5 and 6 tandem cylinders were increased by decreasing the spacing between cylinders. Furthermore, maximum force occurred when standing wave trapped modes happened. Kagemoto *et al*. (2002) experimentally studied the trapped mode for tandem cylinders. They mentioned that for regular waves in trapped mode frequency, the forces were substantially lower than that of linear wave theory because of dissipating effect on cylinder hull boundary layer. There are other reasons for resonance effect reduction even in an ideal fluid:

1) Irregular array of tandem cylinders (because of either spacing or diameter).

2) Ignoring the monochromatic waves.

The first case was studied by Duclos and Clement (2004). They mentioned that a little change in the cylinder's regularity (less than 0.5% in their spacing) was enough to decrease large forces in the resonance mode. The second case was studied by Walker and Taylor (2005).

Kim *et al*. (2007) used the direct boundary element method to calculate forces on single fixed cylinder and two arrays of cylinders with constant diameter. Chen *et al*. (2011a, 2011b) used null field integral equation to study spacing irregularity between cylinders.

In this research, a model was developed using the direct boundary element method to analyze the interaction between waves of encountering angle β with n tandem cylinders. The model was able to find trapped modes and resonance frequencies and relevant forces for different numbers of tandem cylinders. Finally, the effect of changing diameter and separation distance of cylinders was specified.

2 Governing equation

In this section the relevant relations and formulas were presented.

2.1 Formulation of problem

Assuming that the fluid is irrotational and incompressible, the interaction of linear waves with n bottom-fixed vertical cylinders was investigated. The geometry of this problem is displayed in Fig. 1.

The global Cartesian coordinate system (x, y, z) was defined with the origin located at the center of the geometry and on the still-water level where z axis directed vertically. The structure is subjected to a train of regular surface waves of height H, the angular frequency ω and an encounter angle β propagating to the positive x axis. n vertical circular cylinders with radius a, and separation distance D were situated in the water of uniform depth h. Therefore, $2d=D+2a$ was the center to center separation distance between cylinders. The velocity potential can be defined as:

$$\Phi(x,y,z)=\mathrm{Re}(\phi(x,y,z)\mathrm{e}^{-\mathrm{i}\omega t}) \tag{1}$$

where Re denotes the real part of complex expression.

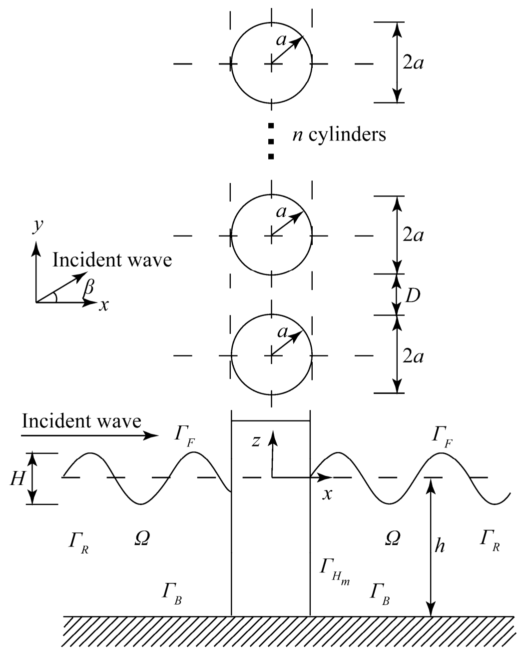

Fig. 1 Definition of *n* vertical circular cylinders

From the linear feature of potential flow, the total velocity potential as in Eq. (1) is a sum of incident and scattered (reflected plus diffracted) waves and is defined as follows:

$$\phi=\phi_I+\phi_D \tag{2}$$

$$\phi_I=-\mathrm{i}\frac{gH}{2\omega}\varphi_I \tag{3}$$

$$\phi_D=-\mathrm{i}\frac{gH}{2\omega}\varphi_D \tag{4}$$

$$\phi_I=\frac{\cosh k(h+z)}{\cosh kh}\mathrm{e}^{\mathrm{i}k(x\cos\beta+y\sin\beta)} \tag{5}$$

where ϕ_I and ϕ_D are incident and scattered wave velocity potentials, respectively. H is the wave height, g is the acceleration due to gravity, and k is the wave number which is the positive real root of the dispersion relation:

$$\omega^2=kg\tanh(kh) \tag{6}$$

Boundary value problems solved by the formulation of scattered wave velocity potential ϕ_D for Laplace equation, free surface, cylinder surface, and bed and radiation boundary conditions are given as respectively:

$$\nabla^2\phi_D=0 \quad \text{in} \quad \Omega \tag{7}$$

$$\frac{\partial\phi_D}{\partial z}-\frac{\omega^2}{g}\phi_D=0 \quad \text{on} \quad \Gamma_F \tag{8}$$

$$\frac{\partial \phi_D}{\partial n} = -\frac{\partial \phi_I}{\partial n} \quad \text{on } \varGamma_{Hm}, \quad m = 1,2,...,N \tag{9}$$

$$\frac{\partial \phi_D}{\partial z} = 0 \quad \text{on} \quad \varGamma_B \tag{10}$$

$$\lim_{R \to \infty} \sqrt{R} \left\{ \frac{\partial \phi_D}{\partial z} - \mathrm{i} k \phi_D \right\} = 0 \quad \text{on} \quad \varGamma_R \tag{11}$$

where $\varOmega$ is the fluid region, $\varGamma_F$ is the free surface, $\varGamma_{Hm}$, m= 1, 2, 3, … is the body surface of the mth cylinder, $\varGamma_B$ is the sea bed, i is the imaginary part $\mathrm{i} = \sqrt{-1}$, $\varGamma_R$ is the vertical boundary at infinity and $R = \sqrt{x^2 + y^2}$. Due to the constant cross section of the cylinders with respect to z, the incident and scattered wave velocity potentials are defined as follows:

$$\phi_D = \frac{\cosh k(h+z)}{\cosh kh} \psi_D(x,y) \tag{12}$$

$$\phi_I = \frac{\cosh k(h+z)}{\cosh kh} \psi_I(x,y) \tag{13}$$

$$\psi_I(x,y) = \mathrm{e}^{\mathrm{i}k(x\cos\beta + y\sin\beta)} \tag{14}$$

If Eq. (12) is substituted into Eq. (7), the boundary value with $\psi_D(x,y)$ is obtained as follows:

$$\nabla^2 \psi_D + k^2 \psi_D = 0 \quad \text{in} \quad \varOmega \tag{15}$$

Substituting Eq. (12) into Eq. (8) and using Eq. (6), free surface boundary condition is automatically satisfied and the remaining conditions are cylinder surface boundary condition and radiation condition. Finally by analyzing the boundary value problems, the scattered wave velocity potential is determined, and wave and wave forces acting on cylinders are calculated.

2.2 Formulation of boundary element method

The fundamental solution G of the Helmholtz equation is defined by:

$$\nabla^2 G + k^2 G + \delta(x-\xi, y-\eta) = 0 \tag{16}$$

where δ is the Dirac Delta function and G is the fundamental solution for Helmholtz equation:

$$G = \frac{i}{4} H_0^{(1)}(kr) \tag{17}$$

in which $r = \sqrt{(x-\xi)^2 + (y-\eta)^2}$ and (ξ,η) and (x,y) are the coordinate of the source point and observation point respectively. $H_0^{(1)}(kr) = J_0(kr) + \mathrm{i}y_0(kr)$ is the Henkel function of the first kind of order zero.

If one approximate. Eq. (17) for $r \to \infty$, on S_∞ then:

$$H_0^{(1)}(kr) = \sqrt{\frac{2}{\pi kr}} \exp\left\{ \mathrm{i}(kr - \frac{\pi}{4}) \right\} \tag{18}$$

$$\frac{\partial H_0^{(1)}(kr)}{\partial n} = \mathrm{i}k \sqrt{\frac{2}{\pi kr}} \exp\left\{ \mathrm{i}(kr - \frac{\pi}{4}) \right\} \tag{19}$$

Using this approximation, radiation boundary condition (Eq. (11)) will be satisfied. Also due to Eq. (12), the sea bed boundary condition Eq. (10) will be satisfied too. The boundary integral problem can be solved only for cylinder surface body using Eq. (20), (see Fig. 2). Finally, the boundary integral equation for n tandem cylinders when the observation point located on cylinder surface boundary S is obtained as follows:

$$\frac{1}{2}\psi_D(x_i,y_i) + \int_{S_{H1}+...+S_{HN}} \psi_D(x_j,y_j) \frac{\partial G(x_i,y_i : x_j,y_j)}{\partial n} \mathrm{d}s \tag{20}$$

where S_m (m=1, 2,…, N) is cylinder's boundary, and (x_j, y_j) and (x_i, y_i) denote the observation and source points, respectively.

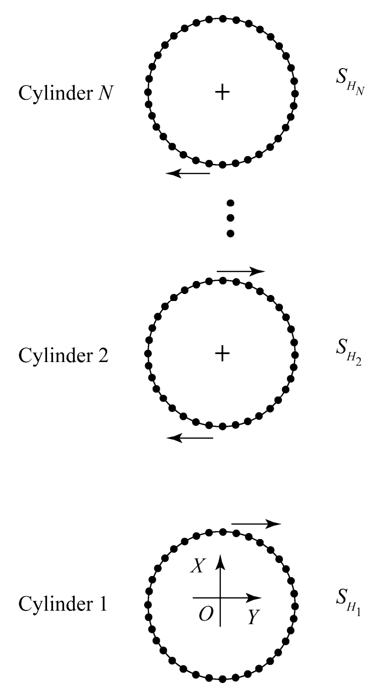

Fig. 2 Configuration of numerical model

2.3 Formulation of wave force

The wave force acting on the vertical circular cylinders is defined as follows:

$$P(x,y,z,t) = \mathrm{Re}\left\{ p(x,y,z)\mathrm{e}^{-\mathrm{i}\omega t} \right\} \tag{21}$$

The Bernoulli equation was used to get the pressure as:

$$p = -\rho \frac{\partial \phi}{\partial t} \tag{22}$$

where ρ is the water density.

The wave force in j direction acting on the cylinder number m is defined as follows:

$$F_j^m = \mathrm{Re}\left\{ f_j^m \mathrm{e}^{-\mathrm{i}\omega t} \right\} \tag{23}$$

The wave force in this direction can be presented using ψ_I and ψ_D as follows:

$$\mathrm{d}f_j^m = \rho g \frac{H}{2} \frac{\cosh k(z+h)}{\cosh kh} \mathrm{d}z \int_{S_{Hm}} (\psi_I + \psi_D) n_j \mathrm{d}s \tag{24}$$

Finally, by integrating Eq. (24) in the z direction, the wave force on the cylinder is defined as follows:

$$f_j^m = \rho g \frac{H}{2} \frac{\tanh kh}{k} \int_{S_{Hm}} (\psi_I + \psi_D) n_j \mathrm{d}s \tag{25}$$

2.4 Singular boundary integral

In Eq. (20), when observation point (x_j, y_j) get together with the source point (x_i, y_i), we have $i=j$ and it leads to R=0. Therefore, Eq. (17) will be singular. In order to solve Eq. (20) using Eq. (17), it is required to overcome this problem. One can use an approximation formula for Eq. (17), when $r \to 0$ as follows:

$$\lim_{R\to\infty} H_0^{(1)}(r) = 1 + \frac{2\mathrm{i}}{\pi}[\ln(r) + \gamma - \ln 2] \tag{26}$$

Substituting Eq. (26) for Eq. (17), in singular condition of Eq. (20), and using the well-known Gauss-Legendre method, this singularity will vanish.

3 Model validation

First of all the model results were validated. This can be completed either by using analytical methods, or numerical methods based on Galerkin-BEM (Newman, 2005) or earlier researches performed based on null-field BEM (Chen *et al*., 2011a, 2011b). In this section, validation of the wave force and trapped modes on middle cylinder of nine tandem cylinders was performed using the analytical method of Walker and Taylor (2005). The incident waves propagate in a direction parallel to the array. Direction of the wave β angle is relevant to X axis (see Fig. 1) Therefore, if $\beta = 0$, then the force on the 1st cylinder is related to S_{H1} and the force on the 2nd cylinder is related to S_{H2}, and so on (Fig. 2). All the results in this research were for $\beta = 0$. It was also important to emphasize that vertical axis presents the normalized force as $F_x / \rho g(H/2)a^2$. So depth of water was not concluded in the normalized process. Moreover, the cylinder's radius and separation distance on horizontal axis was normalized with the wave number. Fig. 3 shows this comparison. For this setup, the existence of trapped modes close to ka=0.5 and 1.0 was evident. This figure also shows very good agreement between two methods for magnitude of the wave load acting on the middle cylinder.

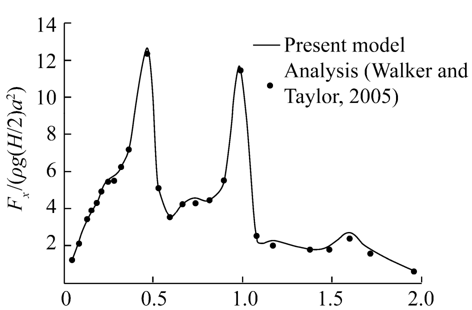

Fig. 3 Comparison between wave forces versus ka on 5th cylinders in a row of 9 tandem cylinders, 2*d*/*a*=6, *h*=10 m for present model and analytical model (Walker and Taylor, 2005)

For better validation of trapped mode effects of present day work, the model normalized amplitude (η / A in which A is incident wave amplitude) of the free surface elevation on the upstream face of middle cylinders in an array of nine cylinders and consequent trapped modes were compared with analytical model of Walker and Taylor (2005). Fig. 4 demonstrates this result. It is evident in this figure that the first and second trapped modes' effects were more critical. This is why the wave elevation in the 1st trapped mode is maximum. Normalized amplitude of the free surface elevation on the upstream face of single isolated cylinder is also represented for better comparison of trapped mode effects and validation. This figure shows very good agreement between results showing the accuracy of the model for estimation of trapped modes.

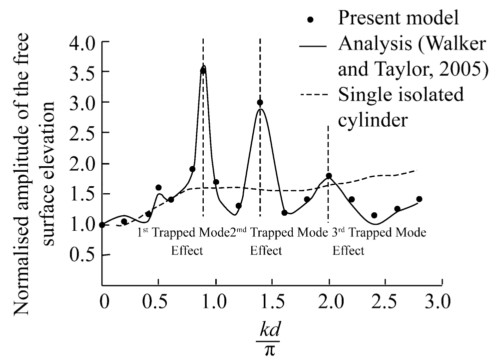

Fig. 4 Comparison of model and analytical results for normalized amplitude and trapped modes of the free surface elevation on the upstream face of middle cylinders in an array of nine cylinders, 2*d*/*a*=4, *β*=0

4 Numerical analysis and remarks

In the next stage of the research, forces on a single cylinder and two tandem cylinders were estimated respectively by the model. The results were then compared in Fig. 5 with those of analytical models for a water depth of 50 meter and *D*/*a*=3. Based on this figure the model results have good agreement with those computed by Han and Ohkusu (1995) and McCamy and Fuchs (1954). It should be noted that due to shielding effects for two adjacent tandem cylinders, the force on the second cylinder was lower than that of the first one. However, the first cylinder was in a trapped mode due to existence of the second one. In general this phenomenon depends on the number of cylinders and their arrangement. For example, for three tandem cylinders, the second cylinder also has critical situation. This is evident as its force is approximately equal to that of the first one. In this setup, wave energy was trapped between the 1st–2nd and 2nd–3rd cylinders while them 3rd cylinder was in shielding effect. The first cylinder was more affected by scattered waves rather than trapped mode, but the second one was affected by both of the trapped modes and scattered waves rigorously. Fig. 6 shows the differences between these three forces for parameters, $\frac{2d}{a}=6$ and water depth h=6, where, $2d$ is the distance between center to center of cylinders ($2d$=D+$2a$, see Fig. 1).

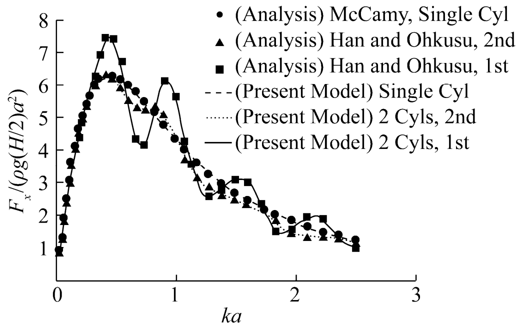

Fig. 5 Comparison between model results and analytical results for wave forces on each cylinders of sets of single and two, *D*/*a*=3

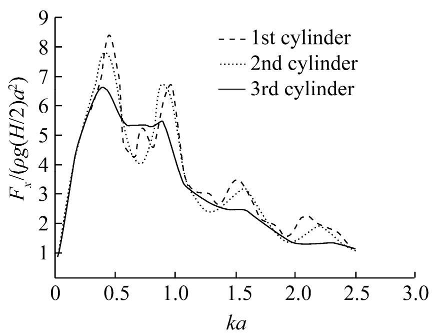

Fig. 6 Wave forces on each cylinder for 3 tandem cylinders, 2*d*/*a*=6, *h*=10 m

Fig. 5 also illustrates that all force diagrams have a maximum about ka=0.5. To investigate more about this case, wave forces on the 1st and 5th cylinders in a row of 9 and on the 1st and 13th cylinders in a row of 21 cylinders were calculated. The results were presented in Fig. 7, for $\frac{2d}{a}=6$ and water depth of h=10 m. It is evident in this figure that by increasing the number of tandem cylinders, the maximum wave forces on cylinders belongs to the middle one. Occurrence of trapped wave energy phenomenon can be justified due to the presence of standing waves in this region. As the number of tandem cylinders increases, critical frequencies occur in a narrower bound. The most critical frequencies occur when d is $\lambda/2\pi$ and $\lambda/4\pi$, respectively. So we can say that the bound limit was lower than $kd = n\pi/2$ (Maniar and Newman, 1997, Newman, 2005). Considering Eq. (4), it was noted that critical frequency in linear wave theory was approximately lower than:

$$\omega_n = \sqrt{\frac{n\pi}{2d} g \tanh\left(\frac{n\pi}{2d} g\right)} \tag{27}$$

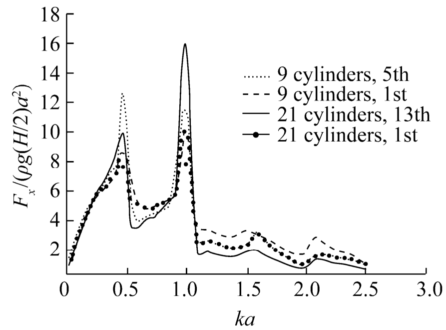

Fig. 7 Wave forces on 1st and 5th cylinders in a row of 9 and on 1st and 13th cylinders in a row of 21 cylinders, 2*d*/*a*=6, *h*=10 m

With regard to the above discussion and Newman studies (Maniar and Newman, 1997, Newman, 2005), there are two direct expressions for trapped mode phenomenon:

1) A single body in a canal with vertical wall.

2) Array of cylinders and considering imaginary vertical wall in the middle distance between two tandem cylinders for reflection of the encountering wave.

Both expressions are applicable in engineering such as deck and pile wharf and bridge design. In Eq. (19), odd numbers n=1, 3, 5,… have zero normal velocity $(\partial\Phi/\partial x=0)$ at vertical plan $y=\pm d, \pm 3d, \ldots$ and they are 180 degree in phase with adjacent cylinders. This is similar to the condition that the wall of canal is located in the mentioned region. If n is even, n=2, 4, 6,… then at vertical plan $y=\pm d, \pm 3d, \ldots$ the potential is zero, $\Phi=0$, and they are in phase with adjacent cylinders. If n is odd, then the physical condition is called Newman and if even, it is called Diricle. It is obvious that the Diricle condition is more critical as the generated standing waves for this condition is complete (e.g. for n=1, $2d=\lambda$ and n=2, $2d=\lambda$). Therefore, trapped energy in Diricle condition was bigger. Furthermore, changing in cylinders radius and distance between cylinders can affect the forces. For example, for 3 tandem cylinders, wave force versus ka on the 2nd cylinder will be decreased as the radius of cylinder increases as seen in Fig. 8. In this figure the distance between center to center of cylinders is constant and water depth is 10 m. For this condition, reflected wave from the first cylinder is complete, so that the trapped mode for 2nd cylinder decreases.

Fig. 9 shows the effect of cylinder's radius on wave force on the 2nd cylinder for various kd in 3 tandem cylinders. This figure shows that wave force on 2nd cylinder fluctuates around that of single cylinder. It means that by increasing the distance between cylinders, the trapped mode will vanish and behave similar to the single cylinder. Furthermore, in this situation wave force on 2nd cylinder will be decreased. The maximum wave force on a cylinder with a=4 m is approximately 1/4 wave force on cylinder with a=1 m, for the same wave encountering and three tandem cylinders.

Vos *et al*. (2007), showed that, after encountering the incident wave with the structure, fluid behavior is not similar to the ideal flow. It leads to a wave run-up lower than those of the experimental results. Therefore nonlinear analysis is required. However, based on Newman studies, linear analysis of diffracted wave force is in good agreement with experimental results.

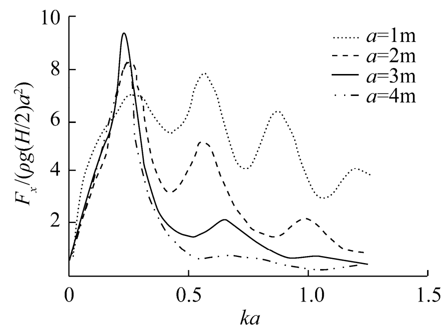

Fig. 8 Wave force on 2nd cylinder versus *ka* in 3 tandem cylinders for different cylinder's radius and constant depth of *h*=10 m

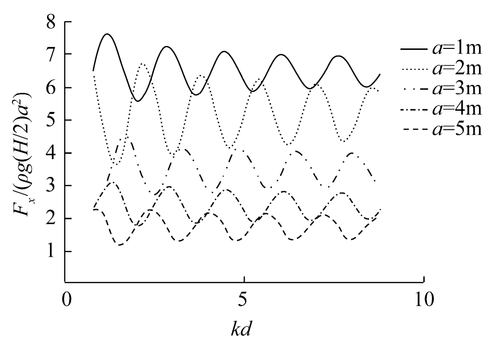

Fig. 9 Wave force on 2nd cylinder versus *kd* in 3 tandem cylinders for different cylinder's radius and constant depth of *h*=10 m.

5 Conclusions

Wave-structure interaction for finite tandem cylinders was studied using linear diffraction theory. Trapped mode was considered in various conditions. Finally, the sensitivity of wave force versus *ka* and *kd*, for varying cylinder's radius *a*, was studied. The results of the inline wave force estimation on *n* tandem cylinders show that the critical cylinder in the row is the middle one for odd numbers of cylinders. Furthermore, the critical trapped mode effect occurs for normalized cylinder radius (*ka*) close to 0.5 and 1.0. The force estimation on the cylinders confirmed that for constant *ka*, force amplitude versus normalized separation distance (*kd*) fluctuates about single cylinder force amplitude. This tends to single cylinder force amplitude as *kd* tends to infinity.

For more accurate results, a nonlinear model for free surface boundary conditions was required, especially for wave run up analysis.

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Symbols

a	Cylinders radius
d	Center to center separation distance

D	Distance between two cylinders
K	Wave number
Γ_{H_m}	mth boundary of cylinders
Γ_B	Sea bed boundary
Γ_R	Far field boundary
Γ_F	Free surface boundary
G	Green function
δ	Dirac Delta function
Ka	Normalized cylinder radius
kd	Normalized center to center separation distance of cylinders
S_{Hm}	mth cylinder's surface
ω	Wave frequency
p	Wave pressure
f_j^m	Wave force on mth cylinder in jth direction
φ_D	Diffraction wave potential
φ_I	Incident wave potential
h	Water depth
$H_0^{(1)}(r)$	Hankel function of the first kind of order zero

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Mohammad Mahdi Abaiee was burned in 1988. He obtained his B.Sc. degree in ship architect and received his M.Sc. degree from Amirkabir University of Technology (Tehran Polytechnic) in Naval Architect in 2012. His main field of interest is hydrodynamics of offshore structure and FSI. His knowledge in BEM referred to his M.Sc. thesis on hydrodynamics of Seastar TLP structures.

Ali Ahmadi was burned in 1987. He obtained his B.Sc. degree in ship architect and received his M.Sc. degree from Sharif University of Technology of Tehran in Naval Architect in 2012. His main field of interest is hydrodynamics of offshore structure and CFD. Ali was developed his BEM knowledge during his M.Sc. program on numerical modeling of TLP structures.