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Robust Adaptive Path Following for Underactuated Surface Vessels with Uncertain Dynamics

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Abstract: A robust adaptive control strategy was developed to force an underactuated surface vessel to follow a reference path, despite the presence of uncertain parameters and unstructured uncertainties including exogenous disturbances and measurement noise. The reference path can be a curve or a straight line. The proposed controller was designed by using Lyapunov's direct method and sliding mode control and backstepping techniques. Because the sway axis of the vessel was not directly actuated, two sliding surfaces were introduced, the first one in terms of the surge motion tracking errors and the second one for the yaw motion tracking errors. The adaptive control law guaranteed the uniform ultimate boundedness of the tracking errors. Numerical simulation results were provided to validate the effectiveness of the proposed controller for path following of underactuated surface vessels.
Keywords: underactuated surface vessels; path following; uncertain parameters; robustness; adaptive control
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1 Introduction

Maneuvering of underactuated surface vessels has long been a representative control problem due to their underactuated nature and inherent nonlinearity, and it has attracted wide attention from the control community for many years. Considering the ship maneuvering problems, such as trajectory tracking and path following issues, the former issue aims at tracking a reference trajectory generated by a virtual vessel, while the latter refers to forcing the vessel to follow a reference path, which not only can be generated by the virtual ship but also is a given path (Do and Pan, 2006).

For the path following problem, the main challenge is that most ships are usually equipped with one or two main propellers for surge motion control along with rudders for yaw motion control of the ship. There are no side thrusters, so the sway axis is not actuated. This configuration is mostly used in marine vehicles (Fossen, 2002). Meanwhile, another challenge of path following is the inherent nonlinearity of the ship dynamics and kinematics with the uncertain parameters and unstructured uncertainties including external disturbances and measurement noise. To overcome these challenges, many different nonlinear design methodologies have been introduced to the underactuated ships. By applying Lyapunov's direct method, two constructive tracking solutions were developed in Jiang (2002), and Do *et al*. (2002) provided a controller for underactuated ships in order to obtain globally exponential stability. Behal *et al*. (2002) developed a global practical tracking controller; Pettersen and Lefeber (2001) proposed a local tracking result based on a recursive technique which was proposed by Jiang and Nijmeijer (1999) for standard chain form systems. Using the cascaded approach, Breivik and Fossen (2004) represented a global tracking control law. More recently, the backstepping method has been popularly used for underactuated ships. In Do *et al*. (2004a, 2004b, 2006), the controllers were designed to force an underactuated surface vessel to follow a predefined path. The stability analysis was investigated relying on Lyapunov's direct method. In addition to Do *et al*. (2004a, 2004b), Do *et al*. (2006) designed a controller to steer the ship to move on the reference path with an adjustable forward speed. A robust adaptive control scheme was proposed for point-to-point navigation of underactuated ships by using a general backstepping technique (Li *et al.*, 2008). Compared with Do *et al*. (2004a, 2004b, 2006), the proposed controller has a certain conciseness and the physical meaning of the tracking errors is much more clear. In Li *et al*. (2009a), a simple control law was presented by using the novel backstepping and feedback dominance, under the presence of model uncertainties, communication delays, and measurement noise. Furthermore, the control design was verified using a model ship in a tank.

The model predictive control (MPC) methodology was first used for ships in Wahl and Gilles (1998), and an MPC rudder roll stabilization control system was presented by Perez (2005). Li *et al*. (2009b) considered the problem of path following for marine surface vessels using the rudder control, and introduced a new MPC methodology based on a linearized 2DOF model for path following implementation

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consideration, while the effectiveness of the MPC controller was validated on a nonlinear 4DOF surface vessel model. To improve the performance of Li *et al*. (2009b), Oh and Sun (2010) proposed an MPC way-point tracking controller combined with line of sight guidance algorithm, and the performance of the controller showed that the designed scheme out-performed those achieved using the MPC method alone. Based on the Serret-Frenet frame which was introduced in Do and Pan (2004b), Wang *et al*. (2009, 2010) proposed a path following controller to force underactuated ships under uncertain parameters to follow a predefined path by using analytic model predictive control and model reference adaptive control.

By using intelligent control, Liu *et al*. (2010) proposed a stable adaptive neural network algorithm for the path following of a 3DOF underactuated ship with parameter uncertainties and disturbances. The sliding mode control method is often used because of its robustness property and insensitivity to the parameters uncertainties (Slotine and Li, 2006). Bu *et al*. (2007) developed a straight line control algorithm by employing an iterative sliding mode control method. Ashrafiuon *et al*. (2008) presented a sliding mode control law for trajectory tracking of underactuated autonomous surface vessels by introducing a first-order sliding surface in terms of surge tracking errors and a second-order sliding surface in terms of lateral motion tracking errors.

Motivated by these recent developments in path following of underactuated surface vessels, this paper presents a robust adaptive sliding mode control law. In fact, the model of underactuated systems is usually uncertain, the environmental disturbances always exist, and the damping matrixes are off-diagonal (Fossen, 2002). By taking into account these challenges, the sliding mode control combined with the backstepping method is used. Two second-order sliding surfaces are introduced, the first sliding surface in terms of path following position errors and the second in terms of orientation errors. The stability analysis is performed based on the Lyapunov theory. The proposed controller can guarantee that all signals of the underactuated system are bounded. Moreover, two kinds of reference paths can be followed, a curve or a straight line. Numerical simulations are provided to validate the effectiveness of the proposed path following controller.

2 Problem statements

2.1 Underactuated surface vessel model

For an ocean vessel moving in 6 DOF, the 6 different motion components are conveniently denoted as surge, sway, heave, roll, pitch, and yaw, see Fig.1 from Perez (2005).

Considering the path following problem of an underactuated surface vessel, the vessel is moving in the horizontal plane, and the heave, roll, and pitch are usually neglected. The mathematical model of an underactuated surface vessel can be rewritten as (Fossen, 2002)

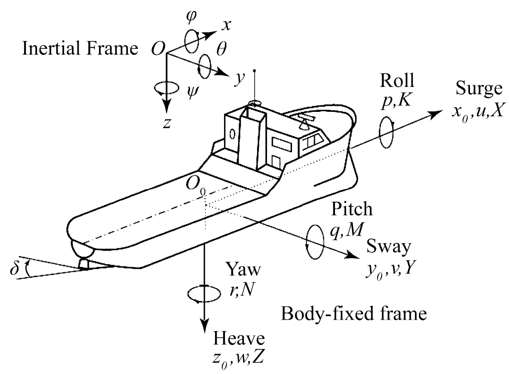


Fig.1 Reference frames and variables for ship motion description

$$\begin{cases}\dot{x} = u\cos\psi - v\sin\psi \\ \dot{y} = u\sin\psi + v\cos\psi \\ \dot{\psi} = r\end{cases} \quad (1)$$

$$\begin{cases}\dot{u} = f_u^{\mathrm{T}}\chi_u(\upsilon) + g_u\tau_u + d_u \\ \dot{v} = f_v^{\mathrm{T}}\chi_v(\upsilon) + d_v \\ \dot{r} = f_r^{\mathrm{T}}\chi_r(\upsilon) + g_r\tau_r + d_r\end{cases} \quad (2)$$

where x , y and ψ denote the surge displacement, sway displacement, and yaw angle in the earth fixed frame; u , v and r are the surge, sway and yaw velocities, respectively; $f_u \in \mathbb{R}^{n_u}$, $f_v \in \mathbb{R}^{n_v}$ and $f_r \in \mathbb{R}^{n_r}$ are unknown constant vectors, n_u , n_v and n_r are known dimensions; $\chi_u(\bullet) \in \mathbb{R}^{n_u}$, $\chi_v(\bullet) \in \mathbb{R}^{n_v}$ and $\chi_r(\bullet) \in \mathbb{R}^{n_r}$ represent the known smooth velocity function vectors; g_u and g_r are unknown nonzero constant control coefficients. Since the control force for sway motion is not available, the available control inputs are the control force for surge motion τ_u and control moment for yaw motion τ_r , and the ship model in (1-2) is underactuated (Do *et al*., 2004a). d_u , d_v and d_r denote the unstructured uncertainties including environmental disturbances induced by wave, wind, and current.

2.2 Path following error dynamics

The general framework of ship path following is shown in Fig.2. For path following control, the position and orientation errors attached to the reference path $\boldsymbol{\Omega}$ are introduced as follows

$$x_e = x - x_d,\ y_e = y - y_d,\ \psi_e = \psi - \psi_d,\ z_e = \sqrt{x_e^2 + y_e^2}\ , \quad (3)$$

where $\psi_d = \arcsin(y_e/z_e)$ represents the desired orientation; z_e is the distance of path following error; x_d and y_d denote the desired displacement in path $\boldsymbol{\Omega}$ of the underactuated vessel.

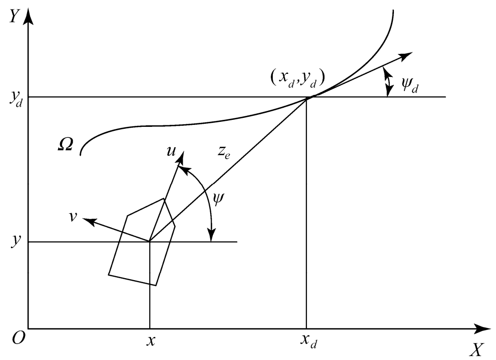


Fig.2 General framework of ship path following

Assumption 1 The reference path $\boldsymbol{\Omega}$ is smooth, and x_d , $\dot{x}_d$, $\ddot{x}_d$, y_d , $\dot{y}_d$, $\ddot{y}_d$, $\dot{\psi}_d$ and $\ddot{\psi}_d$ are all bounded.

Assumption 2 The unstructured uncertainties d_u , d_v and d_r satisfy $|d_u| \le d_{u\max} < \infty$, $|d_v| \le d_{v\max} < \infty$ and $|d_r| \le d_{r\max} < \infty$.

Assumption 3 The sway velocity v is passive-bounded.

Remark 1 For the underactuated nature of the ship, the control force for sway motion is unavailable. Under this consideration, the sway dynamics should be stable. In fact, the sway motion of the underactuated ship satisfies a passive-bounded property (Do *et al*., 2004a, 2004b; Li *et al*., 2008). Detailed analysis is made in section 4 by considering a different case from Li *et al*. (2008).

Control objective: Under Assumptions 1–3, the objective of this paper is to seek the adaptive sliding mode control laws τ_u and τ_r to force an underactuated surface vessel with uncertain parameters and environmental disturbances to follow a reference path $\boldsymbol{\Omega}$.

3 Control design

In this section, the procedure to design an adaptive sliding mode controller for an underactuated surface vessel in the presence of uncertain parameters and environmental disturbances is presented. The sliding mode control is chosen for its robustness property (Slotine and Li, 2006). It can be seen from Eq.(2) that the control force for sway motion is unavailable. The main difficulty is that the surge and yaw motion control law must be designed to control the ship's 3DOF plane motion in surge, sway, and yaw directions. Since position variables cannot be defined in the body-fixed frame, the integral of surge and yaw velocities are used.

3.1 Surge control law τ_u

Step 1. The first equation of (2) can be rewritten as

$$\begin{aligned}\dot{u} &= \left(\hat{f}_u^{\mathrm{T}} + \Delta f_u^{\mathrm{T}}\right)\chi_u(\upsilon) + \left(\hat{g}_u + \Delta g_u\right)\tau_u + \hat{d}_u + \Delta d_u \\ &= \hat{f}_u^{\mathrm{T}}\chi_u(\upsilon) + \hat{g}_u\tau_u + \hat{d}_u + F_u \end{aligned} \tag{4}$$

where $F_u = \Delta f_u^{\mathrm{T}}\chi_u(\upsilon) + \Delta g_u\tau_u + \Delta d_u$, $\Delta(\bullet)$ denotes the uncertain part of $(\bullet)$.

From the equation (3), the result is

$$x_e = z_e\cos\psi_d \text{ , } y_e = z_e\sin\psi_d \text{ .} \tag{5}$$

Define the surge velocity error as follows

$$u_e = u - \alpha_u \text{ .} \tag{6}$$

Consider the following Lyapunov function candidate

$$V_1 = \frac{1}{2}z_e^2 \tag{7}$$

Differentiating (7) by using (2) and (5) yields

$$\begin{aligned}&\dot{V}_1 = z_e\dot{z}_e = x_e\dot{x}_e + y_e\dot{y}_e = \\ &z_e(u_e\cos\psi_e + \alpha_u\cos\psi_e - \\ &v\sin\psi_e - \dot{x}_d\cos\psi_d - \dot{y}_d\sin\psi_d)\end{aligned} \tag{8}$$

The stabilizing function α_u is selected as

$$\alpha_u = \left(\cos\psi_e\right)^{-1}\left(-k_1z_e + v\sin\psi_e + \dot{x}_d\cos\psi_d + \dot{y}_d\sin\psi_d\right) \tag{9}$$

where $k_1 \ge 0$. Substituting (9) into (8), it is easy to get

$$\dot{V}_1 = -k_1z_e^2 + z_eu_e\cos\psi_e \tag{10}$$

Differentiating (6) along with (4) and (9) yields

$$\dot{u}_e = \dot{u} - \dot{\alpha}_u = \hat{f}_u^{\mathrm{T}}\chi_u(\upsilon) + \hat{g}_u\tau_u + \hat{d}_u + F_u - \dot{\alpha}_u \tag{11}$$

Step 2. Adaptive control law design

Considering the underactuated nature of the vessel, the first sliding surface is defined in terms of the surge motion tracking errors

$$s_1 = u_e + \lambda_1\int_0^t u_e(\tau)\mathrm{d}\tau \tag{12}$$

where $\lambda_1 > 0$, taking the time derivative of s_1 leads to

$$\dot{s}_1 = \dot{u}_e + \lambda_1u_e \tag{13}$$

In order to achieve the adaptive sliding mode control law, consider the following Lyapunov function candidate

$$V_2 = \frac{1}{2}s_1^2 + \frac{1}{2}\tilde{F}_u^2\xi_1^{-1}, \xi_1 > 0 \tag{14}$$

where $\tilde{F}_u = F_u - \hat{F}_u$ is the estimated error, $\hat{F}_u$ is the estimated value of F_u .

Taking the time derivative of V_2 by using (10) and (11) yields

$$\begin{aligned}\dot{V}_2 &= s_1\dot{s}_1 - \tilde{F}_u\dot{\hat{F}}_u\xi_1^{-1} + \tilde{F}_u\dot{F}_u\xi_1^{-1} = \\ & s_1\left(\dot{u}_e + \lambda_1 u_e\right) - \tilde{F}_u\dot{\hat{F}}_u\xi_1^{-1} + \tilde{F}_u\dot{F}_u\xi_1^{-1} = \\ & s_1\left(\hat{f}_u^{\mathrm{T}}\chi_u(\upsilon) + \hat{g}_u\tau_u + \hat{d}_u + F_u - \dot{\alpha}_u + \lambda_1 u_e\right) - \\ & \tilde{F}_u\dot{\hat{F}}_u\xi_1^{-1} + \tilde{F}_u\dot{F}_u\xi_1^{-1} = \\ & s_1\left(\hat{f}_u^{\mathrm{T}}\chi_u(\upsilon) + \hat{g}_u\tau_u + \hat{d}_u + \hat{F}_u - \dot{\alpha}_u + \lambda_1 u_e\right) - \\ & \tilde{F}_u\left(\dot{\hat{F}}_u - \xi_1 s_1\right)\xi_1^{-1} + \tilde{F}_u\dot{F}_u\xi_1^{-1}\end{aligned} \tag{15}$$

The surge motion adaptive control law τ_u is chosen as follows:

$$\begin{aligned}\tau_u = (g_u)^{-1}\Big[& -\hat{f}_u^{\mathrm{T}}\chi_u(\upsilon) - \hat{d}_u + \dot{\alpha}_u - \\ & \lambda_1 u_e - \hat{F}_u - k_2\left(s_1 + k_3\,\mathrm{sgn}(s_1)\right)\Big]\end{aligned} \tag{16}$$

where k_2 and k_3 are positive constants.

Then the adaptive law can be selected as

$$\dot{\hat{F}}_u = \xi_1 s_1, \quad \xi_1 > 0 \tag{17}$$

However, since the sign function in Eq.(16) is discontinuous, there must be the chattering in the control system. In order to weaken the chattering, saturation function is used as follows

$$\mathrm{sat}(s_i/\varepsilon_i) = \begin{cases} s_i/\varepsilon_i, |s_i/\varepsilon_i| \le 1 \\ \mathrm{sgn}(s_i), |s_i/\varepsilon_i| > 1 \end{cases}, \quad i = 1, 2, \varepsilon_i > 0 \tag{18}$$

Hence, the adaptive sliding mode control law (16) is rewritten as

$$\begin{aligned}\tau_u = g_u^{-1}\Big[& -\hat{f}_u^{\mathrm{T}}\chi_u(\upsilon) - \hat{d}_u + \dot{\alpha}_u - \lambda_1 u_e - \\ & \hat{F}_u - k_2\left(s_1 + k_3\mathrm{sat}(s_1/\varepsilon_1)\right)\Big]\end{aligned} \tag{19}$$

Substituting Eq.(17) and Eq.(19) into Eq.(15), the result is

$$\begin{aligned}\dot{V}_2 &= s_1\left(-k_2 s_1 - k_2 k_3\mathrm{sat}(s_1/\varepsilon_1)\right) + \tilde{F}_u\dot{F}_u\xi_1^{-1} \le \\ & -k_2 s_1^2 - k_2 k_3|s_1| + \tilde{F}_u\dot{F}_u\xi_1^{-1}\end{aligned} \tag{20}$$

3.2 Yaw motion control law τ_r

Step 1. Similar to Eq.(4), the yaw motion error dynamics can be rewritten as

$$\begin{aligned}\dot{r} &= \left(\hat{f}_r^{\mathrm{T}} + \Delta f_r^{\mathrm{T}}\right)\chi_r(\upsilon) + \left(\hat{g}_r + \Delta g_r\right)\tau_r + \hat{d}_r + \Delta d_r = \\ & \hat{f}_r^{\mathrm{T}}\chi_r(\upsilon) + \hat{g}_r\tau_r + \hat{d}_r + F_r \\ \dot{\psi}_e &= r - \dot{\psi}_d\end{aligned} \tag{21}$$

where $F_r = \Delta f_r^{\mathrm{T}}\chi_r(\upsilon) + \Delta g_r\tau_r + \Delta d_r$

Similar to the previous procedure, the Lyapunov function candidate is defined as follows

$$V_3 = \frac{1}{2}\psi_e^2 \tag{22}$$

Let $r_e = r - \alpha_r$, and the time derivative of Eq.(22) along with Eq.(21) satisfies

$$\dot{V}_3 = \psi_e\dot{\psi}_e = \psi_e\left(r_e + \alpha_r - \dot{\psi}_d\right) \tag{23}$$

Then the stabilizing function α_r is selected as

$$\alpha_r = -k_4\psi_e + \dot{\psi}_d, k_4 > 0 \tag{24}$$

Substituting Eq.(24) into Eq.(23), it is easy to have

$$\dot{V}_3 = -k_4\psi_e^2 + \psi_e r_e \tag{25}$$

Step 2. Adaptive control law design

The second sliding surface is defined in terms of the yaw motion tracking errors

$$s_2 = r_e + \lambda_2\int_0^t r_e(\tau)\mathrm{d}\tau \tag{26}$$

The time derivative of Eq.(26) along the solution of Eq.(21) satisfies

$$\begin{aligned}\dot{s}_2 &= \dot{r}_e + \lambda_2 r_e = \\ & \hat{f}_r^{\mathrm{T}}\chi_r(\upsilon) + \hat{g}_r\tau_r + \hat{d}_r + F_r - \dot{\alpha}_r + \lambda_2 r_e\end{aligned} \tag{27}$$

The Lyapunov function candidate is considered as follows

$$V_4 = \frac{1}{2}s_2^2 + \frac{1}{2}\tilde{F}_r^2\xi_2^{-1} \tag{28}$$

where $\tilde{F}_r = F_r - \hat{F}_r$ is the estimated error, and $\hat{F}_r$ is the estimated value of F_r.

Differentiating Eq.(28) by using Eq.(25) and Eq.(27) yields

$$\begin{aligned}\dot{V}_4 &= s_2\dot{s}_2 - \tilde{F}_r\dot{\hat{F}}_r\xi_2^{-1} + \tilde{F}_r\dot{F}_r\xi_2^{-1} = \\ & s_2\left(\hat{f}_r^{\mathrm{T}}\chi_r(\upsilon) + \hat{g}_r\tau_r + \hat{d}_r + F_r - \dot{\alpha}_r + \lambda_2 r_e\right) - \tilde{F}_r\dot{\hat{F}}_r\xi_2^{-1} + \tilde{F}_r\dot{F}_r\xi_2^{-1} = \\ & s_2\left(\hat{f}_r^{\mathrm{T}}\chi_r(\upsilon) + \hat{g}_r\tau_r + \hat{d}_r + F_r - \dot{\alpha}_r + \lambda_2 r_e\right) - \\ & \tilde{F}_r\left(\dot{\hat{F}}_r - \xi_2 s_2\right)\xi_2^{-1} + \tilde{F}_r\dot{F}_r\xi_2^{-1}\end{aligned} \tag{29}$$

The yaw motion control law τ_r is selected as

$$\begin{aligned}\tau_r = g_r^{-1}\Big[& -\hat{f}_r^{\mathrm{T}}\chi_r(\upsilon) - \hat{d}_r + \dot{\alpha}_r - \lambda_2 r_e - \\ & \hat{F}_r - k_5\left(s_2 + k_6\mathrm{sat}(s_2/\varepsilon_2)\right)\Big]\end{aligned} \tag{30}$$

where k_5 and k_6 are positive constants.

Choosing an adaptive law as follows

$$\dot{\hat{F}}_r = \xi_2 s_2, \xi_2 > 0 \tag{31}$$

By replacing the equations of Eq.(30) and Eq.(31) into Eq.(29) given by

$$\begin{aligned}\dot{V}_4 &= s_2\left(-k_5 s_2 - k_5 k_6\mathrm{sat}(s_2/\varepsilon_2)\right) + \tilde{F}_r\dot{F}_r\xi_2^{-1} \le \\ & -k_5 s_2^2 - k_5 k_6|s_2| + \tilde{F}_r\dot{F}_r\xi_2^{-1}\end{aligned} \tag{32}$$

4 Stability analysis

4.1 Theorem 1.

Assuming that the Assumptions 1–3 hold, the sliding mode control laws τ_u and τ_r are derived as in Eq.(19) and Eq.(30), and adaptation laws are given by Eq.(17) and Eq.(31). The control objective of path following for underactuated surface vessels in the presence of uncertain parameters and unstructured uncertainties is solved, and systems (1) and (2) are asymptotic stability.

Proof. Define a Lyapunov function candidate

$$V_5 = V_2 + V_4 = \frac{1}{2}s_1^2 + \frac{1}{2}s_2^2 + \frac{1}{2}\tilde{F}_u^2\xi_1^{-1} + \frac{1}{2}\tilde{F}_r^2\xi_2^{-1} \tag{33}$$

Differentiating Eq.(33) along with Eq.(20) and Eq.(32), the result is

$$\dot{V}_5 \le -k_2 s_1^2 - k_5 s_2^2 - k_2 k_3 |s_1| - k_5 k_6 |s_2| + \tilde{F}_u \dot{F}_u \xi_1^{-1} + \tilde{F}_r \dot{F}_r \xi_2^{-1} \tag{34}$$

There are two kinds of conditions about the uncertain parameters F_u and F_r, which are discussed as follows.

1) If the uncertainties F_u and F_r are slowly varying with time, the result is $\dot{F}_u = 0$, $\dot{F}_r = 0$, and Eq.(34) is rewritten as

$$\dot{V}_5 \le -k_2 s_1^2 - k_5 s_2^2 - k_2 k_3 |s_1| - k_5 k_6 |s_2| \le 0 \tag{35}$$

According to the sliding mode design and the Lyapunov theory, systems (1) and (2) are asymptotic stability.

2) If the uncertainties F_u and F_r are varying fast with time but with bounded norm, which means that $\tilde{F}_u \dot{F}_u \xi_1^{-1} < 0$, $\tilde{F}_r \dot{F}_r \xi_2^{-1} < 0$, equation (34) is derived as

$$\begin{aligned} \dot{V}_5 \le & -k_2 s_1^2 - k_5 s_2^2 - k_2 k_3 |s_1| - k_5 k_6 |s_2| + \\ & \tilde{F}_u \dot{F}_u \xi_1^{-1} + \tilde{F}_r \dot{F}_r \xi_2^{-1} < 0 \end{aligned} \tag{36}$$

Systems (1) and (2) are uniformly ultimately bounded. It can be seen from (36) that the tracking errors converge to a bounded domain, and it can be adjusted by changing the control parameters k_2, k_3, k_5, k_6, and the adaptation gains ξ_1, ξ_2. Generally, in order to weaken the chattering, the control parameters k_3, k_6 are selected to be smaller and k_2, k_5 are larger. If the adaptation gains ξ_1, ξ_2 are large, it will result in faster adaptation, but it will increase the chattering.

4.2 v-Dynamics stability

According to the underactuated surface vessel (1-2), the control force for sway motion is not available. In order to design the controller, the stability of the vessel's sway dynamics should be guaranteed, which means that the sway velocity of the vessel is passive-bounded (Do and Pan, 2006, Li *et al*., 2006).

To show that v is bounded, the Lyapunov function candidate is defined by $\dot{V}_6 = \frac{1}{2}v^2$, and differentiating it yields

$$\begin{aligned} \dot{V}_6 = & -\frac{m_{11}}{m_{22}}urv - \left(\frac{d_v}{m_{22}} + \sum_{i=2}^{3}\frac{d_{vi}}{m_{22}}|v|^{i-1}\right)v^2 \le \\ & -\rho_2 V_v + \mu_2 \end{aligned} \tag{37}$$

where $\rho_2 = \frac{2d_v}{m_{22}}$, $\mu_2 = -\frac{m_{11}}{m_{22}}urv$.

Let $\lambda_2 = \frac{\mu_2}{\rho_2}$, rewriting (37), the result is

$$V_v \le \lambda_2 + (V_v(0) - \lambda_2)\mathrm{e}^{-\rho_2 t} \tag{38}$$

Hence, the sway velocity is passive-bounded.

5 Numerical simulation

In this section, numerical simulation is provided to show the effectiveness of the proposed control law and the accuracy of stability analysis. In this paper, considering a monohull ship with a mass of $118\times10^3\,\mathrm{kg}$ and a length of $32\,\mathrm{m}$, the dynamics model and its hydrodynamic coefficients with parameters uncertainties are adopted from Do and Pan (2006). The simulation results are convenient to compare with Do and Pan (2006) and Li *et al*. (2006). In the simulation, the reference path is generated by a virtual ship as follows

$$\begin{aligned} \dot{x}_d &= u_d\cos(\psi_d) - v_d\sin(\psi_d) \\ \dot{y}_d &= u_d\sin(\psi_d) + v_d\cos(\psi_d) \\ \dot{\psi}_d &= r_d \\ \dot{v}_d &= f_v^T\chi_v(v_d) \end{aligned} \tag{39}$$

where $\chi_v(v_d) = \left[-u_d r_d, -v_d, -|v_d|v_d, -|v_d|^2 v_d\right]^{\mathrm{T}}$, $u_d = 5$,

$$r_d = \begin{cases} \mathrm{e}^{0.005t/300}, t \le 50s \\ 0, \quad 50s < t \le 170s \\ 0.025, t > 170s \end{cases}$$

The initial conditions are chosen as:

$$[x_d(0), y_d(0), \psi_d(0), v_d(0),] = [0,0,0,0]$$

$$[x(0), y(0), \psi(0), u(0), v(0), r(0)] = [0,100,0,0,0,0]$$

$$d_{u\max 0} = 0.7d_{u\max},\ d_{u\max} = 2,\ d_{r\max 0} = 0.7d_{r\max},\ d_{r\max} = 3$$

$$f_{u0} = 0.7f_u,\ f_{r0} = 0.7f_r,\ g_{u0} = 0.7g_u,\ g_{r0} = 0.7g_r$$

The control parameters selected for the simulation are:

$$k_1 = 0.6,\ k_2 = 2,\ k_3 = 0.1,\ k_4 = 2,\ k_5 = 4,\ k_6 = 0.2,$$

$$\lambda_1 = 3,\ \lambda_2 = 6,\ \varepsilon_1 = 0.01,\ \varepsilon_2 = 0.05,\ \xi_1 = 15,\ \xi_2 = 20.$$

The simulation results of ship path following control are depicted in Figs.3-8. Fig.3 shows the position and the orientation of the vessel path following under the proposed control law. It is clearly seen that the vessel follows a reference path with high accuracy, in the presence of uncertain parameters and unstructured uncertainties. Fig.4 displays the ship path following position errors x_e, y_e, the orientation error ψ_e . The path following velocity errors u_e, v_e, r_e are plotted in Fig.5. From Figs.4 and 5, it can be seen that the path following errors and the velocity errors are simultaneously convergent to zero with fast convergence rate except the sway velocity error v_e which exponentially converges to a small value before the 170 s. But after the 170 s, the sway velocity converges to a varying periodically, because the reference path is a circle.

The control inputs τ_u and τ_r are given in Fig.6, and they converge to a bounded domain. Because the disturbance in yaw motion is larger than the one in the surge dynamics, it is obviously shown that the control moment for yaw motion τ_r is more noise than the control force for surge motion τ_u . There is a larger magnitude in τ_r at the 170 s, since the reference path is changed from a straight line to a circle. From a practical viewpoint, the control inputs τ_u and τ_r are still realistic with the magnitude restrictions. In Li *et al*. (2006), the control input τ_r is much more noise than the one in this paper.

Finally, the estimations f_u , f_r , g_u and g_r are shown in Figs.7–8. The amplitude of f_r and g_r have an obvious change at the 170 s. Because of the larger disturbance in yaw dynamics, it can be seen that the estimation of g_r is much more noise than g_u .

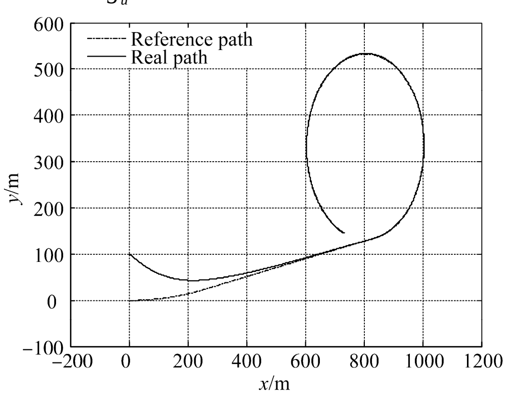


Fig.3 Position and orientation of the vessel in the *xy* plane.

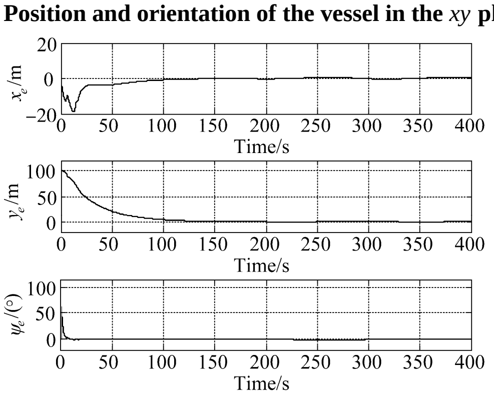


Fig.4 Position and orientation errors of the vessel.

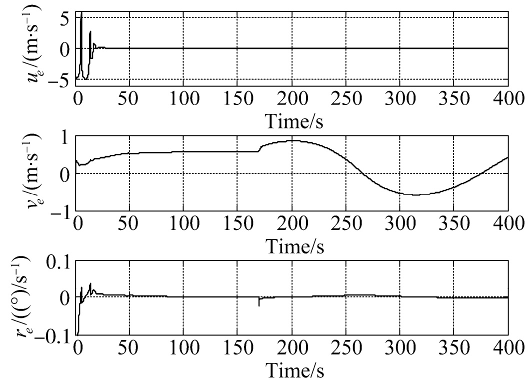


Fig.5 Velocity errors of the vessel, where

$$v_e = \sqrt{u^2 + v^2} - \sqrt{\dot{x}_d^2 + \dot{y}_d^2} \text{ .}$$

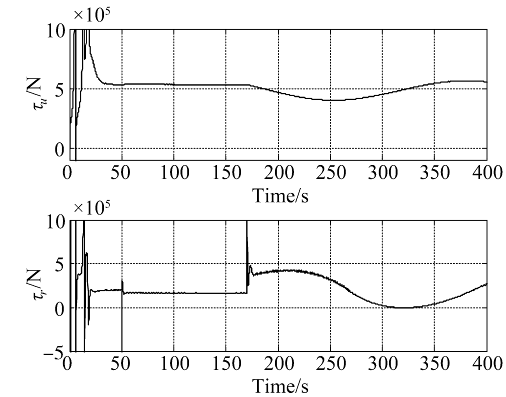


Fig.6 The time responses for the inputs of the vessel.

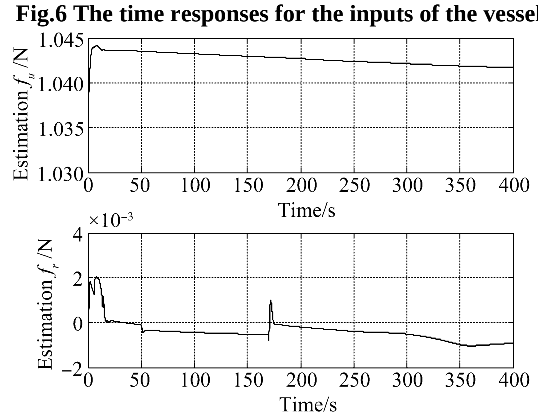


Fig.7 The estimations of F_u and F_r .

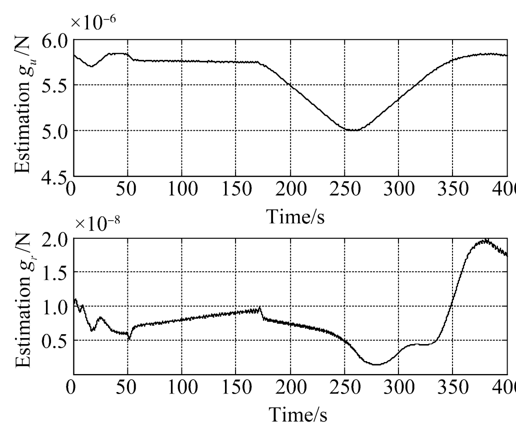


Fig.8 The estimations of g_u and g_r .

6 Conclusions

In this paper, a robust adaptive scheme was proposed for path following of underactuated surface vessels with uncertain parameters and unstructured uncertainties including exogenous disturbances and measurement noise. The sliding mode control method, together with the backstepping design, led to an adaptive sliding mode control law. It was noted that in order to design the sliding mode control law, two second-order sliding surfaces in terms of the path following position errors and orientation errors were chosen. The stability analysis was performed based on the Lyapunov theory. The effectiveness of the designed controller was also validated by the numerical simulations. Based on the ideas of this paper, future work will consider the rudder saturation and rate limits.

References

Ashrafiuon H, Muske KR, McNinch LC, Soltan RA (2008). Sliding mode tracking control of surface vessels. *IEEE Transactions on Industrial Electronics*, **55**(11), 4004-4012.

Behal A, Dawson DM, Dixon WE, Fang Y (2002). Tracking and regulation control of an underactuated surface vessel with nonintegrable dynamics. *IEEE Transactions on Automatic Control*, **47**(3), 495-500.

Bu RX, Liu ZJ, Hu JQ (2007). Straight–path tracking control of underactuated ships using dynamic nonlinear sliding. *Journal of Tsinghua Univ (Sci & Tech)*, **47**(s2), 1880-1883.

Breivik M, Fossen TI (2004). Path following for marine surface vessels. *MTS/IEEE Techno-Ocean*, **4**, 2282-2288.

Do KD, Jiang ZP, Pan J (2002). Universal controllers for stabilization and tracking of underactuated ships. *Systems and Control Letter*, **47**, 299-317.

Do KD, Jiang ZP, Pan J (2004a). Robust adaptive path following of underactuated ships. *Automatica*, **40**, 929-944.

Do KD, Pan J (2004b). State-and output-feedback robust path-following controllers for underactuated ships using Serret–Frenet frame. *Ocean Engineering*, **31**, 587-613.

Do KD, Pan J (2006a). Robust path-following of underactuated ships: Theory and experiments on a model ship. *Ocean Engineering*, **33**, 1354-1372.

Do KD, Pan J (2006b). Global robust adaptive path following of underactuated ships. *Automatica*, **42**, 1713-1722.

Fossen TI (2002). *Marine Control systems, Trondheim*. Norway, Marine Cybernetics.

Jiang ZP, Nijmeijer H (1999). A recursive technique for tracking control of nonholonomic systems in chained form. *IEEE Transactions on Automatic Control*, **44**, 265-279.

Jiang ZP (2002). Global tracking control of underactuated ships by Lyapunove's direct method, *Automatica*, **38**, 301-309.

Li JH, Lee PM, Jun BH , Lim YK(2008). Point-to-point navigation of underactuated ships. *Automatica*, **44**, 3201-3205.

Li Z, Sun J, Oh SR (2009a). Design, analysis and experimental validation of robust nonlinear path following control of marine surface vessels. *Automatica*, **45**, 1649-–1658.

Li Z, Sun J (2009b). Path following for marine surface vessels with rudder and roll constraints: an mpc approach. *American Control Conference*, Paper ThC11.6.

Liu Y, Guo C, Shen ZP, Liu Y, Guo D (2010). Stable adaptive neural network control of path following for underactuated ships. *Control Theory and Applications*, **27**(2), 169-174.

Oh SR, Sun J (2010). Path following of underactuated marine surface vessels using line-of-sight based model predictive control. *Ocean Engineering*, **37**, 289-295.

Pettersen KY, Lefeber E (2001). Way-point tracking control of ships. *In Proceedings of Conference on Decision and Control*, Florida, USA, 940-945.

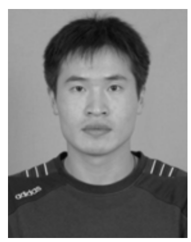
Perez T (2005). *Ship motion control: course keeping and roll stabilisation using rudder and fins*. Springer, Berlin.

Slotine J, Li WP (2006). *Applied Nonlinear Control*. China Machine Press, Beijing, 268-269.

Wahl A, Gilles E (1998). Track-keeping on waterways using model predictive control. *Proceedings of the IFAC Conference on Control Applications in Marine Systems*, Fukuoka, Japan, 149-154.

Wang XF, Zou ZJ, Li TS, Luo WL (2009). Nonlinear model predictive controller with disturbance observer for path following of underactuated ships. *Journal of Wuhan University of Technology (Transportation Science & Engineering)*, **33**(5), 1020-1024.

Wang XF, Zou ZJ, Li TS, Luo WL (2010). Adaptive path following controller of underactuated ships using serret-frenet frame. *Journal of Shanghai Jiaotong University (Science)*, **15**(3), 334-339.



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