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A geometric Model for the Spatial Correlation of an Acoustic Vector Field in Surface-generated Noise

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Abstract: Spatial correlation of sound pressure and particle velocity of the surface noise in horizontally stratified media was demonstrated, with directional noise sources uniformly distributed on the ocean surface. In the evaluation of particle velocity, plane wave approximation was applied to each incident ray. Due to the equivalence of the sound source correlation property and its directivity, solutions for the spatial correlation of the field were transformed into the integration of the coherent function generated by a single directional source. As a typical horizontally stratified media, surface noise in a perfect waveguide was investigated. Correlation coefficients given by normal mode and geometric models show satisfactory agreement. Also, the normalized covariance between sound pressure and the vertical component of particle velocity is proportional to acoustic absorption coefficient, while that of the surface noise in semi-infinitely homogeneous space is zero.
Keywords: spatial correlation; sound pressure and particle velocity; surface noise; stratified media; ray theory
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1 Introduction

Signal-to-noise ratio (SNR) gain of a hydrophone array not only depends on the spatial correlation of the expected signals, but also strongly depends on the coherence structure of the noise (Yu *et al*., 2011). Properties of the noise are very important for hydrophone array design and performance enhancement.

As one of the important background interferences for active and passive sonar systems, ocean ambient noise has been investigated for many years, and it is still a widely discussed topic in underwater acoustics. During the past several decades, many researchers have done a large amount of work to reveal its spatial coherence, and various noise models have been demonstrated in research literature. The earlier models belong to the isotropic volume noise and the surface noise categories presented by Cron and Sherman (1962). Within a decade, another surface noise model in semi-infinitely homogeneous space was given (Cox, 1973). Unfortunately, some important acoustic parameters of the media were neglected in these models, such as sound velocity and properties of the ocean interface. It should be noted that these parameters always have significant effects on sound propagation in shallow water, which leads to varying sound field distribution. Perhaps because of this phenomenon spatial correlation of the surface noise in horizontally stratified media was eventually developed based on a normal mode (Kuperman and Ingenito, 1980). In 1996, surface noise in layered media was studied again and a model based on ray theory was proposed (Harrison, 1996). By that point, research on the spatial correlation of sound pressure for volume noise and surface noise had matured significantly.

With the development and application of the composite acoustic vector sensor, new challenges come into being during the design and performance analysis of a vector sensor array. Excluding the spatial coherence structure of sound pressure, properties of acoustic particle velocity of the interferences are also needed. Similar to the developmental progress of the scalar models, one simple vector model was first built with identical noise sources uniformly distributed on a spherical surface or in a sphere (Hawkes and Nehorai, 2001; Sun *et al*., 2003; Huang *et al*., 2009). Next, spatial correlation of sound pressure and particle velocity of the surface noise in a semi-infinitely homogeneous space was studied (Huang and Yang, 2010). At the same time, all these models were built in an idealized waveguide, and they cannot be used to depict the noise in a real ocean environment. Although one vector model of the surface noise in a range-independent waveguide was presented based on a normal mode (Huang *et al*., 2010), it is very difficult or even impossible to give explicit expressions for the channel with an irregular sound speed profile or irregular bottom interface. Another important reason is the deficiency of the comparable results or benchmark experimental data for model verification. It is necessary to look for a method to verify the current model or develop another new model. This is the motivation of the research where the surface noise in horizontally stratified media is developed based on a geometric model.

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The following contents contain the three sections. In the first section, mathematical formulas of the geometric model are derived. As a numerical example, surface noise in a perfect waveguide is analyzed in the second section and the covariance matrix and correlation coefficients are also presented. Comparisons between the normal mode model and the geometric model are made and some discussions are presented in the last section.

2 Mathematical derivation

According to ray theory (Yang, 2009), the sound field can be regarded as the summation of all eigenrays formulated in the product of the amplitude and complex phase. With respect to a source in a horizontal range r and azimuth angle ϕ in a cylindrical coordinate, complex amplitude of the field observed by a 3-D composite acoustic vector sensor can be given via Euler's formula with plane wave approximation. The latter is often introduced in acoustic vector signal processing and usually met by the practical sonar signal. For simplicity, the characteristic impedance of the media ρc in each velocity component is omitted, which does not alter the coherent property of the field.

$$\boldsymbol{\psi}(z_r, r) = \sum_p A_p(z_r, r, \theta_r) \mathrm{e}^{\mathrm{i}B_p(z_r, r, \theta_r)} \boldsymbol{U} \tag{1}$$

where, $\boldsymbol{U} = \{1 \quad \cos\theta_r \cos\varphi \quad \cos\theta_r \sin\varphi \quad \sin\theta_r\}^{\mathrm{T}}$ is comprised of the directionality functions of the components; z_r is the observing depth; θ_r is the grazing angle of the ray at the observer; $A_p(z_r, r, \theta_r)$ and $B_p(z_r, r, \theta_r)$ are the amplitude and phase of the p th multipath, respectively. For ocean ambient noise, the total pressure can be obtained by summing the wave radiated from each source one by one, but the velocity field cannot be obtained directly via Euler's formula because the final combination directivity of the noise sources is unknown. Fortunately, noise sources are produced stochastically in the ocean. They do not have certain phase and amplitude relationships between each other and can be considered to be incoherent. Even if this is not the case, source correlation can be substituted by its directionality function (Liggett and Jacobson, 1965). This is very important for the derivation of correlation functions, and solutions for the spatial correlation of ocean ambient noise can be transformed into the integration of the correlation functions for a single directional source.

Considering the following noise model, noise sources uniformly distribute on the ocean surface, and each source has unit strength with the vertical directionality function $g(\theta_s)$, where θ_s denotes the ray grazing angle at the source. For a directional source, the sound field can be obtained easily from the expression of the field due to an omni-directional source. By multiplying the directionality function on the right side of Eq.(1) and integrating with respect to the source plane, spatial correlation of the acoustic vector field can be represented as

$$\boldsymbol{\rho}(d, \gamma) = q\int_0^\infty \int_0^{2\pi} \boldsymbol{\psi}(z_1, \boldsymbol{r}_1) \boldsymbol{\psi}^{\mathrm{H}}(z_2, \boldsymbol{r}_2) g^2(\theta_s) r \mathrm{d}r \mathrm{d}\varphi \tag{2}$$

In the above equation, the superscript symbol H denotes the complex conjugate transpose, q is the source number per unit area, d and γ are the separation and elevation angle of the line connecting the two arbitrary points $(\boldsymbol{r}_1, z_1)$ and $(\boldsymbol{r}_2, z_2)$ in space, and the azimuth is assumed to be ϕ_0. Keeping the cylindrical axis passing through the midpoint of the line and establishing its direction downwards, a schematic diagram of the surface noise is illustrated in Fig.1. The inset represents the projection of the two spatial points on xoy plane.

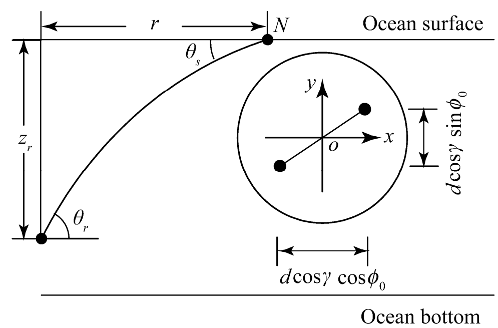

Fig.1 Schematic diagram of the surface noise model

It can be seen that the directionality vector of the acoustic vector sensor is a function of the spatial coordinates. This enhances the integral complexity of the correlation function. Since the separation between two observers is much smaller compared with the ray travel range from the surface or the bottom to the observer, the same type of rays arriving at the observers are parallel. Under this hypothesis, directionality vectors for both sensors are identical and the integration can be simplified greatly. Substituting Eq.(1) into Eq.(2) and combining with the geometric relations, the phase difference between each pair of rays can be treated as $kd\cos\xi$. Here k is the media wave number and ξ is the complement of the incident angle. Similar to the normal mode expression of sound intensity, there are both coherent and incoherent terms in the integrand. As for the phase difference between different types of rays constantly oscillating, coherent terms are not the dominant part in the integrand and their summation can be neglected before the integral calculation. A simplified correlation function can be deduced.

$$\boldsymbol{\rho}(d, \gamma) = \int_0^\infty \int_0^{2\pi} \sum_p \left| A_p(z_r, r, \theta_r) \right|^2 \times \mathrm{e}^{\mathrm{i}kd\cos\xi} \boldsymbol{U}\boldsymbol{U}^{\mathrm{H}} g^2(\theta_s) r \mathrm{d}r \mathrm{d}\varphi \tag{3}$$

where the cosine function of the incident grazing angle can be expressed as the function of elevation angle γ, grazing angle θ_r, azimuth angle ϕ, and ϕ_0 as follows

$$\cos\xi = \sin\theta_r \sin\gamma + \cos\theta_r \cos\gamma \cos(\varphi - \varphi_0) \qquad (4)$$

It is obvious that the square term in Eq.(3) represents ray intensity. We can substitute the term with the acoustic intensity formula given by Liu and Lei (2006), and the integral with respect to the horizontal range r will be transformed into the integral with elevation angle θ_r. For constant sound speed media, its lower and upper boundary values are $\pm\pi/2$. For other complex sound speed profiles, Snell's law can be used to determine the limits. In the following analysis, only the range-independent media with monotonically depth-varied speed is considered. Therefore, it only needs to determine the upper limit and use an unsigned variable $\theta_{\max}$ for denotation. One general expression of the correlation function in horizontally stratified media is finally obtained.

$$\boldsymbol{\rho}(d,\gamma) = \int_0^{2\pi}\int_{-\theta_{\max}}^{+\theta_{\max}} \mathrm{e}^{\mathrm{i}kd\cos\xi} \boldsymbol{Q}\boldsymbol{U}\boldsymbol{U}^{\mathrm{H}} g^2(\theta_s) \times \frac{1}{1-R_s(\theta_s)R_b(\theta_b)e^{-\alpha s_c}} \frac{\cos\theta_r}{|\sin\theta_s|} \mathrm{d}\theta_r \mathrm{d}\varphi \qquad (5)$$

Here, R_s and R_b are the energy reflection coefficients of the wave on the surface and bottom interface, respectively; α is the media absorption coefficient; s_c is the ray travel range in one cycle, and Q is the absorption attenuation for the ray propagating from the surface to the observer or from the observer to the surface via the bottom reflection. Before the analysis of the noise in horizontally stratified media, one special case is investigated. The bottom vanishes or the depth of the media tends to infinity, and the absorption attenuation of the media equals zero. With these hypotheses, the reflection coefficient on the surface and the exponential function of the absorption are equal to a unit, and the reflection coefficient on the bottom is zero. If the media speed is constant, Eq.(5) can be rewritten as

$$\boldsymbol{\rho}(d,\gamma) = \int_0^{2\pi}\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \mathrm{e}^{\mathrm{i}kd\cos\xi} \boldsymbol{U}\boldsymbol{U}^{\mathrm{H}} g^2(\theta_s) \frac{\cos\theta_r}{|\sin\theta_s|} \mathrm{d}\theta_r \mathrm{d}\varphi \qquad (6)$$

In fact, Eq.(6) is the correlation function of the surface noise in semi-infinitely homogeneous space, It is consistent with the result presented by Huang and Yang (2010).

Now the following case is considered. It is supposed that sound speed in the media is constant and the directionality function of the noise source is $\sin\theta_s$, a situation which is often introduced in research literature (Harrison, 1997). The exponential function is also expanded into the series of Bessel functions (Guo, 1965). The integral of ϕ can be calculated first according to the orthogonal property of the trigonometric function. In order to analyze the characteristics of the correlation function more explicitly and conveniently, Eq.(6) is expanded into the separate formulas below. The natural number in the subscript corresponds to sound pressure and three components of particle velocity in order.

$$\rho_{11} = \int_0^{\frac{\pi}{2}} \frac{\mathrm{e}^{\mathrm{i}kd\sin\theta_r\sin\gamma}\mathrm{e}^{-\alpha s_p} + R_b\mathrm{e}^{-\mathrm{i}kd\sin\theta_r\sin\gamma}\mathrm{e}^{-\alpha(s_c-s_p)}}{1-R_sR_b\mathrm{e}^{-\alpha s_c}} \times 2\pi\mathrm{J}_0(kd\cos\theta_r\cos\gamma)\sin\theta_s\cos\theta_r\mathrm{d}\theta_r$$

$$\rho_{22} = \int_0^{\frac{\pi}{2}} \frac{\mathrm{e}^{\mathrm{i}kd\sin\theta_r\sin\gamma}\mathrm{e}^{-\alpha s_p} + R_b\mathrm{e}^{-\mathrm{i}kd\sin\theta_r\sin\gamma}\mathrm{e}^{-\alpha(s_c-s_p)}}{1-R_sR_b\mathrm{e}^{-\alpha s_c}} \times \pi[\mathrm{J}_0(kd\cos\theta_r\cos\gamma) - \mathrm{J}_2(kd\cos\theta_r\cos\gamma)\cos(2\varphi_0)] \times \sin\theta_s\cos^3\theta_r\mathrm{d}\theta_r$$

$$\rho_{33} = \int_0^{\frac{\pi}{2}} \frac{\mathrm{e}^{\mathrm{i}kd\sin\theta_r\sin\gamma}\mathrm{e}^{-\alpha s_p} + R_b\mathrm{e}^{-\mathrm{i}kd\sin\theta_r\sin\gamma}\mathrm{e}^{-\alpha(s_c-s_p)}}{1-R_sR_b\mathrm{e}^{-\alpha s_c}} \times \pi[\mathrm{J}_0(kd\cos\theta_r\cos\gamma) + \mathrm{J}_2(kd\cos\theta_r\cos\gamma)\cos(2\varphi_0)] \times \sin\theta_s\cos^3\theta_r\mathrm{d}\theta_r$$

$$\rho_{44} = \int_0^{\frac{\pi}{2}} \frac{\mathrm{e}^{\mathrm{i}kd\sin\theta_r\sin\gamma}\mathrm{e}^{-\alpha s_p} + R_b\mathrm{e}^{-\mathrm{i}kd\sin\theta_r\sin\gamma}\mathrm{e}^{-\alpha(s_c-s_p)}}{1-R_sR_b\mathrm{e}^{-\alpha s_c}} \times 2\pi\mathrm{J}_0(kd\cos\theta_r\cos\gamma)\sin\theta_s\sin^2\theta_r\cos\theta_r\mathrm{d}\theta_r$$

$$\rho_{12} = \int_0^{\frac{\pi}{2}} \frac{\mathrm{e}^{\mathrm{i}kd\sin\theta_r\sin\gamma}\mathrm{e}^{-\alpha s_p} + R_b\mathrm{e}^{-\mathrm{i}kd\sin\theta_r\sin\gamma}\mathrm{e}^{-\alpha(s_c-s_p)}}{1-R_sR_b\mathrm{e}^{-\alpha s_c}} \times 2\pi\mathrm{i}\cos\varphi_0\mathrm{J}_1(kd\cos\theta_r\cos\gamma)\sin\theta_s\cos^2\theta_r\mathrm{d}\theta_r$$

$$\rho_{13} = \int_0^{\frac{\pi}{2}} \frac{\mathrm{e}^{\mathrm{i}kd\sin\theta_r\sin\gamma}\mathrm{e}^{-\alpha s_p} + R_b\mathrm{e}^{-\mathrm{i}kd\sin\theta_r\sin\gamma}\mathrm{e}^{-\alpha(s_c-s_p)}}{1-R_sR_b\mathrm{e}^{-\alpha s_c}} \times 2\pi\mathrm{i}\sin\varphi_0\mathrm{J}_1(kd\cos\theta_r\cos\gamma)\sin\theta_s\cos^2\theta_r\mathrm{d}\theta_r$$

$$\rho_{14} = \int_0^{\frac{\pi}{2}} \frac{\mathrm{e}^{\mathrm{i}kd\sin\theta_r\sin\gamma}\mathrm{e}^{-\alpha s_p} - R_b\mathrm{e}^{-\mathrm{i}kd\sin\theta_r\sin\gamma}\mathrm{e}^{-\alpha(s_c-s_p)}}{1-R_sR_b\mathrm{e}^{-\alpha s_c}} \times 2\pi\mathrm{J}_0(kd\cos\theta_r\cos\gamma)\sin\theta_s\sin\theta_r\cos\theta_r\mathrm{d}\theta_r$$

$$\rho_{23} = -\int_0^{\frac{\pi}{2}} \frac{\mathrm{e}^{\mathrm{i}kd\sin\theta_r\sin\gamma}\mathrm{e}^{-\alpha s_p} + R_b\mathrm{e}^{-\mathrm{i}kd\sin\theta_r\sin\gamma}\mathrm{e}^{-\alpha(s_c-s_p)}}{1-R_sR_b\mathrm{e}^{-\alpha s_c}} \times \pi\sin(2\varphi_0)\mathrm{J}_2(kd\cos\theta_r\cos\gamma)\sin\theta_s\cos^3\theta_r\mathrm{d}\theta_r$$

$$\rho_{24} = \int_0^{\frac{\pi}{2}} \frac{\mathrm{e}^{\mathrm{i}kd\sin\theta_r\sin\gamma}\mathrm{e}^{-\alpha s_p} - R_b\mathrm{e}^{-\mathrm{i}kd\sin\theta_r\sin\gamma}\mathrm{e}^{-\alpha(s_c-s_p)}}{1-R_sR_b\mathrm{e}^{-\alpha s_c}} \times 2\pi\mathrm{i}\cos\varphi_0\mathrm{J}_1(kd\cos\theta_r\cos\gamma)\sin\theta_s\sin\theta_r\cos^2\theta_r\mathrm{d}\theta_r$$

$$\rho_{34} = \int_0^{\frac{\pi}{2}} \frac{\mathrm{e}^{\mathrm{i}kd\sin\theta_r\sin\gamma}\mathrm{e}^{-\alpha s_p} - R_b\mathrm{e}^{-\mathrm{i}kd\sin\theta_r\sin\gamma}\mathrm{e}^{-\alpha(s_c-s_p)}}{1-R_sR_b\mathrm{e}^{-\alpha s_c}} \times 2\pi\mathrm{i}\sin\varphi_0\mathrm{J}_1(kd\cos\theta_r\cos\gamma)\sin\theta_s\sin\theta_r\cos^2\theta_r\mathrm{d}\theta_r$$

Here, s_p is the ray travel range from the surface to the observer. The above functions imply that not only the directional properties of the source may influence the structure of the field, but the acoustic parameters of the boundaries and water volume will also have a significant effect on areas such as reflection coefficients, the sound speed profile, and absorption coefficient of the media. With the power order of the directionality function increasing, correlation functions of the field oscillate more quickly. But no matter how complicated the directionality function is, correlation functions always have the characteristics of the first kind of Bessel function. These results show once again that ocean environmental parameters will affect the correlation properties of the noise field.

3 Surface noise in a perfect waveguide

3.1 Theoretical analysis

As a simple but representative case, surface noise in a perfect waveguide is introduced, with a pressure release surface above and a rigid bottom below. Sound speed is also constant. Although this case does not exist anywhere in the world, it is fit for the model verification and the comparison can be made with the normal mode model presented by Huang *et al*. (2010). On the other hand, the absorption coefficient of the media is very small for low frequency sound; thus, the following equations are obtained.

$$R_s = R_b = 1, \quad \theta_s = \theta_r, \quad s_c = \frac{2H}{\sin\theta_r}, \quad 1-\mathrm{e}^{-\alpha s_c} \approx \alpha s_c \tag{7}$$

Here H is the media depth. Substitute Eq.(7) into the separate formulas of Eq.(6) and omit the constant term of $\pi/\alpha H$, the real correlation functions are given.

$$\rho_{11} = 2\int_0^{\frac{\pi}{2}} \cos(kd\sin\theta_r\sin\gamma)\mathrm{J}_0(kd\cos\theta_r\cos\gamma)\times \sin^2\theta_r\cos\theta_r\mathrm{d}\theta_r$$

$$\rho_{22} = \int_0^{\frac{\pi}{2}} \cos(kd\sin\theta_r\sin\gamma)\sin^2\theta_r\cos^3\theta_r\mathrm{d}\theta_r\times [\mathrm{J}_0(kd\cos\theta_r\cos\gamma)-\mathrm{J}_2(kd\cos\theta_r\cos\gamma)\cos(2\varphi_0)]$$

$$\rho_{33} = \int_0^{\frac{\pi}{2}} \cos(kd\sin\theta_r\sin\gamma)\sin^2\theta_r\cos^3\theta_r\mathrm{d}\theta_r\times [\mathrm{J}_0(kd\cos\theta_r\cos\gamma)+\mathrm{J}_2(kd\cos\theta_r\cos\gamma)\cos(2\varphi_0)]$$

$$\rho_{44} = \int_0^{\frac{\pi}{2}} \cos(kd\sin\theta_r\sin\gamma)\sin^4\theta_r\cos\theta_r\mathrm{d}\theta_r\times 2\mathrm{J}_0(kd\cos\theta_r\cos\gamma)$$

$$\rho_{12} = -\int_0^{\frac{\pi}{2}} \sin(kd\sin\theta_r\sin\gamma)\sin\theta_r\cos^2\theta_r\mathrm{d}\theta_r\times 2\alpha(H-z_r)\cos\varphi_0\mathrm{J}_1(kd\cos\theta_r\cos\gamma)$$

$$\rho_{13} = -\int_0^{\frac{\pi}{2}} \sin(kd\sin\theta_r\sin\gamma)\sin\theta_r\cos^2\theta_r\mathrm{d}\theta_r\times 2\alpha(H-z_r)\sin\varphi_0\mathrm{J}_1(kd\cos\theta_r\cos\gamma)$$

$$\rho_{14} = \int_0^{\frac{\pi}{2}} \cos(kd\sin\theta_r\sin\gamma)\sin^2\theta_r\cos\theta_r\mathrm{d}\theta_r\times 2\alpha(H-z_r)\mathrm{J}_0(kd\cos\theta_r\cos\gamma)$$

$$\rho_{23} = -\int_0^{\frac{\pi}{2}} \cos(kd\sin\theta_r\sin\gamma)\sin^2\theta_r\cos^3\theta_r\mathrm{d}\theta_r\times \sin(2\varphi_0)\mathrm{J}_2(kd\cos\theta_r\cos\gamma)$$

$$\rho_{24} = -\int_0^{\frac{\pi}{2}} \sin(kd\sin\theta_r\sin\gamma)\sin^3\theta_r\cos^2\theta_r\mathrm{d}\theta_r\times 2\cos\varphi_0\mathrm{J}_1(kd\cos\theta_r\cos\gamma)$$

$$\rho_{34} = -\int_0^{\frac{\pi}{2}} \sin(kd\sin\theta_r\sin\gamma)\sin^3\theta_r\cos^2\theta_r\mathrm{d}\theta_r\times 2\sin\varphi_0\mathrm{J}_1(kd\cos\theta_r\cos\gamma)$$

All of the above formulas show that the coherence structure of the field does not depend on the absolute range or depth. In the normal mode model, the cross spectral density functions are of the functions of source and receiver depth. Variance of the separate quantities and covariance between particle velocity components decreases with the increase of the media depth and absorption coefficient linearly, while covariance between sound pressure and particle velocity is incoherent to the absorption coefficient, but linearly decreases with the increase of the observer depth. Another interesting feature worthy of emphasizing is that cross-correlation between the sound pressure and vertical component of particle velocity is identical to the auto-correlation of sound pressure. The proportional constant is just the covariance of the former. Letting d equal zero, the covariance matrix of the noise received by a three-dimensional single vector sensor can be obtained. It is consistent with the results calculated with the normal mode model.

$$R = \begin{bmatrix} 1 & 0 & 0 & x \\ 0 & \frac{1}{5} & 0 & 0 \\ 0 & 0 & \frac{1}{5} & 0 \\ x & 0 & 0 & \frac{3}{5} \end{bmatrix}, \quad x = \alpha(H-z_r) \tag{8}$$

Eq.(8) indicates that the normalized covariance between the sound pressure and vertical component of particle velocity is proportional to the media absorption coefficient, while the quantity of the surface noise in a semi-infinitely homogeneous space is a nonzero constant. Although the result is derived from the ideal model, it can be predicted that covariance of the real ocean surface noise has analogous characteristics for which it will be helpful to invert the media absorption coefficient.

3.2 Numerical simulation

Considering a numerical example, the following parameters are used. Media depth is 50 m, and sound speed is 1 500 m/s. The noise source radiates a harmonic sound wave at the frequency of 4 000 Hz. Referring to the sound absorption coefficient of seawater (Liu and Lei, 2006), the magnitude at this frequency is about 10^{-4} dB/m. It is small enough to satisfy the above approximation in the mathematical derivation. First, the analysis of the horizontal correlation properties of the field with normal mode model (Huang *et al*., 2010) and geometric model are taken. Because the cross spectral density functions in the normal mode model are of the function of the two observer depths, sound fields at two extreme depths are chosen in the following analysis. One is near to the surface and the other is close to the bottom. The horizontal correlation coefficients of the acoustic vector field at different depths are illustrated in Figs.2–4. Results obtained from the geometric model are shown with a dash line, and those obtained from the normal mode model at 5 m and 40 m are presented with a solid line and dotted line, respectively.

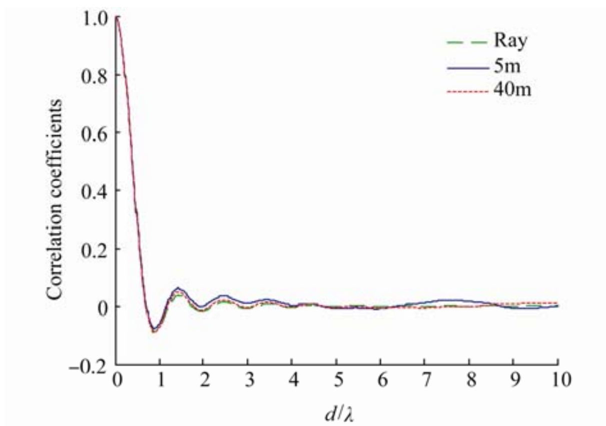

Fig.2 Horizontal correlation of sound pressure at different depth

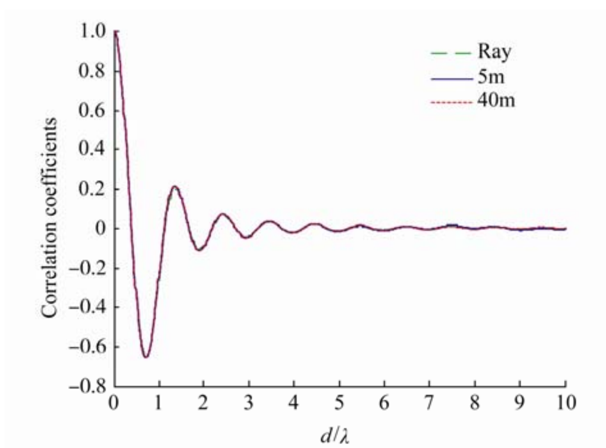

Fig.3 Horizontal correlation of horizontal component of particle velocity at different depth

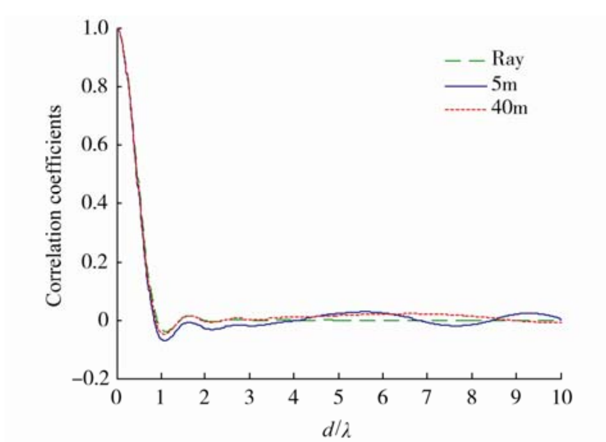

Fig.4 Horizontal correlation of vertical component of particle velocity at different depth

Correlation coefficients of the horizontal component of particle velocity agree well with each other for each depth. When the separation of the two observers is greater than one wavelength, correlation coefficients of sound pressure and the vertical component of the particle velocity deviate slightly from the normal mode model. However, if the observers are far from the boundaries, results of both models are consistent. For the vertical coherence of the field, analytical solutions can be obtained and results are shown in Figs.5–7. Whether the reference depth increases or decreases, the two models agree well with each other.

$$\rho_{11} = 2\frac{\sin(kd)}{kd} + 4\frac{\cos(kd)}{(kd)^2} - 4\frac{\sin(kd)}{(kd)^3}$$

$$\rho_{22} = -2\frac{\cos(kd)}{(kd)^2} + 10\frac{\sin(kd)}{(kd)^3} + 24\frac{\cos(kd)}{(kd)^4} - 24\frac{\sin(kd)}{(kd)^5}$$

$$\rho_{44} = 2\frac{\sin(kd)}{kd} + 8\frac{\cos(kd)}{(kd)^2} - 24\frac{\sin(kd)}{(kd)^3} - 48\frac{\cos(kd)}{(kd)^4} + 48\frac{\sin(kd)}{(kd)^5}$$

$$\rho_{14} = \alpha(H - z_r)\rho_{11}$$

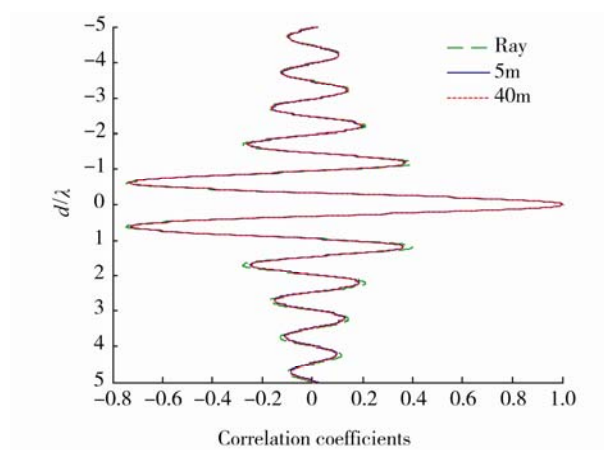

Fig.5 Vertical correlation of sound pressure at different observing depth

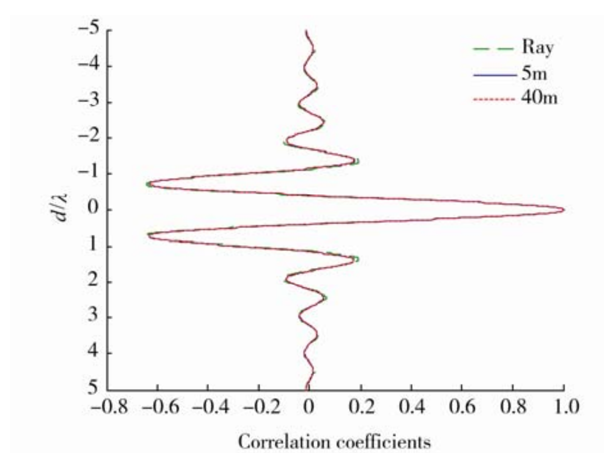

Fig.6 Vertical correlation of horizontal component of particle velocity at different observing depth

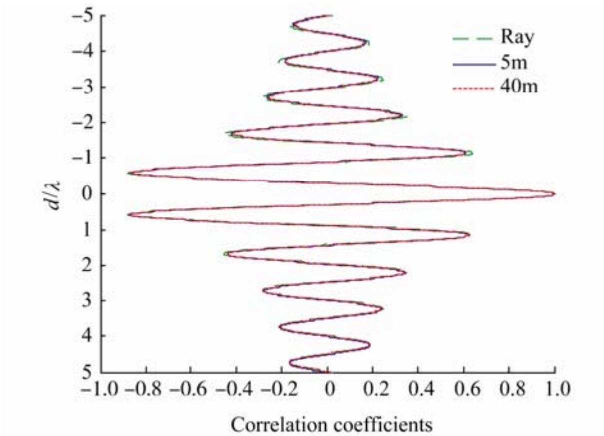

Fig.7 Vertical correlation of vertical component of particle velocity at different observing depth

Comparisons from these figures indicate that oscillation attenuation of the correlation functions in a vertical direction is much slower than in a horizontal direction. From this point of view, signal detection in a horizontal direction would be better.

4 Conclusions

An acoustic vector model for the surface noise in a horizontally stratified media has been developed based on ray theory. With respect to plane wave approximation for each eigenray in a far field and the properties of the noise source in a real ocean environment, integration formulas of the spatial correlation were derived and the coherence structure of the surface noise was obtained. According to a numerical example of the surface noise in a perfect waveguide, comparisons between the normal mode model and the geometric model were made. The feasibility of the geometric model developed in the paper was verified.

In horizontally stratified media, only for several kinds of sound speed profiles can the normal mode model give analytical solutions, while the geometric model will have no particular requirement for the profiles. It is not necessary to know the separate acoustic parameters of the boundaries, except for the energy reflection coefficient. For most cases, although only the integral expressions of the correlation functions can be obtained, all the integrals are of a single dimension and easily calculated via numerical methods. In some special cases, analytical solutions can be given. Moreover, the geometric model is intuitionistic. The cross correlation between the sound pressure and vertical component of particle velocity is the same as the auto correlation of the pressure. The normalized covariance is proportional to the media absorption coefficient, which is different from that of the surface noise in a semi-infinitely homogeneous space.

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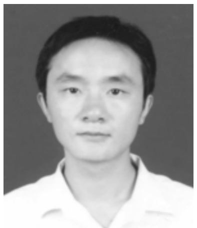

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