J. Marine Sci. Appl. (2012) 11: 74-82
DOI: 10.1007/s11804-012-1108-3

A Nonlinear Restoring Effect Study of Mooring System and its Application

Jian Zhang[1*], Huilong Ren[1] and Lijie Zhang[2]

1. College of Shipbuilding Engineering, Harbin Engineering University, Harbin 150001, China
2. Tianjin Centrans Shipping Management Co., Ltd., Tianjin 300000, China

Abstract: Mooring system plays an important role in station keeping of floating offshore structures. Coupled analysis on mooring-buoy interactions has been increasingly studied in recent years. At present, chains and wire ropes are widely used in offshore engineering practice. On the basis of mooring line statics, an explicit formulation of single mooring chain/wire rope stiffness coefficients and mooring stiffness matrix of the mooring system were derived in this article, taking into account the horizontal restoring force, vertical restoring force and their coupling terms. The nonlinearity of mooring stiffness was analyzed, and the influences of various parameters, such as material, displacement, pre-tension and water depth, were investigated. Finally some application cases of the mooring stiffness in hydrodynamic calculation were presented. Data shows that this kind of stiffness can reckon in linear and nonlinear forces of mooring system. Also, the stiffness can be used in hydrodynamic analysis to get the eigenfrequency of slow drift motions.
Keywords: coupled analysis; nonlinear restoring force; mooring system; lower frequency drift motion
Article ID: 1671-9433(2012)01-0074-09

1 Introduction

It is inevitable for a floating offshore structure to encounter external disturbance caused by ocean environment. Floating structures such as semi-submersible, FPSO, TLP, and SPAR must gain the ability to keep position, especially when they are in normal operation service. For the marine structures stated above, higher order wave loads are important. Second order hydrodynamic computation has become a hot-point for recent years, for example, Chen *et al*. (2006, 2009), Simos *et al*. (2008) and Ma *et al*. (2008). Unfortunately the buoy cannot withstand the drift forces all by itself, so a positioning system must be employed. Mooring system and dynamic positioning system are two main forms in ocean engineering practice. Coupled dynamic analysis between floating marine structures and flexible members such as mooring lines and risers, is a challenging task in the field of ocean engineering. Some investigations can be found, such as Kim *et al*. (2003, 2005), Carrett (2005), Cunff *et al*. (2008) and Low *et al*. (2008). Unlike free floating bodies, buoys with positioning system such as mooring lines will possess different mechanical behaviors. Mass, damping and stiffness are three main parameters in the differential equations of motion. Taking into account the mooring effects on a floating body, the mooring forces should be transformed into some forms of parameters stated above and be added in the motion equation. How and what to transform becomes the main distinction among so many methods, Ren *et al*. (2003) and Loukogeorgaki *et al*. (2005). The restoring effect of mooring system is investigated in this paper. Similar to hydrostatic restoring forces, mooring force related to the buoy's displacement can be transformed into mooring stiffness and can be calculated at its equilibrium point, Zhang (2008). For linear hydrodynamic analysis in frequency domain, any physical quantity should be linear or be linearized. However, mooring stiffness is nonlinear in essence, so the tangent or differential stiffness is employed. Due to the nonlinearity, the mooring stiffness will have a great change if the buoy's motion is large. For example, if the buoy shifts to a new equilibrium position due to drift forces, mooring stiffness must be calculated again. Many researchers have contributed their effort into studying the nonlinearity of mooring stiffness for so many years, for example, Huang *et al*. (1999), Kim *et al*. (2001), Couliard *et al*. (2001), Ren *et al*. (2003), Kim *et al*. (2003,2005), Loukogeorgaki *et al*. (2005), Carrett (2005), Cunff *et al*. (2008), Zhang (2008) and Low *et al*. (2008). So far chains and wire ropes are widely used in mooring system. An explicit mooring stiffness formulation of chain/wire rope is derived in this paper, which can be applied in offshore engineering design practice.

2 Mooring stiffness of single mooring line and mooring system

Similar to the calculation of hydrostatic restoring coefficients, mooring stiffness can also be calculated at its static equilibrium position. For a small length of chain/wire rope ds, the submerged weight per unit length is w. The following basic equations can be found in any book about mooring line mechanics covering axial stretch, so there is no

Received date: 2011-07-18.
Foundation item: Supported by the National Natural Science Foundation of China under Grant No.51079034.
***Corresponding author Email:** zhangjian_102@sina.com

need to give details here.

$$\mathrm{d}x = \mathrm{d}s(1+\varepsilon)\cos\phi \tag{1}$$

$$\mathrm{d}z = \mathrm{d}s(1+\varepsilon)\sin\varphi \tag{2}$$

$$\cos\varphi = \frac{T_H}{T} \tag{3}$$

$$\sin\varphi = \frac{T_V}{T} \tag{4}$$

$$\varepsilon = \frac{T}{EA} \tag{5}$$

$$T^2 = T_H^2 + T_V^2 \tag{6}$$

$$T_H = wa \tag{7}$$

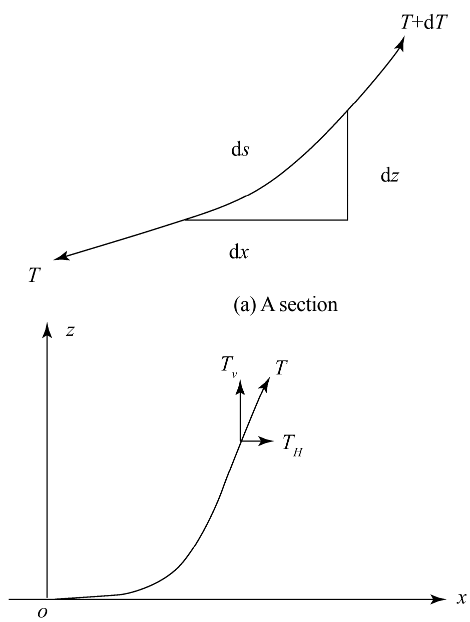
$$T_V = ws \tag{8}$$

where T, T_H, T_V, s and EA are axial tension, horizontal tension, vertical tension, unstretched length and the stiffness per unit length respectively. The coordinates (x, z) satisfy the following equation of parameter s.

$$\begin{aligned} x &= \frac{T_H}{EA}s + \frac{T_H}{w}\mathrm{sh}^{-1}\left(\frac{ws}{T_H}\right) \\ z &= \frac{w}{2EA}s^2 + \sqrt{\left(\frac{T_H}{w}\right)^2 + s^2} - \frac{T_H}{w} \end{aligned} \tag{9}$$

The restoring effect of mooring line can be considered as a spring, while the stiffness is nonlinear in essence. Thus, a tangent or differential stiffness is used so that it can be applied in frequency analysis or other linear methods.

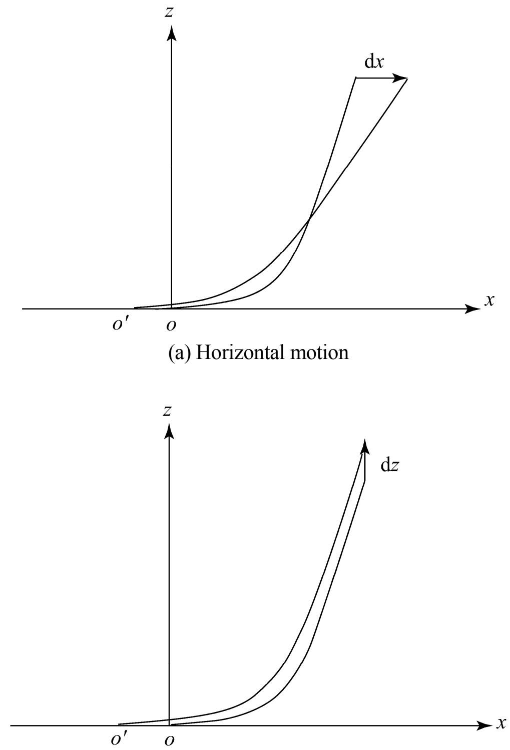
Let $(\mathrm{d}x \quad \mathrm{d}z)^{\mathrm{T}}$ be the displacement of mooring line's top, $(\mathrm{d}T_H \quad \mathrm{d}T_V)^{\mathrm{T}}$ be the restoring forces in the horizontal and vertical directions.



(a) A section

(b) Entire line

Fig.1 Geometry construction of single mooring line



(a) Horizontal motion

(b) Vertical motion

Fig.2 Top motion of mooring line

If the fairlead's displacement dx occurs, the Eq.(9) will be transformed as follows

$$\begin{cases} x+\delta x+\Delta = \dfrac{T_H+\delta T_H}{EA}(s+\delta s) + \dfrac{T_H+\delta T_H}{w}\mathrm{sh}^{-1}\left[\dfrac{w(s+\delta s)}{T_H+\delta T_H}\right] \\ z = \dfrac{w}{2EA}(s+\delta s)^2 + \sqrt{\left(\dfrac{T_H+\delta T_H}{w}\right)^2 + (s+\delta s)^2} - \dfrac{T_H+\delta T_H}{w} \\ \Delta = \left(1+\dfrac{T_H}{EA}\right)\delta s \end{cases} \tag{10}$$

Combining Eq.(9) with Eq.(10), and with $B=\dfrac{s}{a}$, the following equation can be obtained:

$$\frac{\mathrm{d}T_H}{\mathrm{d}s} = \lim_{\delta s\to 0}\frac{\delta T_H}{\delta s} = \frac{\dfrac{ws}{EA}\sqrt{1+B^2}+B}{\sqrt{1+B^2}-1} \tag{11}$$

$$\frac{\mathrm{d}x}{\mathrm{d}s} = \lim_{\delta s\to 0}\frac{\delta x}{\delta s} = \frac{w\sqrt{1+B^2}}{\mathrm{sh}^{-1}(B)\sqrt{1+B^2} + \dfrac{ws}{EA}\sqrt{1+B^2} + B - \dfrac{\left(\sqrt{1+B^2}-1\right)^2}{\dfrac{ws}{EA}\sqrt{1+B^2}+B}} \tag{12}$$

If the fairlead's displacement dz occurs, Eq.(9) will be transformed as follows:

$$\begin{cases} x+\Delta=\dfrac{T_H+\delta T_H}{EA}(s+\delta s)+\dfrac{T_H+\delta T_H}{w}\mathrm{sh}^{-1}\left[\dfrac{w(s+\delta s)}{T_H+\delta T_H}\right] \\ z+\delta z=\dfrac{w}{2EA}(s+\delta s)^2+\sqrt{\left(\dfrac{T_H+\delta T_H}{w}\right)^2+(s+\delta s)^2}-\dfrac{T_H+\delta T_H}{w} \\ \Delta=\left(1+\dfrac{T_H}{EA}\right)\delta s \end{cases} \tag{13}$$

Combining Eq.(13) with Eq.(9), the following equation results:

$$\frac{\mathrm{d}T_H}{\mathrm{d}s}=\lim_{\delta s\to 0}\frac{\delta T_H}{\delta s}=\frac{w\cdot\left(\sqrt{1+B^2}-1\right)}{\left(\dfrac{ws}{EA}+\mathrm{sh}^{-1}B\right)\sqrt{1+B^2}+B} \tag{14}$$

$$\frac{\mathrm{d}z}{\mathrm{d}s}=\lim_{\delta s\to 0}\frac{\delta z}{\delta s}=\frac{1}{\sqrt{1+B^2}}\left(\frac{ws}{EA}\sqrt{1+B^2}+B-\frac{\left(\sqrt{1+B^2}-1\right)^2}{\mathrm{sh}^{-1}B\sqrt{1+B^2}+\dfrac{ws}{EA}\sqrt{1+B^2}-B}\right) \tag{15}$$

The differential stiffness coefficients of a single mooring line are horizontal stiffness k_{xx}, vertical stiffness k_{zz} and their coupled terms k_{xz}, k_{zx}, shown as follows:

$$k_{xx}=\frac{\partial T_H}{\partial x}=\frac{\dfrac{\mathrm{d}T_H}{\mathrm{d}s}}{\dfrac{\mathrm{d}x}{\mathrm{d}s}} \tag{16}$$

$$k_{zx}=\frac{\partial T_V}{\partial x}=w\frac{\mathrm{d}s}{\mathrm{d}x}=\frac{w}{\dfrac{\mathrm{d}x}{\mathrm{d}s}} \tag{17}$$

$$k_{xz}=\frac{\partial T_H}{\partial z}=\frac{\dfrac{\mathrm{d}T_H}{\mathrm{d}s}}{\dfrac{\mathrm{d}z}{\mathrm{d}s}} \tag{18}$$

$$k_{zz}=\frac{\partial T_V}{\partial z}=w\frac{\mathrm{d}s}{\mathrm{d}z}=\frac{w}{\dfrac{\mathrm{d}z}{\mathrm{d}s}} \tag{19}$$

Substitute Eq.(11), (12), (14) and (15) into above equations, written in matrix form.

$$\boldsymbol{K}=\begin{bmatrix}k_{xx} & k_{zx}\\ k_{xz} & k_{zz}\end{bmatrix}=k_{xx}\begin{bmatrix}1 & \dfrac{\sqrt{1+B^2}-1}{\dfrac{ws}{EA}\sqrt{1+B^2}+B}\\ \dfrac{\sqrt{1+B^2}-1}{\dfrac{ws}{EA}\sqrt{1+B^2}+B} & \dfrac{1}{1+\dfrac{\mathrm{sh}^{-1}B\sqrt{1+B^2}-2B}{\dfrac{ws}{EA}\sqrt{1+B^2}+B}}\end{bmatrix} \tag{20}$$

In which

$$k_{xx}=\frac{w\sqrt{1+B^2}}{\left(\mathrm{sh}^{-1}B+\dfrac{ws}{EA}\right)\sqrt{1+B^2}-B-\dfrac{\left(\sqrt{1+B^2}-1\right)^2}{\dfrac{ws}{EA}\sqrt{1+B^2}+B}} \tag{21}$$

When $EA\to\infty$, or the influence of elasticity can be neglected, then, the stiffness matrix of a single mooring line tends to be

$$\boldsymbol{K}=\begin{bmatrix}k_{xx} & k_{xz}\\ k_{zx} & k_{zz}\end{bmatrix}=k_{xx}\begin{bmatrix}1 & \mathrm{th}\,D\\ \mathrm{th}\,D & \dfrac{E}{\mathrm{th}\,E}-1\end{bmatrix} \tag{22}$$

where $k_{xx}=\dfrac{w}{2(D-\mathrm{th}D)}$, $E=\dfrac{x}{a}$, $D=\dfrac{E}{2}$.

Let the ith fairlead's location be $\boldsymbol{r}=(x\quad y\quad z)^{\mathrm{T}}$, and the ith mooring line's orientation angle be θ. So some elements of the mooring stiffness matrix $\boldsymbol{K}^i$ are given as follows,

$$K_{11}=k_{xx}\cos^2\theta \tag{23}$$

$$K_{33}=k_{zz} \tag{24}$$

$$K_{55}=z^2k_{xx}\cos^2\theta+x^2k_{zz}-2xzk_{xz}\cos\theta \tag{25}$$

$$K_{12}=K_{21}=k_{xx}\cos\theta\sin\theta \tag{26}$$

$$K_{23}=K_{32}=k_{xz}\sin\theta \tag{27}$$

$$K_{34}=K_{43}=-zk_{xz}\sin\theta+yk_{zz} \tag{28}$$

$$\begin{gathered}K_{45}=K_{54}=-z^2k_{xx}\cos\theta\sin\theta+\\ xzk_{xz}\sin\theta+yzk_{xz}\cos\theta-xyk_{zz}\end{gathered} \tag{29}$$

$$\begin{gathered}K_{56}=K_{65}=-yzk_{xx}\cos^2\theta+\\ xzk_{xx}\cos\theta\sin\theta+xyk_{xz}\cos\theta-x^2k_{xz}\sin\theta\end{gathered} \tag{30}$$

For the entire mooring system consisting of N mooring lines, mooring stiffness matrix is

$$\boldsymbol{K}=\sum_{i=1}^{N}\boldsymbol{K}^i \tag{31}$$

Details of Eq.(22)'s derivation and each element in matrix $\boldsymbol{K}$ can be referred to Zhang (2008).

The parameter w and EA can be selected as follows

Table 1 Parameter selection of mooring lines

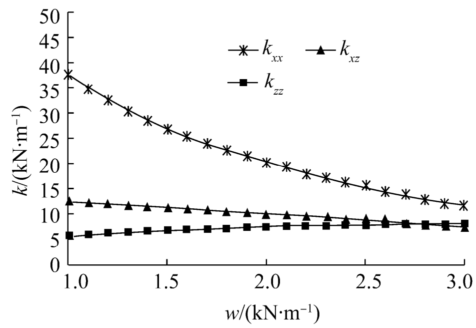
Construction	w /(N·m^{-1})	EA/N
Chain	0.187 5D^2	90 000D^2
Wire rope(six strand)	0.034D^2	45 000D^2
Wire rope(spiral strand)	0.043D^2	90 000D^2

D is diameter (in mm).

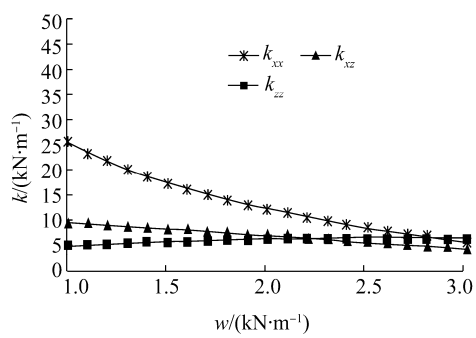
It is clear that the order of $\frac{w}{EA}$ is O(10^{-6}) for the chain and O(10^{-7}) for the wire rope. Usually the order of s is O(10^3–10^4), so $\frac{ws}{EA}$ is small. This explains why it is still effective to neglect elasticity for steel chains and wire ropes under some circumstances.

3 Sensitivity analysis of several parameters

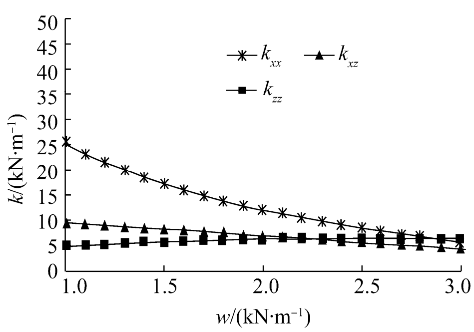
For a single mooring line, sensitivity analysis of mooring stiffness k_{xx}, k_{xz} (or k_{zx}) and k_{zz} is performed, regarding the following parameters, displacement x, water depth h, pretension T_H and the submerged weight per unit length w, seen from Fig.3 to Fig.6.



(a) h=100 m, T_H=500 kN

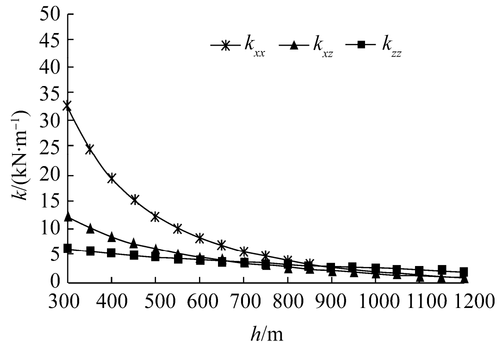


(b) h=500 m, T_H=2 000 kN

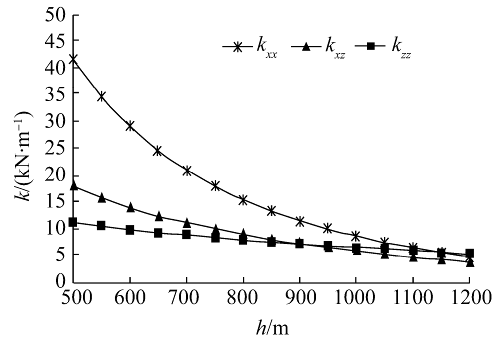


(c) h=1 000 m, T_H=4 000 kN

Fig.3 Sensitivity analysis of submerged weight

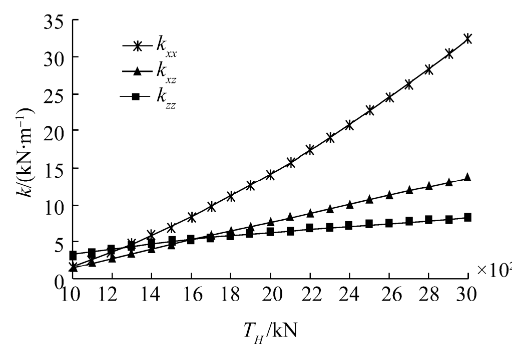


(a) w=1.2 kN/m, T_H=1 500 kN

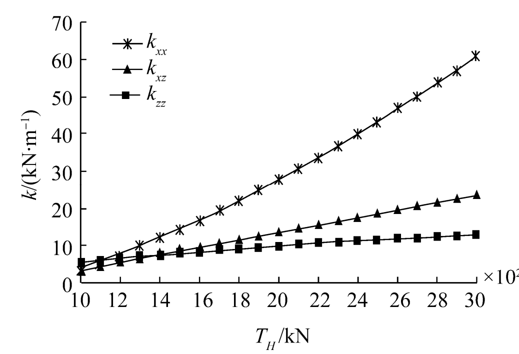


(b) w=2.5 kN/m, T_H=4 000 kN

Fig.4 Sensitivity analysis of water depth

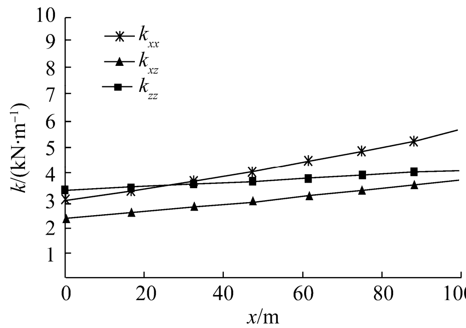


(a) w=1.8 kN/m, h=500 m

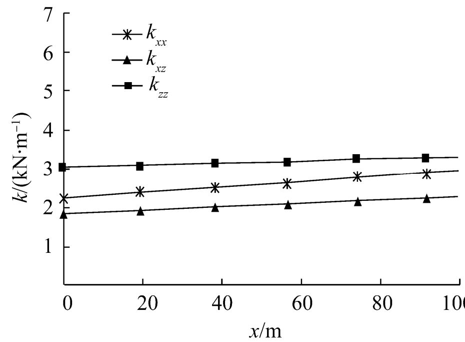


(b) w=2.6 kN/m, h=300 m

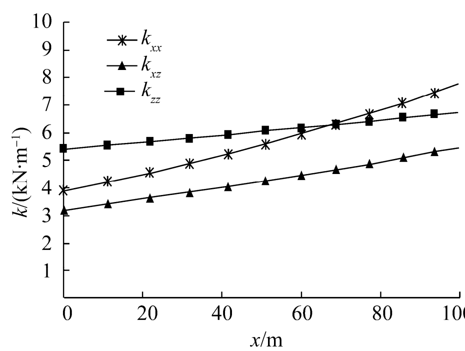
Fig.5 Sensitivity analysis of pretesion



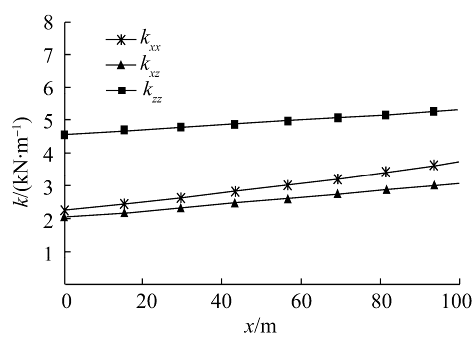
(a) w=1.5 kN/m, h=300 m, T_H =600 kN



(b) w=1.5 kN/m, h=800 m, T_H =1 500 kN



(c) w=2.7 kN/m, h=300 m, T_H =1 000 kN



(d) w=2.7 kN/m, h=500 m, T_H =1 500 kN

Fig.6 Sensitivity analysis of displacement

We can find that the mooring efficacy is reduced as water depth increases. Pretension is key factor to mooring efficacy. However submerged weight per unit length causes different trends for horizontal and vertical stiffness, because vertical stiffness is controlled by vertical tension, or weight, while horizontal stiffness is controlled by horizontal tension. It is also clearly seen that the mooring stiffness is nonlinear, while for not large displacement the mooring stiffness can be expressed by a linear function of displacement. So the Taylor series expansion of nonlinear mooring stiffness at the first order was applied. The derivation is shown as follows

$$\boldsymbol{K}=\begin{bmatrix} k_{xx}+x\dfrac{\partial k_{xx}}{\partial x}+z\dfrac{\partial k_{xx}}{\partial z} & k_{xz}+x\dfrac{\partial k_{xz}}{\partial x}+z\dfrac{\partial k_{xz}}{\partial z} \\ k_{zx}+x\dfrac{\partial k_{zx}}{\partial x}+z\dfrac{\partial k_{zx}}{\partial z} & k_{zz}+x\dfrac{\partial k_{zz}}{\partial x}+z\dfrac{\partial k_{zz}}{\partial z} \end{bmatrix}=$$
$$\begin{bmatrix} k_{xx} & k_{xz} \\ k_{zx} & k_{zz} \end{bmatrix}+x\begin{bmatrix} \dfrac{\partial k_{xx}}{\partial x} & \dfrac{\partial k_{xz}}{\partial x} \\ \dfrac{\partial k_{zx}}{\partial x} & \dfrac{\partial k_{zz}}{\partial x} \end{bmatrix}+z\begin{bmatrix} \dfrac{\partial k_{xx}}{\partial z} & \dfrac{\partial k_{xz}}{\partial z} \\ \dfrac{\partial k_{zx}}{\partial z} & \dfrac{\partial k_{zz}}{\partial z} \end{bmatrix}= \quad (32)$$
$$\boldsymbol{K}_0+x\boldsymbol{K}_x+z\boldsymbol{K}_z$$

$$\boldsymbol{K}_x=\frac{\partial k_{xx}}{\partial x}\begin{bmatrix} 1 & \mathrm{th}D \\ \mathrm{th}D & \dfrac{E}{\mathrm{th}E}-1 \end{bmatrix}+k_{xx}\begin{bmatrix} 0 & \dfrac{1-\mathrm{th}^2 D}{2} \\ \dfrac{1-\mathrm{th}^2 D}{2} & \dfrac{\mathrm{sh}E\,\mathrm{ch}E-E}{\mathrm{sh}E} \end{bmatrix}E_x \quad (33)$$

$$E_x=-\frac{1}{a}\cdot\frac{1}{2\left(\dfrac{D}{\mathrm{th}D}-1\right)} \quad (34)$$

$$\boldsymbol{K}_x=-\frac{wE_x}{4\left(\dfrac{D}{\mathrm{th}D}-1\right)^2}\begin{bmatrix} 1 & \dfrac{\mathrm{sh}E-E}{\mathrm{ch}E-1} \\ \dfrac{\mathrm{sh}E-E}{\mathrm{ch}E-1} & \left(\dfrac{\mathrm{sh}E-E}{\mathrm{ch}E-1}\right)^2 \end{bmatrix}=$$
$$\frac{w}{8a\left(\dfrac{D}{\mathrm{th}D}-1\right)^3}\begin{bmatrix} 1 & \dfrac{\mathrm{sh}E-E}{\mathrm{ch}E-1} \\ \dfrac{\mathrm{sh}E-E}{\mathrm{ch}E-1} & \left(\dfrac{\mathrm{sh}E-E}{\mathrm{ch}E-1}\right)^2 \end{bmatrix} \quad (35)$$

Let

$$f=\frac{\mathrm{sh}E-E}{\mathrm{ch}E-1} \quad (36)$$

$$\boldsymbol{K}_x=\frac{w}{8a\left(\dfrac{D}{\mathrm{th}D}-1\right)^3}\begin{bmatrix} 1 & f \\ f & f^2 \end{bmatrix} \quad (37)$$

Similar as Eq.(35),

$$\boldsymbol{K}_z=-\frac{wE_z}{4\left(\dfrac{D}{\mathrm{th}D}-1\right)^2}\begin{bmatrix} 1 & f \\ f & f^2 \end{bmatrix} \quad (38)$$

$$E_z=f\cdot E_x \quad (39)$$

$$\boldsymbol{K}_z=-\frac{wfE_x}{4\left(\dfrac{D}{\mathrm{th}D}-1\right)^2}\begin{bmatrix} 1 & f \\ f & f^2 \end{bmatrix}=f\boldsymbol{K}_x \quad (40)$$

So the Taylor series expansion of nonlinear mooring stiffness is

$$\boldsymbol{K}=\boldsymbol{K}_0+x\cdot\boldsymbol{K}_x+zf\cdot\boldsymbol{K}_x=\boldsymbol{K}_0+(x+zf)\cdot\boldsymbol{K}_x \quad (41)$$

It can be easily found that if first order derivative of mooring stiffness is considered, the stiffness will be a linear function of motion, and the restoring forces will be a square function of motion. Therefore, the mooring stiffness is nonlinear in essence.

For free floating structures working at sea, the motion equation is as follows:

$$(\boldsymbol{M}+\boldsymbol{A})\ddot{\boldsymbol{\eta}}(t)+\boldsymbol{B}\dot{\boldsymbol{\eta}}(t)+\boldsymbol{C}\boldsymbol{\eta}(t)=\boldsymbol{f}(t) \tag{42}$$

While for moored floating structures, the mooring stiffness $\boldsymbol{K}$ can be added into equation (42) as follows:

$$(\boldsymbol{M}+\boldsymbol{A})\ddot{\boldsymbol{\eta}}(t)+\boldsymbol{B}\dot{\boldsymbol{\eta}}(t)+(\boldsymbol{C}+\boldsymbol{K})\boldsymbol{\eta}(t)=\boldsymbol{f}(t) \tag{43}$$

4 Application cases

A semi-submersible operation in South China Sea was selected as a computation example. The mooring stiffness used here is constant, and the hydrodynamic computation is performed by WALCS, which employs BEM based on linear theory of 3-D potential flow in frequency domain. For the purpose of studying mooring stiffness, the RAOs of the semi-submersible's motion in moored and free-floating condition are computed respectively, with incident waves of 0°, 30°, 60° and 90° considered, as can be seen from Figs.7–10. Some parameters of the mooring system are as follows.

Table 2 Basic parameters of semi-submersible

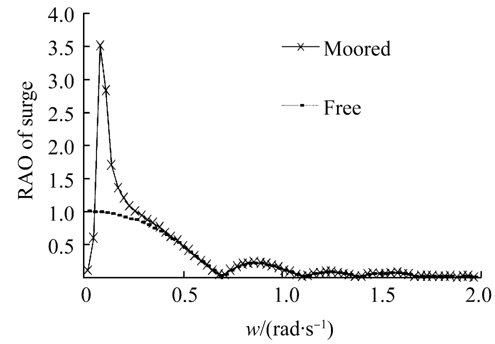
item	value
Main deck (L×W×H) /m^3	90×74×40
Pontoon (L×W×H) /m^3	90×15×6.4
Draft/m	22.86
Weight/t	28 000
Center of weight/m	(0,0,22)

Table 3 Basic parameters of mooring lines

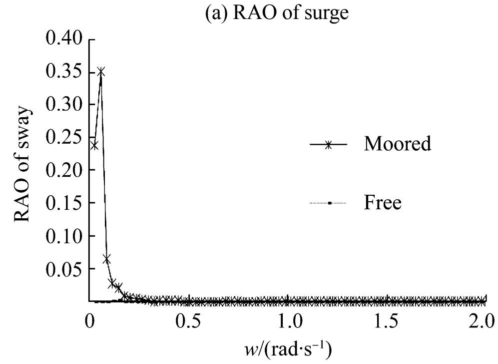
H/m	w /(kN·m^{-1})	T_H/kN
350	2.73	2/222

Table 4 Arrangement of mooring lines

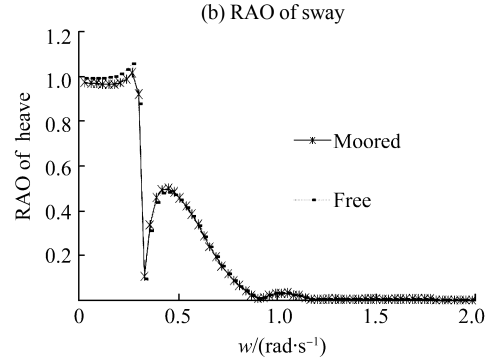
No.	Fairlead Cord./m	Angle/(°)
1	(45.1,29.7,−6.1)	0
2	(45.1,32.2, −6.1)	0
3	(35.5,37.1, −6.1)	90
4	(−34.3,37.1, −6.1)	90
5	(−45.1,32.2, −6.1)	180
6	(−45.1,29.7, −6.1)	180
7	(−45.1, −29.7, −6.1)	180
8	(−45.1, −32.2, −6.1)	180
9	(−34.3, −37.1, −6.1)	270
10	(35.5, −37.1, −6.1)	270
11	(45.1, −32.2, −6.1)	0
12	(45.1, −29.7, −6.1)	0



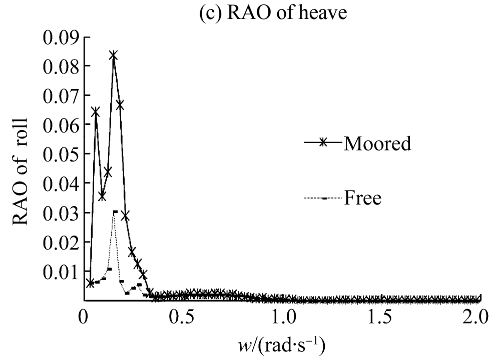
(a) RAO of surge



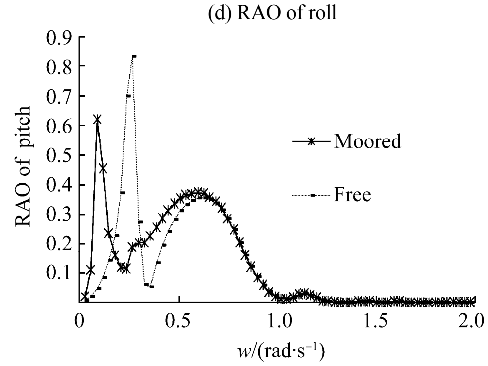
(b) RAO of sway



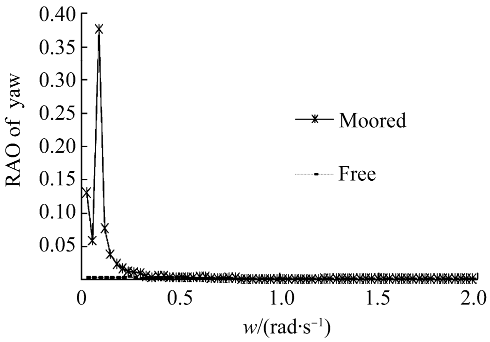
(c) RAO of heave



(d) RAO of roll

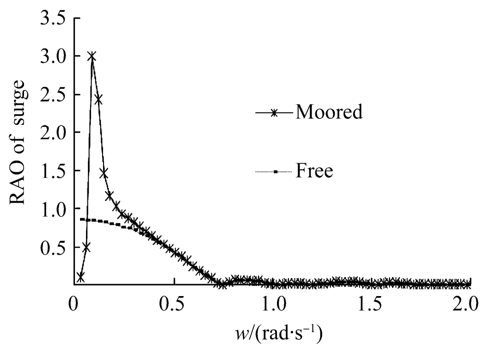


(e) RAO of pitch

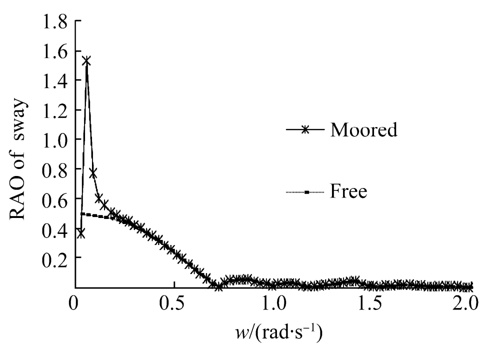


(f) RAO of yaw

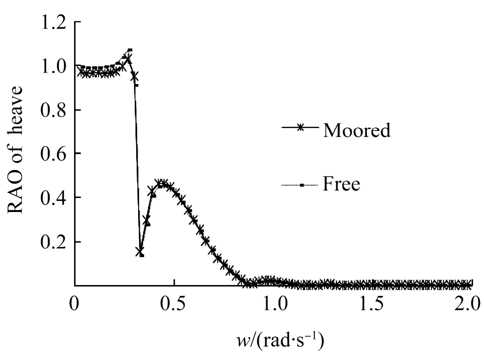
Fig.7 RAO of motion with 0º incident wave



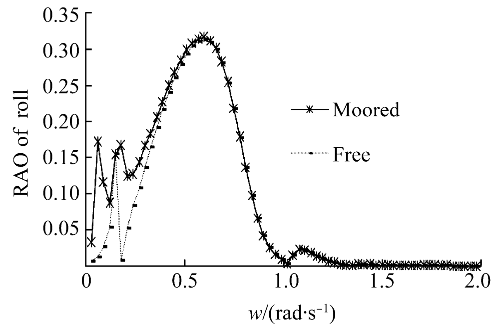
(a) RAO of surge



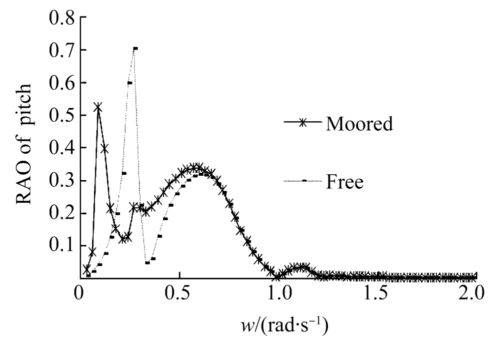
(b) RAO of sway



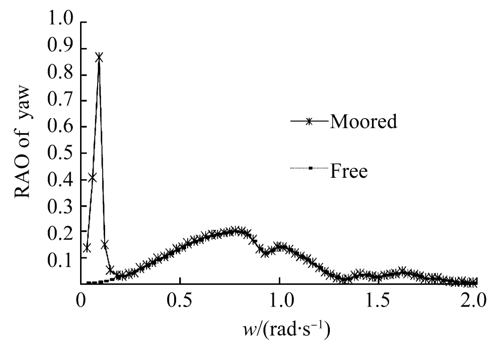
(c) RAO of heave



(d) RAO of roll

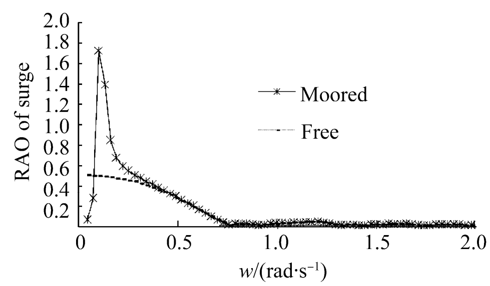


(e) RAO of pitch

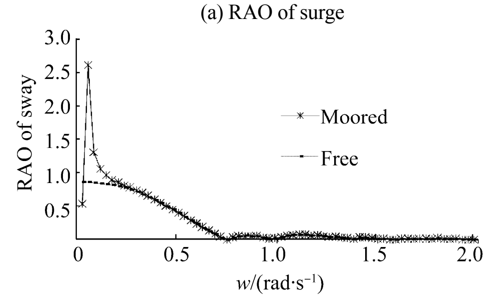


(f) RAO of yaw

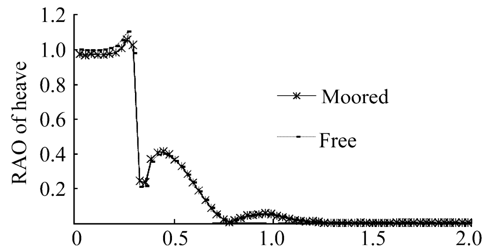
Fig.8 RAO of motion with 30º incident wave



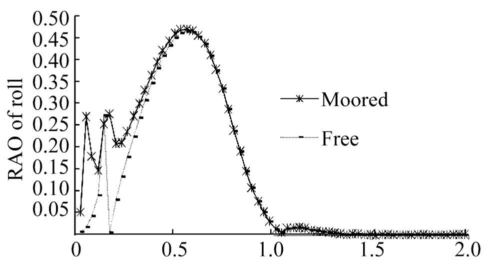
(a) RAO of surge



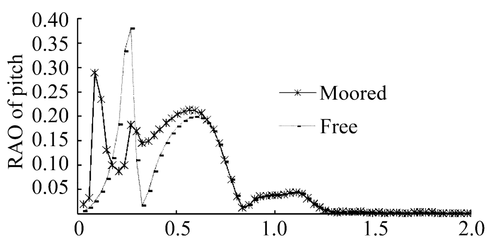
(b) RAO of sway



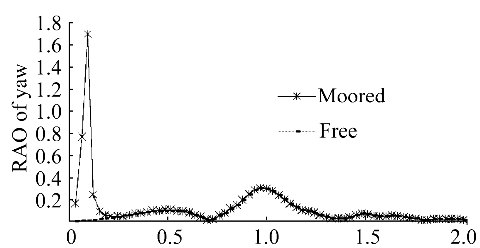
(c) RAO of heave



(d) RAO of roll

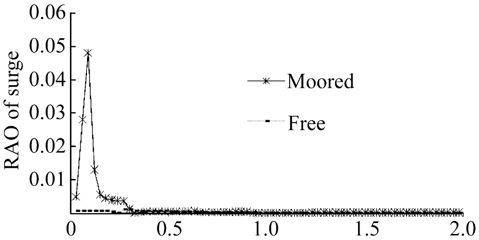


(e) RAO of pitch

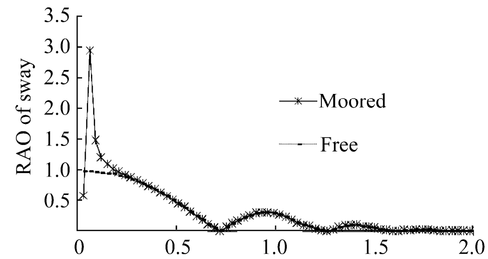


(f) RAO of yaw

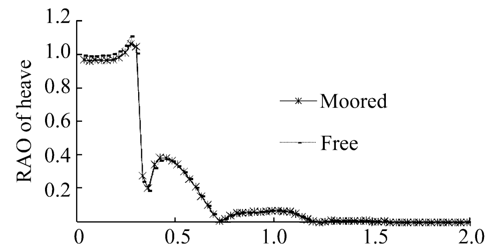
Fig.9 RAO of motion with 60º incident wave



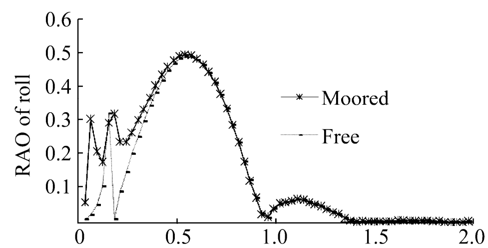
(a) RAO of surge



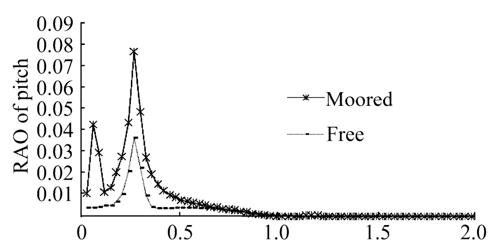
(b) RAO of sway



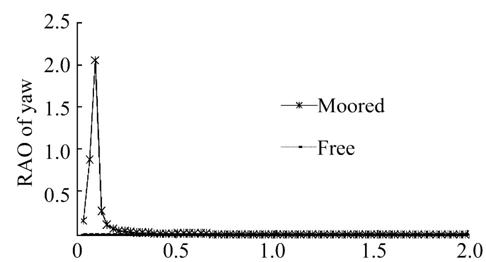
(c) RAO of heave



(d) RAO of roll



(e) RAO of pitch



(f) RAO of yaw

Fig.10 RAO of motion with 90º incident wave

It is well known that there are only three kinds of restoring forces for 6 DOF motions, or hydrostatic restoring coefficients of heave, roll, and pitch are nonzero. So there are only three significant modes for floating bodies under a free condition. Generally, mooring stiffness coefficients of all the six displacements are nonzero. Meanwhile, floating bodies such as semi-submersibles usually possess large mass and small stiffness, so floating structures with a mooring system will possess low natural frequency. For this reason, there is great difference between moored and free conditions at the low frequency region. However, the effect is not obvious on heave motion due to a large hydrostatic restoring coefficient compared with the mooring stiffness coefficient.

The RAOs showed above are very large near the resonance frequency, partially because the mooring damping is not reckoned in. On the other hand, data shows that for slow drift force at lower frequency, normal hydrodynamic damping is not enough to restrain slow drift resonance, and other kinds of damping must be employed, such as mooring damping. The slow drift problem is not mainly concerned here, only mooring stiffness is investigated.

5 Conclusions

An explicit formulation of mooring stiffness is derived in this article. It can be conveniently applied in offshore engineering practice, such as in hydrodynamic analysis. The comparison of a semi-submersible's RAOs in free floating and moored condition shows that mooring stiffness not only influence the platform's motion in a horizontal plane greatly (no restoring forces in free surge, sway and yaw), but also the roll, pitch and heave motion. An application of the mooring stiffness formulation can be made in a lower frequency drift problem, because the resonance frequency can be forecasted.

References

Chen XB (2006). Middle-field formulation for the computation of wave-drift loads. *Journal of Engineering Mathematics*. **59**, 61-82.

Chen XB, Rezende F (2009). Efficient computations of second-order low-frequency wave load. *International Conference on Ocean, Offshore and Arctic Engineering*, Honolulu, Hawaii, USA, 79522.

Couliard PY, Langley RS (2001). Nonlinear dynamics of deep-water moorings. *International Conference on Offshore Mechanics and Arctic Engineering*, Rio de Janeiro, 1020.

Cunff CL, Ryu S, Heurtier JM, Duggal AS (2008). Frequency-domain calculations of moored vessel motion including low frequency effect. *International Conference on Offshore Mechanics and Arctic Engineering, Estoril, Portugal*, 57632.

Garrett DL (2005). Coupled analysis of floating production system. *Ocean Engineering*, **32**(2), 802-816.

Huang XL, Fan J, Zhu XY (1999). The comparison between the frequency method and time domain method in the dynamic analysis of a mooring line. *International Conference on Offshore Mechanics and Arctic Engineering*, St. Johns, Canada, 4061.

Kim BK, Michael MB (2001). Nonlinear dynamics and stability of spread mooring with riser. *Applied Ocean Research*, **23**(1), 111-123.

Kim MH, Koo BJ, Mercier RM, Ward EG (2005). Vessel/mooring/riser coupled dynamic analysis of a turret-moored FPSO compared with OTRC experiment. *Ocean Engineering*, **32**(3), 1780-1802.

Li Runpei, Wang Zhinong (1992). *Strength analysis of ocean platforms*. Shanghai Jiao Tong University Press, Shanghai, 1-50. (in Chinese)

Loukogeorgaki E, Angelides DC (2005). Stiffness of mooring lines and performance of floating breakwater in three dimensions. *Applied Ocean Research*, **27**(3), 187–208.

Low YM, Langley RS (2008). A hybrid time/frequency domain approach for efficient coupled analysis of vessel/mooring/riser dynamics. *Ocean Engineering*, **35**(2), 433-446.

Ma Shan, Shi S, Kim MH (2008). Second-order low-frequency wave forces on a SPM offloading tanker in shallow water. *International Conference on Offshore Mechanics and Arctic Engineering*, Estoril, Portugal, 58048.

Ren Huilong, Zhang Haibin, Dai Yuzhi, Song Jingzheng (2003). 3-D computational method of wave loads on turret moored FPSO tankers. *Journal of Marine Science and Application*, **2**(1), 22-25.

Tahar A, Kim MH (2003). Hull/mooring/riser coupled dynamic analysis and sensitivity study of a tanker-based FPSO. *Applied Ocean Research*, **25**(2), 367-382.

Zhang Jian (2008). Coupled dynamic analysis of platform and flexible members in frequency domain. Master thesis, Harbin Engineering University, 35-43. (in Chinese)

Jian Zhang was born in 1982. He is a PhD candidate of Harbin Engineering University. His current research interests include coupled analisys between floating structures and mooring lines.

Huilong Ren was born in 1965. He is a professor of Harbin Engineering University. His current research interests include wave loads, structural strength and reliability.