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The Stress Combination Method for the Fatigue Assessment of the Hatch Corner of a Bulk Carrier Based on Equivalent Waves

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Abstract: The stress combination method for the fatigue assessment of the hatch corner of a bulk carrier was investigated based on equivalent waves. The principles of the equivalent waves of ship structures were given, including the determination of the dominant load parameter, heading, frequency, and amplitude of the equivalent regular waves. The dominant load parameters of the hatch corner of a bulk carrier were identified by the structural stress response analysis, and then a series of equivalent regular waves were defined based on these parameters. A combination method of the structural stress ranges under the different equivalent waves was developed for the fatigue analysis. The combination factors were obtained by least square regression analysis with the stress ranges derived from spectral fatigue analysis as the target value. The proposed method was applied to the hatch corner of another bulk carrier as an example. This shows that the results from the equivalent wave approach agree well with those from the spectral fatigue analysis. The workload is reduced substantially. This method can be referenced in the fatigue assessment of the hatch corner of a bulk carrier.

Keywords: stress combination method; equivalent wave; bulk carrier; hatch corner; fatigue strength assessment; ship structures
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1 Introduction

Fatigue as a mode of structural failure is a well known problem to ship owners and designers. Very intensive work has been performed to analyze fatigue failures (Feng, 2006). According to the rules of several classification societies (DNV, 2001; ABS, 2002; GL, 2000), simplified procedures and direct calculations are used to carry out fatigue analysis on ship structures. Since the wave loads and structural stresses are expressed in experience equations, the simplified method can easily be used. A more accurate method is direct fatigue analysis, including the spectral analysis and equivalent wave approach. The direct fatigue analysis of ship structures involves the calculation of wave-induced load through program and structural response analysis by finite element analysis. However, in the direct fatigue analysis, the spectral fatigue analysis requires a heavy workload and is time-consuming. Hence, the equivalent wave approach is worthy of further study Although the equivalent wave method is widely used to assess the overall yielding strength or buckling strength of ships and offshore structures (Zhan and Gu, 2002; Wang, 2008; Chen and Zhong, 2008; Gu *et al*., 1998; Feng *et al*., 2009), relatively little research has been conducted on fatigue assessment based on the equivalent wave approach. An equivalent wave approach has been proposed for the fatigue assessment of a Ro/Ro vessel (Feng and Ren, 2005). A permissible stress method, which was adopted under the guidance of the China classification society, for fatigue strength of a ship hull structure based on an equivalent wave is given (Zhan *et al*., 2009). By the equivalent wave approach, the components of loads or stresses can be combined to achieve good results with a low workload. In this paper, a stress combination method for the fatigue assessment of the hatch corner of a bulk carrier is investigated based on equivalent waves.

2 Principles of the equivalent wave approach

The equivalent wave is a regular wave under which the response of the ship structures corresponds to the long term response induced by the dominant loads. The outline of the approach is as follows (Dai *et al*., 2007),

The max value of the combination of variant loads occurs when one of the variant loads reaches its max value and the transient values of the other variant loads are taken, written as

$$Q_m = \max_{i=z}^{n}[\max_{T}(Q_i(t)) + \sum_{j=1}^{n} Q_j(t)] \tag{1}$$

where $\max_{T}(Q_i(t))$ is the maximum value of the load $Q_i(t)$ in the service period T, and $Q_j(t)$ is the transient value of other loads.

The component of wave loads, called the dominant load parameter, which has the greatest impact on the strength of

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the ship structures needs to be identified. It normally refers to the sectional wave loads and ship motions, including the wave induced vertical bending moment, vertical shear force, horizontal bending moment, horizontal shear force, torsion, and acceleration.

The long term value of the dominant load parameter X in the service period needs to be determined based on the wave spectrum and the wave scatter diagram.

The response amplitude operators of the dominant load parameter need to be calculated at a series of different frequencies and headings. The frequency ω and heading α corresponding to the maximum response amplitude operator of the dominant load parameter are taken as the frequency and heading of the equivalent wave.

The wave length of the equivalent wave λ is written as

$$\lambda = \frac{2\pi g}{\omega^2} \tag{2}$$

where g is the acceleration of gravity and ω the frequency of the equivalent wave.

Based on the long term value X of the dominant load parameter, the wave amplitude η of the equivalent wave is defined as

$$\eta = \frac{X}{A} \tag{3}$$

where X is the long term value of the dominant load parameter, and A the maximum response amplitude operator of the dominant load parameter corresponding to the frequency ω and heading α.

According to the theory of the regular wave, all components of the loads can be calculated or superposed based on the equivalent wave defined as above.

3 The reference stress ranges for fatigue assessment

The long term probability density function of the fatigue stress range S of ship structures can be described by Weibull distribution (Soares and Moan, 1991; Hu and Chen, 1996),

$$f_S(S) = \frac{h}{q}\left(\frac{S}{q}\right)^{h-1} \exp\left[-\left(\frac{S}{q}\right)^h\right] \tag{4}$$

where h and q are respectively the shape parameter and scale parameter of the Weibull distribution.

The scale parameter of the Weibull distribution is expressed as

$$q = \frac{S_0}{(\ln N_0)^{1/h}} \tag{5}$$

where N_0 is the return period of the fatigue loads, taken as 10^4 (IACS, 2006). S_0 is the stress range corresponding to the level of probability of exceedance 10^{-4}.

In accordance with the Miner rules, the cumulative damage D of the ship structures in the design life can be written as

$$D = \frac{N_L}{A} q^m \Gamma\left(1 + \frac{m}{h}\right) \tag{6}$$

$$N_L = \frac{0.85 T_L}{4 \log L} \tag{7}$$

where m and A are the parameters of the $S-N$ curve. $\Gamma(\cdot)$ is the Gamma function. N_L is the total number of cycles (IACS, 2006). T_L is the design life. L is the ship length.

Assuming the damage obtained by spectral fatigue analysis is D_s, the corresponding reference stress range S_0 can be derived based on Eq.(5) and Eq.(6) as

$$S_0 = (\ln N_0)^{1/h} \left[\frac{D_S A}{N_L \Gamma\left(1 + \frac{m}{h}\right)} \right]^{1/m} \tag{8}$$

where, S_0 is the reference stress range, N_0 is the return period of the fatigue loads, h is the shape parameter of the Weibull distribution, and m and A are the parameters of the $S-N$ curve. $\Gamma(\cdot)$ is the Gamma function, N_L the total number of cycles (IACS, 2006), and D_s the damage obtained by spectral fatigue analysis.

4 The stress ranges combination based on equivalent waves

It can be found that the damage from Eq.(6) is the same as the damage obtained by spectral fatigue analysis if the stress range achieved by the equivalent waves is equal to the reference stress range S_0 in Eq.(8). Consequently, the objective is how the reference stress range S_0 can be achieved by the equivalent waves. Because the fatigue stress ranges are induced simultaneously by various load components, the long term probability density function of the fatigue stress ranges need to be represented by the combination of the fatigue stress ranges induced by various load components. In view of this, a kind of combination method is developed.

An analysis is made of the stress response of the structural detail. Based on the structural stress response, the dominant load parameters of the structural detail are identified. In terms of the identified dominant load parameters, a group of equivalent waves is obtained in accordance with the principle of an equivalent wave. The stress ranges under the different equivalent waves are calculated. With the reference stress ranges derived from spectral fatigue analysis as the target value, the stress ranges resulting from different equivalent waves are combined applying a series of weight factors obtained by the least square regression analysis.

It can be generalized that the reference stress range S_0 is combined with the stress range from different equivalent waves defined by different dominant load parameters,

$$S_0 = \sqrt{A_1 S_1{}^2 + A_2 S_2{}^2 + A_3 S_3{}^2 + \ldots} \tag{9}$$

where S_1 , S_2 , S_3 are the stress ranges from equivalent waves based on different dominant load parameters. A_1 , A_2 , A_3 are the combination factors obtained by regression analysis.

5 The equivalent waves for the fatigue assessment of the hatch corner of a bulk carrier

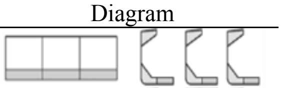
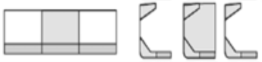
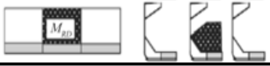
5.1 Fatigue reference stress ranges

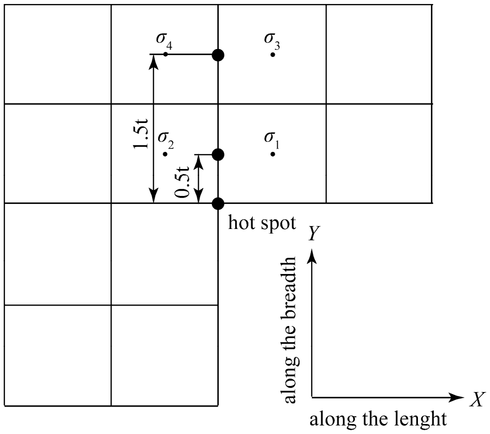
The hatch corners of the midship of five bulk carriers with a single shell and double bottoms are used to carry out the spectral fatigue analysis. The main dimensions of the ships are listed in Table 1. Four loading conditions typical of bulk carriers are considered, as shown in Table 2. In the spectral fatigue calculation, the world-wide scatter diagram and Pierson-Moskowitz wave spectrum (DNV, 2001) are adopted. The headings are from 0º to 360º with intervals of 30º. Wave frequencies are from 0.1 rad/s to 1.8 rad/s with intervals of 0.1 rad/s. The fatigue hot spot stress is obtained through the extrapolation of the stresses, as shown in Fig.1 along with the global finite element model of the ship with built-in fine mesh at the structural details of the hatch corner (IACS, 2006). An E class $S-N$ curve is applied (CCS, 2007). The stress range derived by Eq.(8), where the damage D_s is calculated by the spectral fatigue analysis method, is regarded as the target value of the reference stress ranges. The target values of the fatigue reference stress ranges of the hatch corner of the midship section are derived based on the results from the spectral fatigue analysis, as shown in Table 3.

Table 1 Main dimension of the five bulk carriers m

Ship	Length	Breadth	Depth
1	183.0	32.26	17.0
2	176.8	30.50	15.8
3	215.0	32.20	18.3
4	216.0	32.26	18.9
5	217.0	32.26	18.3

Table 2 Diagram of loading conditions

Loading condition	Diagram
Light ballast	
Heavy ballast	
Homogeneous	
Alternate	



$$\sigma_{\text{HOTSPOT}} = 1.5 \times \left(\frac{\sigma_1 + \sigma_2}{2}\right) - 0.5 \times \left(\frac{\sigma_3 + \sigma_4}{2}\right)$$

Fig.1 The extrapolation of hot spot stress

Table 3 Target value of reference stress ranges MPa

Ship	Light	Heavy	Homogeneous	Alternate
1	233.57	274.17	293.08	354.95
2	170.94	208.01	228.35	200.99
3	250.58	226.22	233.44	313.38
4	284.26	376.63	364.92	342.18
5	220.68	264.34	302.65	267.36

5.2 The dominant load parameters for hatch corner

The dominant load parameters can be determined by the stress responses of the hatch corner. The hatch corner is on the deck, where the stress induced by the vertical wave bending moment is great. The hatch corner is also close to the side of the ship hull, where the stress induced by the horizontal wave bending moment is large (Hansen and Winterstein, 1995). Because of the opening of the hatch on the deck, the stress induced by the torsion is significant. In addition, it is found that the stress caused by the vertical wave shearing force is comparatively large and cannot be neglected. It is concluded that the vertical wave bending moment, the horizontal wave bending moment, the torsion, and the vertical shearing force are applicable as the dominant load parameters for the fatigue assessment of the hatch corner of the midship section. Therefore, Eq.(9) can be rewritten as,

$$S_0 = \sqrt{A_{M_V} S_{M_V}^2 + A_{M_H} S_{M_H}^2 + A_{TM} S_{TM}^2 + A_{F_V} S_{F_V}^2} \tag{10}$$

where S_0 is the reference stress range; S_{M_V}, S_{M_H}, S_{TM}, S_{F_V} are respectively the stress ranges under the different equivalent waves based on the midship sectional wave vertical bending moment M_V, horizontal bending moment M_H, torsion TM, and vertical shearing force F_V; A_{M_V}, A_{M_H}, A_{TM}, and A_{F_V} are the corresponding combination factors.

5.3 Stress combination based on the equivalent waves

According to the above identified four dominant load parameters, the equivalent waves can be defined in terms of the principle of the equivalent waves in Section 1. Based on the equivalent waves, the stress ranges of the hatch corners of the five ships can be calculated under the four loading conditions. For any one of the loading conditions, the stress ranges of the hatch corner of the midship section are regressed by the least square method based on Eq.(10), with the target values of the five ships given in Table 3. The combination factors are obtained in the four loading conditions, respectively, as shown in Table 4. It must be pointed out that the stress ranges should be based on a single side of the ship because the stress responses of the ship structures are not symmetric in an oblique sea. The combination factors given in Table 4 are based on the port of the ship.

Table 4 Combination factors of stress ranges

Combination factors	Light	Heavy	Homogeneous	Alternate
A_{M_V}	0.38	1.16	0.41	0.84
A_{M_H}	1.89	−0.01	2.06	−0.92
A_{TM}	0.36	−0.21	−2.18	0.39
A_{F_V}	0.38	−0.56	1.73	2.05

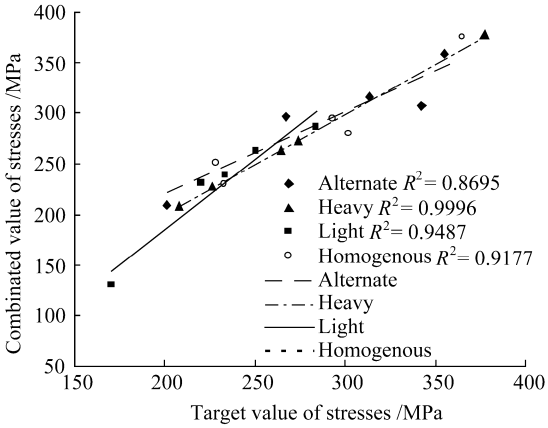


Fig.2 The regression between the target value of the stresses and the combined value of the stresses

The regression between the target values of the stresses given in Table 3 and the combined values of the stresses with the combination factors given in Table 4 is acceptable as shown in Fig.2 for each of the four loading conditions, e.g., light ballast, heavy ballast, homogenous, and alternate loading conditions.

5.4 Procedure of the equivalent wave method for the fatigue assessment of the hatch corner

In general, the following procedure of the equivalent wave method for the fatigue assessment of the hatch corner of the midship in the world-wide scatter diagram can be adopted. For any one certain loading condition, the wave parameters including heading, wave frequency, and wave amplitude of one equivalent wave are to be determined based on one of the dominant load parameters, the midship sectional vertical wave bending moment, the horizontal wave bending moment, the torsion, or the vertical wave shearing force. Under the equivalent wave, the wave loads and ship motion are calculated and applied to the finite element model of ship structures. The stress ranges of the hatch corner (port) then can be obtained by finite element analysis. The calculation of the stress ranges under the other equivalent waves defined by different dominant load parameters are performed in the same manner. The reference stress ranges by the equivalent waves are further combined with the factors given in Table 4. The resultant stress ranges can be used to make the fatigue assessment of the hatch corner.

6 Numerical example

The fatigue assessment of the midship hatch corner of a bulk carrier in the world-wide scatter diagram is used here as an example. The ship length is 182.00 m, the breadth is 32.26m, the depth is 16.67 m, and the draft is 11.92 m. The global finite element model of the entire ship is built as shown in Fig.3. The very fine local mesh of the hatch corner (Port) is shown in Fig.4.

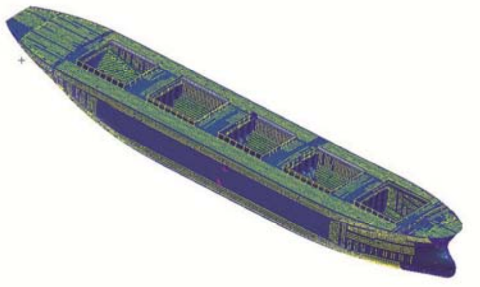


Fig.3 The finite element model of the entire ship

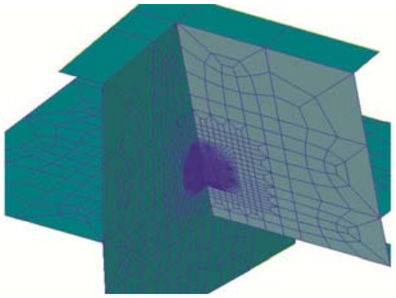


Fig.4 The very fine mesh of hatch corner

The level of probability of exceedance corresponding to the long term dominant load parameter is taken as 10^{-4}. The equivalent waves are determined by the midship sectional vertical wave bending moment, horizontal wave bending moment, vertical wave shearing force, and the torsion under each loading condition. The parameters of the equivalent waves are obtained by the wave loads program calculation, as shown in Table 5.

Table 5 Parameter of the equivalent waves

DLP	Param.	Light	Heavy	Homo.	Alter.
M_V	Fre./(rad·s^{-1})	0.6	0.6	0.8	0.8
	Head /(°)	0	0	120	120
	Amp. /m	4.17	4.64	3.92	4.04
M_H	Fre./(rad·s^{-1})	0.8	0.9	0.8	0.8
	Head /(°)	90	60	60	90
	Amp. /m	1.32	2.40	2.74	1.35
TM	Fre./(rad·s^{-1})	0.8	0.7	0.6	0.7
	Head /(°)	120	90	90	90
	Amp. /m	2.67	2.05	3.35	1.29
F_V	Fre./(rad·s^{-1})	0.7	0.7	0.6	0.7
	Head /(°)	180	0	0	0
	Amp. /m	2.86	3.17	3.54	3.76

The stress distributions of the ship structures induced by the equivalent waves determined by the midship sectional vertical wave bending moment and vertical wave bending moment in heavy ballast condition are given here as the example, as shown in Fig.5 and Fig.6. A typical stress distribution around the hatch corner is shown in Fig.7.

The hot spot stress ranges of the hatch corner (port) of the bulk carrier are calculated based on the equivalent waves, as shown in Table 6.

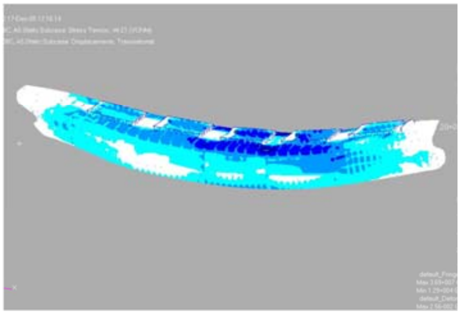


Fig.5 The stress distribution induced by the equivalent waves determined by the midship sectional vertical wave bending moment in heavy ballast condition

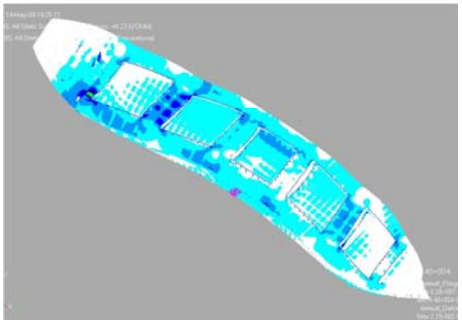


Fig.6 The stress distribution induced by the equivalent waves determined by the midship sectional horizontal wave bending moment in heavy ballast condition

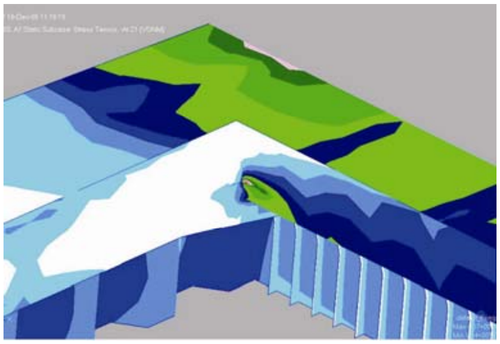


Fig.7 A typical stress distribution around the hatch corner

Table 6 Stress range obtained by equivalent waves MPa

Stress range	Light	Heavy	Homogeneous	Alternate
S_{M_V}	305.53	445.32	89.15	242.88
S_{M_H}	104.89	235.59	102.48	116.58
S_{TM}	108.57	152.36	222.47	111.01
S_{F_V}	147.66	197.77	353.31	235.48

The reference stress ranges by the equivalent waves, as shown in Table 7, can be obtained by the combination of the structural stress ranges given in Table 6, with respect to the factors in Table 4. The comparison shows that results from the equivalent wave approach agree well with those from the spectral fatigue analysis.

Table 7 Comparison of stress ranges MPa

Loading conditions	Equivalent wave	Spectral analysis
Light ballast	262.76	285.82
Heavy ballast	449.96	455.57
Homogeneous	364.79	378.17
Alternative	393.99	389.57

Although the results are satisfactory, it must be realized that they are not very conservative because the number of the ships is limited and the total length of the ships is around two hundred meters.

7 Conclusions

The stress combination method for the fatigue assessment of the hatch corner of a bulk carrier was investigated based on equivalent waves. Based on a certain number of dominant load parameters, the equivalent waves were determined. The structural stress ranges of the hatch corner by the equivalent waves were fitted to the reference stress ranges from the spectral analysis. The following conclusions were drawn.

1) The dominant load parameters are identified by analysis of the structural stress responses. It is found that the midship sectional vertical wave bending moment, horizontal wave bending moment, torsion, and the vertical wave shearing force are applicable as the dominant load parameters of the hatch corner of the midship of a bulk carrier.

2) The combination factors of the structural stress ranges (port) under different equivalent waves in the four typical loading conditions are achieved.

3) The fatigue assessment of the hatch corner in the world-wide scatter diagram can be performed based on the four equivalent waves in each loading condition. The workload of the fatigue assessment is reduced substantially, in contrast to the time-consuming calculation of the spectral fatigue analysis.

4) The approach developed here can be referenced in the fatigue strength assessment of the hatch corner of a bulk carrier.

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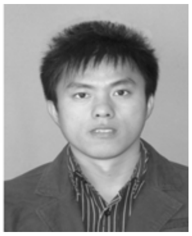

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