

A Free Surface Frequency Domain Green Function with Viscous Dissipation and Partial Reflections from Side Walls

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Abstract: The free-surface Green function method is widely used in solving the radiation or diffraction problems caused by a ship or ocean structure oscillating on the waves. In the context of inviscid potential flow, hydrodynamic problems such as multi-body interaction and tank side wall effect cannot be properly dealt with based on the traditional free-surface frequency domain Green function method, in which the water viscosity is omitted and the energy dissipation effect is absent. In this paper, an open-sea Green function with viscous dissipation was presented within the theory of visco-potential flow. Then the tank Green function with a partial reflection from the side walls in wave tanks was formulated as a formal sum of open-sea Green functions representing the infinite images between two parallel side walls of the source in the tank. The new far-field characteristics of the tank Green function is vitally important for improving the validity of side-wall effects evaluation, which can be used in supervising the tank model tests.

Keywords: Green function; viscous dissipation; side wall effect; partial reflection

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1 Introduction

The evaluation of wave loads on ships and ocean structures is vitally important for ship design and safe operation. The wave-body interaction problem can be solved by model tests in a wave basin or theoretical and numerical analysis, in which the ship and ocean structure are considered to be a rigid body with specified geometry and gravity distribution.

The theoretical and numerical analysis of a wave-body interaction problem is often in the context of so-called inviscid potential flow theory. In potential flow theory, the viscosity of the water is omitted and a velocity potential used to describe the water flow is introduced. Based on the perturbation method, the steady state wave-body interaction problems can be reorganized into first-order and second-order boundary value problems with radiation and diffraction potential. The Green function method is a powerful tool to solve this velocity potential boundary value problem, and has been proven to be very successful in the process of prediction of wave loads on a single floating body (Chen, 2004). However, the results are not satisfactory when this free surface frequency domain Green function is applied to solve the interaction between a multi-body and waves. The failure is mainly due to the false prediction of the water flow in the gap between two bodies. Where resonance appeared in numerical simulation, it actually doesn't exist because water viscosity will bring energy dissipation in the real world.

Chen (2004) tried to eliminate the above numerical resonance phenomena in multi-body hydrodynamics by introducing a fictitious force in the momentum equation to represent the energy dissipation of various sources without modifying the inviscid and irrotational properties in the flow field. However, the selection of the artificial damping coefficient in Chen's method is not universal. Qin and Shen (2010) presented another way to represent the energy dissipation by modifying the free surface Green function, and that method will be formulated and discussed in this paper. The new formulation of the free surface Green function with viscous dissipation can not only be used in multi-body hydrodynamics, but also can be applied to construct the so-called tank Green function to evaluate the side wall effect when doing model testing in a wave tank.

Model testing in a wave basin is an effective way to predict the wave loads on floating bodies for the purpose of designing ship and ocean structures. However, it is well known that hydrodynamic forces measured in wave tanks sometimes exhibit large scatter compared to the expected results in the open sea due to wave reflections from side walls. A number of studies have been performed in order to evaluate the side wall effect in wave tanks. Most of them, such as Taylor and Hung (1985), Yeung and Sphaier (1989), Kashiwagi (1991), and McIver (1993), give analytical or semi-analytical solutions using eigenfunction expansion or multipoles with limitation to bodies of simple geometry such as vertical cylinders placed in the center of the wave tank. A few studies, such as Linton (1993), Chen (1994) and Xia (2002), deal with the formulation and numerical computation of tank Green functions. The inviscid potential flow free of any dissipation

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was modeled in these previous studies. The present study on the Green function including the dissipation effect is a detailed and systematic presentation based on content that was partly published in Qin and Shen (2010) and Shen and Qin (2011), and is expected to give important insight on the realistic effect of water viscosity and side walls in wave tanks.

2 Open-sea Green function with viscous dissipation

2.1 Definition of Green function with viscous dissipation

The lower half-space filled with water limited on the top by the water air interface is considered here. The bottom of the water is assumed to be flat and the water depth is denoted as h . A right-hand Cartesian coordinate system is defined by placing the $o-xy$ plane coinciding with the undisturbed free surface and the oz axis oriented positively upward. Under the assumption of fairly perfect fluid by Chen and Dias (2010), the Green function $G(M, M')$ representing the velocity potential at a field point $M(x, y, z)$ in the wave tank due to a pulsating source of unit strength located at the point $M'(x', y', z')$ should satisfy the following set of equation:

$$\nabla^2 G(M, M') = \delta(M - M') \quad (1)$$

$$G_z - \bar{k}G - i4\alpha G_{zz} = 0 \quad z = 0 \quad (2)$$

$$G_z = 0 \quad z = -h \quad (3)$$

where $\delta(\cdot)$ is the Dirac function and the parameters \bar{k} , α are defined as

$$\bar{k} = \omega^2/g, \quad \alpha = \mu\omega/\rho g$$

with ω the frequency of the pulsating source, g the acceleration of the gravity, μ the fluid viscosity and ρ is the fluid density.

In addition, the Green function $G(M, M')$ must satisfy a radiation condition which states that, at infinity, $G(M, M')$ is associated only with waves that propagate away from the source.

2.2 Integral and series expression of Green function

The general solution of the open-sea Green function which satisfies Eqs.(1)–(3) can be written as

$$4\pi G(M, M') = -\frac{1}{|MM'|} - \frac{1}{|MM'|} - H(r, z, z') \quad (4)$$

where $\bar{M}'(x', y', -z' - 2h)$ is the symmetrical point of $M'(x', y', z')$ with respect to sea bed $z = -h$. The free surface term $H(r, z, z')$ under the cylindrical coordinates with $r = \sqrt{(x - x')^2 + (y - y')^2}$ should satisfy

$$\nabla^2 H = 0 \quad (5)$$

$$\left(\frac{\partial}{\partial z} - \bar{k} - i4\alpha\frac{\partial^2}{\partial z^2}\right)H = -\left(\frac{\partial}{\partial z} - \bar{k} - i4\alpha\frac{\partial^2}{\partial z^2}\right)\left(\frac{1}{r} + \frac{1}{r'}\right) \quad z = 0 \quad (6)$$

$$\frac{\partial H}{\partial z} = 0 \quad z = -h \quad (7)$$

By the standard Fourier-Hankel transformation of (5)–(7), we get the following expression

$$H(r, z, z') = 2 \int_0^\infty \frac{(k + \bar{k} + i4\alpha k^2)e^{-kh}}{k \sinh kh - (\bar{k} + i4\alpha k^2) \cosh kh} \times \cosh k(z + h) \cosh k(z' + h) J_0(kr) dk \quad (8)$$

with $J_0(\cdot)$ the zeroth-order Bessel function of the first kind defined in Abramowitz and Stegun(1967).

So the integral representation of the Green function with viscous dissipation is as follows:

$$4\pi G(M, M') = -\frac{1}{r} - \frac{1}{r'} - \int_0^\infty f(k, h) \cosh k(z + h) \cosh k(z' + h) J_0(kr) dk \quad (9)$$

$$\text{where } f(k, h) = \frac{2(k + \bar{k} + i4\alpha k^2)e^{-kh}}{k \sinh kh - (\bar{k} + i4\alpha k^2) \cosh kh}.$$

The integral representation of the Green function expressed in Eq.(9) is not convenient for characteristics analysis and numerical calculation. According to the new dispersion relation (10), the roots of wave numbers are not located at real and imaginary axis as described in Fig.1, which is different with the inviscid flow.

$$k \tanh kh - (\bar{k} + i4\alpha k^2) = 0 \quad (10)$$

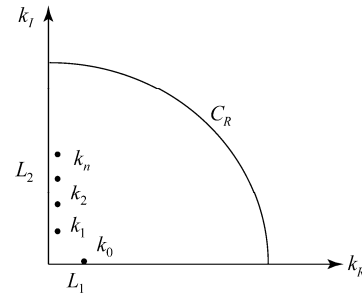


Fig.1 Locations of the roots

According to perturbation theory, the complex roots of the dispersion relation can be expressed as $k = k_R + ik_I$. Where

$$k_0 = (k_{0R} + \gamma k_{0R}^{(1)} + \gamma^2 k_{0R}^{(2)} + \dots) + i(\gamma k_{0I}^{(1)} + \gamma^2 k_{0I}^{(2)} + \dots) \quad (11)$$

$$k_n = (\gamma k_{nR}^{(1)} + \gamma^2 k_{nR}^{(2)} + \dots) + i(k_{nI} + \gamma k_{nI}^{(1)} + \gamma^2 k_{nI}^{(2)} + \dots) \quad (12)$$

where $\gamma = 4\alpha$.

By omitting the terms of order $O(\alpha^2)$ or higher, and using the same technique as in John (1950) to evaluate H , the open-sea Green function with viscous effect can be expressed as

$$G(M, M') = -\frac{ik_0}{\sinh 2k_0 h + 2k_0 h} \times \cosh k_0(z+h) \cdot \cosh k_0(z'+h) \cdot H_0[(k_0 + i4\alpha k_{0I})r] - \sum_{n=1}^{\infty} \frac{2k_n / \pi}{\sin 2k_n h + 2k_n h} \times \cos k_n(z+h) \cdot \cos k_n(z'+h) \cdot K_0[(k_n - i4\alpha k_{nI})r] \quad (13)$$

in which $H_0(\cdot)$ and $K_0(\cdot)$ are the zeroth-order Hankel function of the first kind and the zeroth-order modified Bessel function of the second kind defined in Abramowitz and Stegun (1967), the wave numbers k_0 and k_n for $n \geq 1$ are the roots of the classical dispersion equations:

$$k_0 \tan k_0 h = \bar{k} \quad k_n \tanh k_n h = \bar{k} \quad (14)$$

respectively, while the values of k_{0I} and k_{nI} are defined by

$$k_{0I} = \frac{k_0^2 (\cosh 2k_0 h + 1)}{\sinh 2k_0 h + 2k_0 h} \quad (15)$$

$$k_{nI} = \frac{k_n^2 (\cos 2k_n h + 1)}{\sin 2k_n h + 2k_n h} \quad n = 1, 2, \dots \quad (16)$$

In (13), the first term on the right hand side involving H_0 is often called the wave component while the second term with the sum involving K_0 is evanescent.

2.3 Far Field characteristics

Unlike the inviscid potential flow, the far-field behavior of the velocity potential represented by the Hankel function in (13) is

$$H_0(k_0 r + i4\alpha k_{0I} r) \approx e^{-4\alpha k_{0I} r} \sqrt{2 / [\pi(k_0 + i4\alpha k_{0I})r]} \exp(ik_0 r - i\pi/4) \quad (17)$$

For $r \rightarrow \infty$. The decay factor $e^{-4\alpha k_{0I} r}$ represents the dissipation effect of fluid viscosity which is absent in the classical inviscid potential flow. As illustrated in Fig.2, the magnitude of the complex Hankel function depicted by its real and imaginary parts decreases as $O(1/\sqrt{k_0 r})$ without viscous dissipation ($\alpha = 0$) while it decays much faster in

the order of $O(e^{-4\alpha k_{0I} r} / \sqrt{k_0 r})$ with viscous dissipation ($\alpha \neq 0$). The Green function with dissipation effect must be particularly interesting in the solution of wave diffraction and radiation around one or several floating bodies.

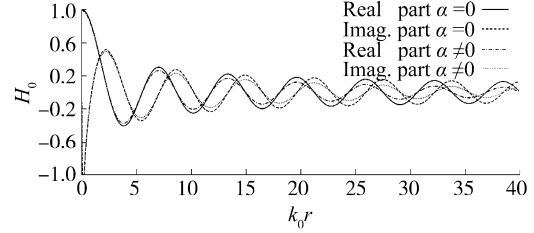


Fig.2 Free surface waves with and without viscous dissipation

3 Tank Green function with partial reflection from side walls

3.1 Definition and construction of tank Green function

In part 2 the open sea case was studied and a new formulation of the free surface frequency domain Green function with viscous dissipation was developed. However, in a water basin, there are some differences due to the existence of the two side walls. In this restricted flow region a so-called tank Green function is constructed to take the side wall effect into consideration. Here, a right-hand Cartesian coordinate system is defined by placing the xoy plane coinciding with the undisturbed free surface and the oz axis is oriented positively upwards. The ox axis is coincident with the center plane of the tank whose width is denoted by b . Under the assumption of fairly perfect fluid (Chen and Dias, 2010), the TGF $G(M, M')$ representing the velocity potential at a field point $M(x, y, z)$ in the wave tank due to a pulsating source of unit strength located at the point $M'(x', y', z')$, should satisfy not only Eqs.(1)–(3) and radiation condition, but also the reflection conditions on two side walls (the reflection coefficients of two side walls are denoted as a_L, a_R):

$$G_y(M, M') = \beta_L G_y^0(M, M') \quad y = b/2 \quad (18)$$

$$G_y(M, M') = \beta_R G_y^0(M, M') \quad y = -b/2 \quad (19)$$

where $\beta_L = 1 - a_L$, $\beta_R = 1 - a_R$.

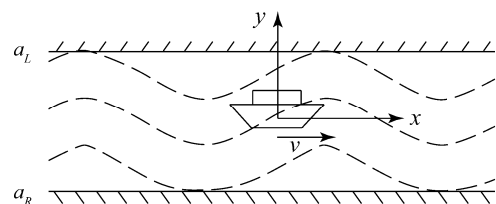


Fig.3 Model test in wave tank

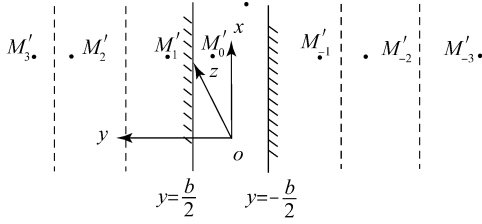


Fig.4 Images of the source point

The solution of TGF that includes the partial reflection effect from side walls can be obtained by considering an infinite number of images (Fig.4) of the source between two parallel side-walls, that is

$$G(M, M'_0) = G_0^0(M, M'_0) + \sum_{n=1}^{\infty} a_L^n a_R^n \left[\begin{aligned} &G_{2n}^0(M, M'_{2n}) + G_{-2n}^0(M, M'_{-2n}) \\ &+ a_R^{-1} G_{2n-1}^0(M, M'_{2n-1}) + a_L^{-1} G_{-2n+1}^0(M, M'_{-2n+1}) \end{aligned} \right] \quad (20)$$

where $G_0^0(M, M'_0)$ is the open-sea Green function with viscous dissipation as shown in (13). $G_{\pm n}^0(M, M'_{\pm n})$ is the velocity potential in M due to the n th image of the source point in $M'_n(x', y'_n, z')$.

$$y'_n = (-1)^n y' + nb' \quad (21)$$

The direct computation of the infinite series is slowly convergent especially when the partial reflection factor is close to 1, which means total reflection of the waves against side walls. The tank Green function can be regrouped into two parts which has been proven to be more computationally efficient:

$$G = G^F + G^H \quad (22)$$

with G^F a finite series

$$G^F(M, M'_0) = G_0^0(M, M'_0) + \sum_{n=1}^N a_L^n a_R^n \left[\begin{aligned} &G_{2n}^0(M, M'_{2n}) + G_{-2n}^0(M, M'_{-2n}) \\ &+ a_R^{-1} G_{2n-1}^0(M, M'_{2n-1}) + a_L^{-1} G_{-2n+1}^0(M, M'_{-2n+1}) \end{aligned} \right] \quad (23)$$

where

$$G_n^0 = Z_0(z) H_0(k_0 r_n + iak_{0l}) + \sum_{m=1}^{\infty} Z_m(z) K_0(k_m r_n + iak_{ml})$$

$$Z_0(z) = \frac{-ik_0}{2k_0 h + \sinh 2k_0 h} \cosh k_0(z+h) \cosh k_0(z'+h)$$

$$Z_m(z) = \frac{-2k_m / \pi}{2k_m h + \sin 2k_m h} \cosh k_m(z+h) \cosh k_m(z'+h)$$

and the remaining part by the truncated infinite series

$$G^H = \sum_{n=N+1}^{\infty} a_L^n a_R^n \left[\begin{aligned} &G_{2n}^0(M, M'_{2n}) + G_{-2n}^0(M, M'_{-2n}) \\ &+ a_R^{-1} G_{2n-1}^0(M, M'_{2n-1}) + a_L^{-1} G_{-2n+1}^0(M, M'_{-2n+1}) \end{aligned} \right] \quad (24)$$

which represents the contribution of the source image far from the field point.

In the following section, the decomposition of G^H into two single integrals and their numerical evaluation are presented.

3.2 Asymptotic part of TGF and integral representations

Similar to the method pointed out by Chen (1994) and adopted in Shen and Qin (2011), assuming that the lowest number $2N$ in (24) is large enough to neglect the evanescent part of the open-sea Green function, the asymptotic part G^H can be rewritten as the sum including two infinite single integrals:

$$G^H = Z_0(z) a_L^{N+1} a_R^{N+1} \times \sum_{n=0}^{\infty} a_L^n a_R^n \left[H_0(R_{n1}) + H_0(R_{n2}) + a_R^{-1} H_0(R_{n3}) + a_L^{-1} H_0(R_{n4}) \right] \quad (25)$$

in which

$$R_{nl} = \sqrt{X^2 + Y_{nl}^2}, \quad l=1,2,3,4$$

$$X = (k_0 + i4\alpha k_{0l})(x - x'), \quad Y_{nl} = 2B(n + y_l)$$

$$B = (k_0 + i4\alpha k_{0l})b$$

$$y_1 = N + 1 - (y - y') / 2b$$

$$y_2 = N + 1 + (y - y') / 2b$$

$$y_3 = N + 1/2 - (y + y') / 2b$$

$$y_4 = N + 1/2 + (y + y') / 2b$$

Applying the following identity:

$$H_0(R_{nl}) = \sum_{m=0}^{\infty} \epsilon_m (-1)^m J_{2m}(X) H_{2m}(Y_{nl}) \quad (26)$$

where $\epsilon_0 = 1$, $\epsilon_m = 2$ when $m \geq 1$.

Furthermore, assuming Y_{nl} to be large, then

$$H_{2m}(Y_{nl}) = (-1)^m e^{iY_{nl}} \{c_1 Y_{nl}^{-1/2} + ic_2 Y_{nl}^{-3/2}\} + E_{mn} \quad (27)$$

where $c_1 = e^{-i\pi/4} \sqrt{2/\pi}$, $c_2 = c_1(16m^2 - 1)/8$.

The error term is

$$E_{mn} < c_1 \frac{(16m^2 - 1)(16m^2 - 9)}{128} Y_{nl}^{-5/2}$$

Introducing (27) into (26) and then into (25) resulted in

$$G^H = Z_0(z) a_L^{N+1} a_R^{N+1} \times \left\{ \sum_{l=1}^2 c(y_l, B) \left[I_1(y_l, B, a_L, a_R) + id(X) I_2(y_l, B, a_L, a_R) / B \right] + \sum_{l=3}^4 c(y_l, B) \left[a_R^{-1} I_1(y_l, B, a_L, a_R) + ia_L^{-1} d(X) I_2(y_l, B, a_L, a_R) / B \right] \right\} \quad (28)$$

in which

$$d(X) = \sum_{m=0}^{\infty} \varepsilon_m J_{2m}(X) (16m^2 - 1) / 8 = (4X^2 - 1) / 8$$

$$c(y_l, B) = e^{i(2By_l - \pi/4)} / (\pi B^{1/2})$$

$$I_1(y_l, B, a_L, a_R) = \sqrt{\pi} \sum_{n=0}^{\infty} \frac{a_L^n a_R^n e^{i2Bn}}{(n + y_l)^{1/2}}$$

$$I_2(y_l, B, a_L, a_R) = \frac{\sqrt{\pi}}{2} \sum_{n=0}^{\infty} \frac{a_L^n a_R^n e^{i2Bn}}{(n + y_l)^{3/2}}$$

I_1 and I_2 are defined and transformed into the following integral representations:

$$I_1(y_l, B, a_L, a_R) = \int_0^{\infty} \frac{t^{-1/2} e^{-y_l t}}{1 - a_L a_R e^{i2B-t}} dt \quad (29)$$

$$I_2(y_l, B, a_L, a_R) = \int_0^{\infty} \frac{t^{1/2} e^{-y_l t}}{1 - a_L a_R e^{i2B-t}} dt \quad (30)$$

Using the Taylor development of e^{-t} in the denominator of two infinite integrals, the single integrals I_1 and I_2 are finally approximated as follows:

$$\tilde{I}_1 = \pi e^A \operatorname{erfc}(\sqrt{A}) / \sqrt{(1 - e^{i2B}) e^{i2B}} \quad (31)$$

$$\tilde{I}_2 = \left[\sqrt{\pi/A} - \pi e^A \operatorname{erfc}(\sqrt{A}) \right] \sqrt{(1 - e^{i2B}) / e^{i6B}} \quad (32)$$

with $\operatorname{erfc}(\cdot)$ as the complementary error function and

$$A = y_l (a_L^{-1} a_R^{-1} e^{-i2B} - 1)$$

Note that the single integral I_2 is always convergent since its major value \tilde{I}_2 is finite regardless of the value of B .

The function e^{i2B} present in (31) and (32) has the property:

$$\left| e^{i2B} \right| = \left| e^{i2k_0 b - 8\alpha k_0 b} \right| < 1 \quad (33)$$

Since $8\alpha k_0 b > 0$, the value of \tilde{I}_1 is always finite and the original integral I_1 convergent. Without taking account of the viscous effect by putting $\alpha = 0$, the value \tilde{I}_1 tends to infinity for a set of discrete values as shown in Chen (1994), which are associated with the resonance modes of transversal waves between two vertical walls.

4 Conclusions

In this paper, the integral expression of the open-sea Green function with dissipation was formulated. Next, its series representation was obtained based on the perturbation solution of the dispersion equation and the transformation of a partial fraction to an infinite series. A Green function with

dissipation was decomposed into a sum of the near-field term and the far-field term. Compared to the traditional inviscid Green function, the far-field term decreases much faster with an exponential decay factor introduced by the fluid viscosity, which is particularly relevant to the solution of wave radiation and diffraction around one or several floating bodies.

The tank Green function with viscous dissipation and partial reflection from a side wall constructed with singularity images can be used for evaluation of the side-wall effect and supervision of the tank model test. The asymptotic part defined by the truncated infinite series is transformed into a sum including two single integrals. The analysis of the single integral and its analytical expression shows that it is finite due to the decay factor associated with the dissipation, unlike the classical TGF which is singular at the wave numbers associated with the resonance of transversal waves between two parallel side walls. These new far-field characteristics of the tank Green function with dissipation will be vitally important for improving the validity of the side-wall effects evaluation, which can be used in supervising the tank model tests.

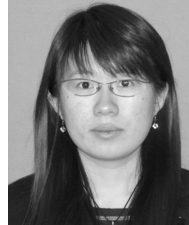
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