

# Prediction of Planing Craft Motion Based on Grey System Theory

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**Abstract:** In order to minimize the harm caused by the instability of a planing craft, a motion prediction model is essential. This paper analyzed the feasibility of using an MGM(1,N) model in grey system theory to predict planing craft motion and carried out the numerical simulation experiment. According to the characteristics of planing craft motion, a recurrence formula was proposed of the parameter matrix of an MGM(1,N) model. Using this formula, data can be updated in real-time without increasing computational complexity significantly. The results of numerical simulation show that using an MGM(1,N) model to predict planing motion is feasible and useful for prediction. So the method proposed in this study can reflect the planing craft motion mechanism successfully, and has rational and effective functions of forecasting and analyzing trends.

**Keywords:** planing craft; MGM(1,N) model; recurrence formula; short-time prediction

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## 1 Introduction

Planing craft is a kind of high speed craft which rests most of its weight on hydrodynamic pressure produced by hull while sailing. Nowadays, planing craft has been widely used for its advantages, such as excellent maneuverability, low cost, and so on. However, because the impact force on hull of planing craft by wave is large, the dolphin movement will appear in high sea state. It causes the seaworthiness of planing craft decreasing seriously and its sailing sea area is limited greatly. In addition, the planing craft has a high requirement on the control of vertical gravity center. Therefore, the short-time prediction technology is needed to predict motion of planing craft exactly and real-timely. According to the result of prediction, corresponding measures can be taken and effective automatic control systems of planing craft also can be designed. These measures and automatic control systems will greatly improve the stability and speed of planing craft. And most important, it will reduce the rate of accidents greatly.

Affected by many kinds of factors, the motion of planing craft is complex in actual sea condition. The coupled six-degree motions consist of a complex system. The motion characteristics of planing craft in actual sea condition, especially in severe sea, is generally random and nonlinear. Therefore, it is difficult to give a mathematical

model to predict the motion of planing craft. At present, the main methods predicting longitudinal motion of planing craft include the methods based on Strip Theory, such as modified strip method and Zarnic's nonlinear model, the method of solving Navier-Stokes equation, the method of time series analysis, and so on. Because the effect of hydrodynamic lift is ignored, the accurate prediction effect of the motion of planing craft can not be achieved by strip theory. The motion simulation of high speed ship through solving Navier-Stokes equation needs to build moving grids, special skills and large computation. So it is difficult to achieve real-time prediction and the precision needs to be improved further. The method of time series analysis needs mass data and has low prediction precision.

To sum up, at present there is no effective method to predict the motion of planing craft precisely. According to the motion characteristics of planing craft in practical wave, this paper researches the prediction effect of high speed planing craft using MGM(1,N) model in grey system theory. According to real-time requirement of the prediction, this paper also proposes a recurrence formula of parameter matrix in MGM(1,N) model.

## 2 Grey system theory and recurrence MGM(1,N) model

### 2.1 Grey system theory

Grey system theory was founded by Prof. J.L.Deng in 1982, taking the uncertain system of "small sample", "poor information", i.e. "partial information known, partial information unknown" as the research object. Through creating and excavating the limited information, grey

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system theory helps us to understand the real world, describe its evolvement rule and grasp its running behavior exactly. Because grey prediction model can compute and presume according to the limited information and does not have special requirement and limit to the observational data, it has been widely applied.

Because planing craft has the characteristics of high speed and randomness, it is unrealistic to predict the motion of planing craft using a mass of original data. This condition just accords with the characteristics of grey system theory. therefore, the method using grey system theory to predict the motion of planing craft will have the important theoretical significance and practical meaning.

## 2.2 MGM(1,N) model

MGM(1,N) model is Multivariable Grey Model. Its form is with  $N$ -element first-order ordinary differential equation system. MGM(1,N) model is the extension of GM(1,1) model in multi-degree freedom, but not the simple combination of GM(1,1) models. And it is also different from GM(1,N) model.

Like other ships, because of the complexity of actual sea condition, the motion of planing craft is very complex. It is the superposition of several simple motions. The motion of each degree of freedom is not independent and there exists coupling influence among them. For example, the pitch usually goes with the heave and both of them can not be generated individually. By only considering the subsystem composed of resistance, pitch and heave of planing craft, a MGM(1,N) model was built. The recurrence formula of parameter matrix of MGM(1,N) model's was proposed.

Suppose the original data matrix consisted of  $N$  sequences and observed value of  $m$  time points is  $X^{(0)} = [x_i^{(0)}(k)]_{m \times N}$ , and the corresponding accumulated generating matrix is  $[x_i^{(1)}(k)]_{m \times N}$ , namely  $x_i^{(1)}(k) = \sum_{j=1}^k x_i^{(0)}(j)$ , ( $i = 1, 2, \dots, N$ ;  $k = 1, 2, \dots, m$ ). MGM(1,N) model is the  $N$ -element first-order ordinary differential equation system of these generating sequences.

$$\begin{cases} \frac{dx_1^{(1)}}{dt} = a_{11}x_1^{(1)} + a_{12}x_2^{(1)} + \dots + a_{1N}x_N^{(1)} + b_1 \\ \frac{dx_2^{(1)}}{dt} = a_{21}x_1^{(1)} + a_{22}x_2^{(1)} + \dots + a_{2N}x_N^{(1)} + b_2 \\ \vdots \\ \frac{dx_N^{(1)}}{dt} = a_{N1}x_1^{(1)} + a_{N2}x_2^{(1)} + \dots + a_{NN}x_N^{(1)} + b_N \end{cases} \quad (1)$$

Let

$$X^{(0)}(k) = [x_1^{(0)}(k), x_2^{(0)}(k), \dots, x_N^{(0)}(k)]^T, \\ X^{(1)}(k) = [x_1^{(1)}(k), x_2^{(1)}(k), \dots, x_N^{(1)}(k)]^T (k = 1, 2, \dots, m),$$

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1N} \\ a_{21} & a_{22} & \dots & a_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ a_{N1} & a_{N2} & \dots & a_{NN} \end{bmatrix}, \quad B = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_N \end{bmatrix}, \quad M = [A, B]^T,$$

the differential equation system above can be written as follows:

$$\frac{dX^{(1)}}{dt} = AX^{(1)} + B \quad (2)$$

Using least square method, the estimated value of  $M$  can be got:

$$\hat{M} = [\hat{A}, \hat{B}]^T = (L^T L)^{-1} L^T Y \quad (3)$$

where

$$L = \begin{bmatrix} 0.5(x_1^{(1)}(2) + x_1^{(1)}(1)) & \dots & 0.5(x_N^{(1)}(2) + x_N^{(1)}(1)) & 1 \\ 0.5(x_1^{(1)}(3) + x_1^{(1)}(2)) & \dots & 0.5(x_N^{(1)}(3) + x_N^{(1)}(2)) & 1 \\ \vdots & \ddots & \vdots & \vdots \\ 0.5(x_1^{(1)}(m) + x_1^{(1)}(m-1)) & \dots & 0.5(x_N^{(1)}(m) + x_N^{(1)}(m-1)) & 1 \end{bmatrix}, \\ Y = \begin{bmatrix} x_1^{(0)}(2) & \dots & x_N^{(0)}(2) \\ x_1^{(0)}(3) & \dots & x_N^{(0)}(3) \\ \vdots & \ddots & \vdots \\ x_1^{(0)}(m) & \dots & x_N^{(0)}(m) \end{bmatrix}.$$

The fitting and prediction formula of this  $N$  dimensional vector can be obtained by solving this differential equation system and inverse operation of accumulative generation:

$$\begin{cases} \hat{X}^{(0)}(k+1) = e^{\hat{A}k} (E - e^{-\hat{A}})(X^{(0)}(1) + \hat{A}^{-1}\hat{B}), \quad k = 1, 2, \dots \\ \hat{X}^{(0)}(1) = X^{(0)}(1). \end{cases} \quad (3)$$

where

$$e^{\hat{A}k} = E + \hat{A}k + \frac{\hat{A}^2}{2!}k^2 + \frac{\hat{A}^3}{3!}k^3 + \dots = E + \sum_{n=1}^{\infty} \frac{\hat{A}^n}{n!}k^n.$$

## 2.3 Recurrence formula of parameter matrix of MGM(1,N) model

While using MGM(1,N) model to predict the motion of high speed planing craft, the newly obtained data also need to be dealt real-timely. Every new group of data should be combined with the former data. Simultaneously, because of the characteristic of grey system that the more original data may lead to a worse prediction effect, the prior data should be eliminated after the new data was added into the model. When the original data was changed, the parameter matrix of MGM(1,N) model should be recalculated. The parameter matrix of MGM(1,N) model is estimated by least square method. The parameter matrix is estimated once only using all data. Based on section 2.2, the amount of calculation of MGM(1,N) model mainly depends on the solution of the parameter matrix  $\hat{M} = [\hat{A}, \hat{B}]^T$ . If the parameter matrix is recalculated using least square method, because the computational process includes the inverse and multiplication of matrix, the amount of calculation will be

great and affect the real-time requirement of prediction. In order to ensure that the amount of calculation will not obviously increase after data updated, a simple and effective method to estimate the parameter matrix was proposed.

In order to prove the recurrence formula proposed by this paper, the related lemma is listed in the following at first.

**Lemma** Suppose  $F$  and  $G$  are  $r$  and  $m$  order inverse square matrix separately,  $H$  and  $K$  are  $r \times m$  and  $m \times r$  order matrix separately, then when  $F - HG^{-1}K$  is inverse, the following equation satisfies:

$$(F - HG^{-1}K)^{-1} = F^{-1} + F^{-1}H(G - KF^{-1}H)^{-1}KF^{-1} \quad (4)$$

**Theorem 1** Suppose the estimated parameter matrix of MGM(1,  $N$ ) model built by taking the observed value of the former  $m$  frames as the original data is  $\hat{M}_m$ , and the estimated parameter matrix of MGM(1,  $N$ ) model built by taking the observed value of the former  $m+1$  frames as the original data is  $\hat{M}_{m+1}$ , then relationship between  $\hat{M}_m$  and  $\hat{M}_{m+1}$  satisfies:

$$\hat{M}_{m+1} = \hat{M}_m + H_{m+1}[y(m+1) - l(m+1)\hat{M}_m] \quad (5)$$

where

$$\begin{aligned} H_{m+1} &= \frac{C_m l^T(m+1)}{1 + l(m+1)C_m l^T(m+1)}, \\ y(m+1) &= [x_1^{(0)}(m+1), \dots, x_N^{(0)}(m+1)], \\ l(m+1) &= [0.5(x_1^{(1)}(m+1) + x_1^{(1)}(m)), \dots, \\ &\quad 0.5(x_N^{(1)}(m+1) + x_N^{(1)}(m)), 1]. \end{aligned}$$

**Proof** After the  $(m+1)$ th group of data were added into the original data, the parameter matrix of MGM(1,  $N$ ) model is

$$\hat{M}_{m+1} = (L_{m+1}^T L_{m+1})^{-1} L_{m+1}^T Y_{m+1},$$

then

$$\begin{aligned} C_{m+1} &= (L_{m+1}^T L_{m+1})^{-1} = \left( \begin{bmatrix} L_m \\ l(m+1) \end{bmatrix} \begin{bmatrix} L_m^T \\ l(m+1) \end{bmatrix} \right)^{-1} = \\ &= [L_m^T L_m + L^T(m+1)l(m+1)]^{-1} = \\ &= [C_m^{-1} + L^T(m+1)l(m+1)]^{-1}, \end{aligned}$$

In (4), let  $F = C_m^{-1}$ ,  $H = l^T(m+1)$ ,  $G = -1$ ,  $K = l(m+1)$ , then

$$\begin{aligned} C_{m+1} &= C_m + C_m l^T(m+1) \cdot [-1 - l(m+1)C_m l^T(m+1)]^{-1} l(m+1)C_m \\ &= C_m - H_{m+1} l(m+1)C_m = [E - H_{m+1} l(m+1)]C_m \end{aligned}$$

where

$$H_{m+1} = \frac{C_m l^T(m+1)}{1 + l(m+1)C_m l^T(m+1)} \quad \text{and } E \text{ is } N+1 \text{ order identity matrix.}$$

Because of the characteristic of grey system that the more original data may lead to a worse prediction effect, the prior data should be eliminated after the new data is added into the model. The following theorem can be proved by the same method.

**Theorem 2** Suppose the estimated parameter matrix of MGM(1,  $N$ ) model, built by taking the observed value of the former  $m+1$  groups as the original data, is  $\hat{M}_{m+1}$ , after eliminating the 1<sup>st</sup> frame data of the  $m+1$  frames, the estimated parameter matrix of MGM(1,  $N$ ) model built by taking the observed value of the later  $m$  frames as the original data is  $\hat{M}_{2 \sim m+1} = [\hat{A}_{2 \sim m+1}, \hat{B}_{2 \sim m+1}]^T$ , suppose

$$\begin{aligned} \hat{M}_{2 \sim m+1}^* &= [\hat{A}_{2 \sim m+1}^*, \hat{B}_{2 \sim m+1}^*]^T \\ &= \hat{M}_{m+1} + H_{2 \sim m+1}[y(2) - l(2)\hat{M}_{m+1}], \end{aligned}$$

then

$$\begin{cases} \hat{A}_{2 \sim m+1} = \hat{A}_{2 \sim m+1}^* \\ \hat{B}_{2 \sim m+1} = \hat{B}_{2 \sim m+1}^* + \hat{A}_{2 \sim m+1}^* (X^{(0)}(1))^T \end{cases} \quad (6)$$

where

$$\begin{aligned} H_{2 \sim m+1} &= \frac{C_{m+1} l^T(2)}{-1 + l(2)C_{m+1} l^T(2)}, \\ X^{(0)}(1) &= [x_1^{(0)}(1), \dots, x_N^{(0)}(1)], \\ l(2) &= [z_1^{(1)}(2), \dots, z_N^{(1)}(2), 1], \\ y(2) &= [x_1^{(0)}(2), \dots, x_N^{(0)}(2)]. \end{aligned}$$

Applying Theorem 1 and Theorem 2 synthetically, the data can be updated in the process of building MGM(1,  $N$ ) model, thus the parameter matrix can also be updated, and the purpose of improving prediction precision can be also achieved. Because the inverse matrix does not need to be recalculated in the process of solving the parameter matrix applying Theorem 1 and Theorem 2, the computational complexity is not increased significantly.

### 3 Experiment of numerical simulation

Taking the experimental data of the resistance, the heave, and the pitch of a certain planing craft encountering 3.8 m wavelength of wave as an example, the correctness of the recurrence formula of parameter matrix of MGM(1,  $N$ ) model proposed by this paper and the feasibility of using MGM(1,  $N$ ) model to predict planing craft motion are verified. According to the experiment, the variation period of the 3 variables is about 0.45s, and the frequency of data acquisition is 25Hz, therefore the cycle of the model can be set at 11. A series of numerical experiments shows that the MGM(1,  $N$ ) model built by taking the data of two or three periods can got a better prediction effect. Taking the experimental value of the two former periods as the original data, a MGM(1,3) model is built, and the estimated parameter matrix is

$$\hat{A} = \begin{bmatrix} 0.657 & -0.678 & -3.582 \\ -0.011 & -0.447 & -0.964 \\ 0.150 & 0.122 & -0.198 \end{bmatrix}, \hat{B} = \begin{bmatrix} 7.616 \\ -9.474 \\ 3.929 \end{bmatrix}$$

$$\varepsilon_i(k) = \frac{\hat{x}_i^{(0)}(k) - x_i^{(0)}(k)}{\max_{1 \leq k \leq m} \{x_i^{(0)}(k)\}} \times 100\% .$$

The result of fitting can be shown in Table 1 and Fig.1, where the definition of relative error is

Table 1 Fitting result of MGM(1,3) model in 3.8m wavelength

Time	Resistance /kg			Heave /cm			Pitch /(°)		
	Actual value	Fitted value	Relative error	Actual value	Fitted value	Relative error	Actual value	Fitted value	Relative error
0.00	7.568	7.568	0.000	-9.594	-9.594	0.000	3.954	3.954	0.000
0.04	5.968	7.099	-6.813	-9.018	-8.821	-1.994	3.177	3.336	-3.363
0.08	5.921	6.438	-3.113	-7.867	-8.031	1.656	2.525	2.687	-3.408
0.12	7.003	6.975	0.169	-5.997	-7.108	11.239	1.811	2.226	-8.779
0.16	8.275	8.558	-1.706	-6.717	-6.319	-4.023	1.622	2.088	-9.855
0.20	11.476	10.729	4.501	-5.422	-5.893	4.770	2.631	2.312	6.727
0.24	14.490	12.861	9.804	-6.141	-5.954	-1.898	3.723	2.834	18.792
0.28	16.373	14.341	12.236	-7.004	-6.484	-5.271	3.975	3.503	9.984
0.32	16.608	14.741	11.245	-7.580	-7.330	-2.524	4.459	4.126	7.027
0.36	13.454	13.947	-2.969	-8.875	-8.250	-6.319	4.711	4.524	3.951
0.40	10.393	12.190	-10.816	-9.450	-8.978	-4.780	4.732	4.582	3.176
0.44	7.474	9.978	-15.075	-9.882	-9.304	-5.849	4.059	4.283	-4.724
0.48	6.156	7.951	-10.811	-9.450	-9.135	-3.192	3.450	3.715	-5.592
0.52	6.156	6.696	-3.254	-8.155	-8.520	3.692	2.778	3.041	-5.569
0.56	6.674	6.576	0.589	-7.004	-7.638	6.416	1.979	2.457	-10.106
0.60	8.463	7.626	5.041	-5.853	-6.745	9.021	1.580	2.132	-11.670
0.64	10.346	9.543	4.837	-5.278	-6.098	8.302	1.706	2.160	-9.586
0.68	12.936	11.774	6.993	-5.853	-5.886	0.331	3.009	2.532	10.075
0.72	15.431	13.677	10.565	-6.141	-6.170	0.292	3.639	3.142	10.505
0.76	15.290	14.701	3.547	-7.292	-6.868	-4.287	3.954	3.814	2.971
0.80	14.395	14.553	-0.946	-8.443	-7.780	-6.705	4.627	4.353	5.780
0.84	11.382	13.276	-11.402	-9.018	-8.643	-3.800	4.711	4.605	2.233

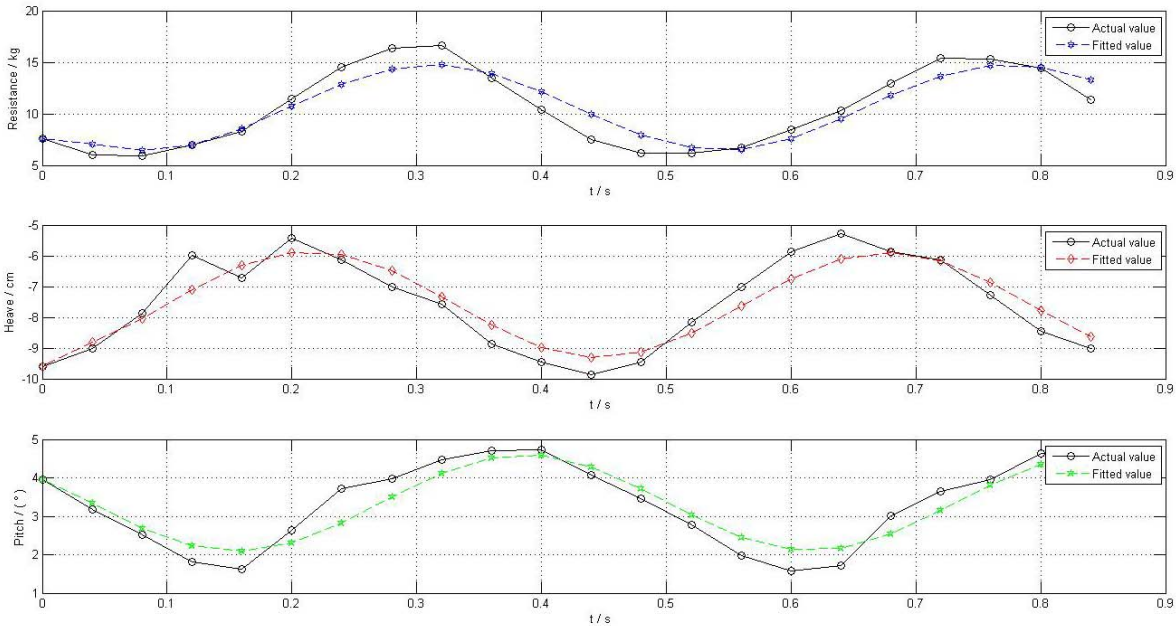


Fig.1 Curves of fitting effect

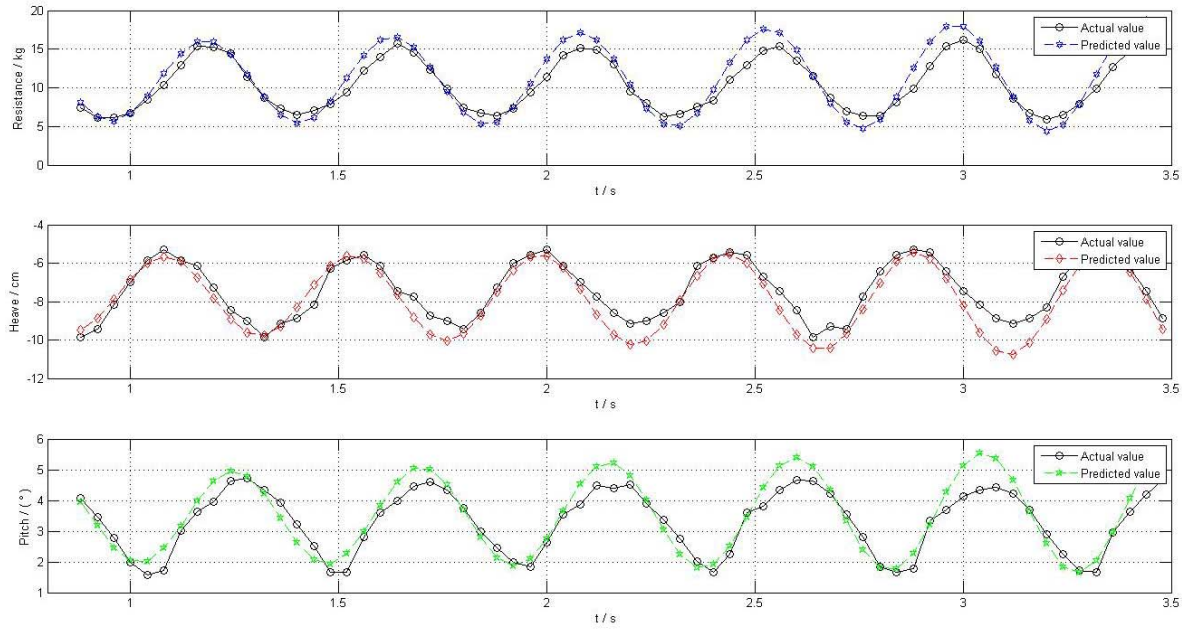


Fig.2 Curves of prediction

The result of prediction can be shown in Fig.2. As shown in Fig.2, the model built by this paper can reflect the trend of each variable, and can predict the data of 53 groups with the relative error less than 20%.

Then taking the observed value from the 2<sup>nd</sup> to the 23<sup>rd</sup> time point as original data, a new MGM(1,3) is built and the parameter matrix calculated by the recurrence formula proposed by this paper is

$$\hat{A} = \begin{bmatrix} 0.669 & -0.708 & -3.683 \\ -0.014 & -0.435 & -0.933 \\ 0.153 & 0.111 & -0.232 \end{bmatrix}, \quad \hat{B} = \begin{bmatrix} 6.182 \\ -8.313 \\ 2.937 \end{bmatrix}.$$

Having been checked, the parameter matrix of  $\hat{A}$  and  $\hat{B}$  calculated by the recurrence formula is equal to the parameter matrix recalculated by directly using the observed values from the 2<sup>nd</sup> to the 23<sup>rd</sup> time point. Thus the correctness of the recurrence formula of MGM(1,*N*) model's parameter matrix proposed by this paper is verified. Meanwhile, in order to verify the universality of this model, the MGM(1,*N*) models using the data acquired in the other wavelength were also built. The result of numerical experiment shows that the satisfactory effect can be obtained using the data acquired in the wavelength less than 4.8m, that is to say, the MGM(1,*N*) model built by taking the data of 2 to 3 periods as the original data can predict the data of 2 to 3 periods with the relative error less than 20%. While Zhu (2007) built a time series model taking the data of 200 time points, which can only predict the data of 20 time points with the relative error less than 20%.

While using MGM(1,*N*) model to predict the planing craft

motion, every group of newly obtained data should be added into the original data and reject the data of the former group. Then the new parameter matrix can be calculated using the recurrence formula proposed by this paper. In this way, data can be updated real-timely without increasing computational complexity significantly.

## 4 Conclusions

In this paper, MGM(1,*N*) model was firstly applied into the prediction of high speed planing craft short-time motion and satisfactory effect was obtained. Because of the characteristics of planing craft such as high speed and randomness and the priority principle of new information in Grey System Theory, while using MGM(1,*N*) model to predict the planing craft short-time motion, every new group of data should be combined with the former data and the prior data should be eliminated. Based on above reasons, the recurrence formula of parameter matrix of MGM(1,*N*) model was proposed, and the correctness and effectiveness of the method was also verified by the numerical simulation experiment. However, the numerical simulation experiment is only based on the experimental data of the planing craft model in numerical wave tank, and may have certain differences with the motion of planing craft sailing in real wave. So the results should be verified further.

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