J. Marine Sci. Appl. (2011) 10: 220-225
DOI: 10.1007/s11804-011-1062-5

Loop Soliton Solutions of a Short Wave Model for a Degasperis-Procesi Equation

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Abstract: An analytic method, i.e. the homotopy analysis method, was applied for constructing the solutions of the short waves model equations associated with the Degasperis-Procesi (DP) shallow water waves equation. The explicit analytic solutions of loop soliton governing the propagation of short waves were obtained. By means of the transformation of independent variables, an analysis one-loop soliton solution expressed by a series of exponential functions was obtained, which agreed well with the exact solution. The results reveal the validity and great potential of the homotopy analysis method in solving complicated solitary water wave problems.
Keywords: homotopy analysis method; one-loop soliton; explicit analytic solution; nonlinearity; Degasperis-Procesi equation
Article ID: 1671-9433(2011)02-0220-06

1 Introduction

The discovery of soliton solutions to the Korteweg de Vries (KdV) equation, a model equation arising as a nonlinear approximation to the governing equations for water waves, had a profound impact on hydrodynamic research. In recent years, two nonlinear equations which also arose as approximations to the governing equations for water waves, the Camassa-Holm (CH) equation (Camassa and Holm, 1993) and the Degasperis- Procesi (DP) equation (Degasperis and Procesi, 1999) had attracted a lot of attention due to their integrable structure as infinite bi-Hamiltonian systems and to the fact that their solitary wave solutions are solitons.

Both model equations are higher-order approximations to the governing equations for water waves than KdV (Constantin and Lannes, 2009; Kolev, 2009) and this feature makes it possible for them to capture one important phenomenon that can not be modeled within the KdV approximation. The Degasperis-Procesi (DP) equation has looplike soliton and thus it is not easy to solve it.
We consider the propagation of short wave for the Degasperis-Procesi (DP) equation.

$$u_{txx} + 3\kappa^3 u_x + 3u_x u_{xx} + uu_{xxx} = 0 \tag{1}$$

Solitons are solitary waves (that is, traveling waves moving at constant speed without change of shape and such that their profile is asymptotically flat) with the property that they recover their shape and speed after interaction with another wave of the same type.

At the same time, finding explicit analytic solutions of nonlinear partial differential equations (NLPDEs) is extremely important in mathematical physics. In recent years, many powerful methods have been developed to construct explicit analytic solution of NLPDEs. Liao (Liao SJ, 1992) employed the basic ideas of the homotopy in topology to propose method for nonlinear problems, namely homotopy analysis method (HAM) (Liao, 2010; Liao and Campo, 2002; Liao, 2003a; Liao, 2003b). This method has been successfully applied to solving many types of nonlinear problems (Molabahrami and Khania, 2009; Sajid and Hayat, 2008; Rashidi and Dinarvand, 2009; Dinarvand and Rashidi, 2010).

Homotopy analysis method is applied to solve such a multiple-valued nonlinear problem with the one-loop soliton solution. The soliton solution solved by the homotopy analysis method is verified by the exact one given in (Matsuno, 2006). This further demonstrates the validity and effectiveness of the homotopy analysis method in solving complicated nonlinear solitary wave problems.

2 The basic idea of homotopy analysis method

To illustrate the basic ideas of this method, we consider the

Received date: 2010-07-20.
Foundation item: Supported by the Natural Science Foundation of China under the grant 11026165 and 11072053 and Doctaral Fund of Ministry of Education of China under the grant 20100041120037 and the Fundamental Research Funds for the Central Universities.
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following nonlinear differential equation:

$$\mathscr{N}(u(t))=0 \tag{2}$$

where $\mathscr{N}$ is a nonlinear differential operator, t denote independent variables, $u(t)$ is an unknown function. For simplicity, boundary or initial conditions are not considered here. Based on the constructed zero-order deformation equation by Liao (Abbasbandy, 2008), we give the following zero-order deformation equation in the similar way

$$(1-q)\mathscr{L}[\phi(t;q)-u_0(t)]=qhH(t)\mathscr{N}[\phi(t;q)] \tag{3}$$

where $q\in[0,1]$ is the embedding parameter, h is a nonzero auxiliary parameter, $H(t)$ is a nonzero auxiliary function, $\mathscr{L}$ is an auxiliary linear operator, $u_0(t)$ is an initial guess of $u(t)$, $\phi(t;q)$ is an unknown function on independent variables t, and q. It is important that one has great freedom to choose auxiliary parameter h in homotopy analysis method (HAM). When q = 0 and q = 1, we have from the zero-order deformation Eq.(2) that $\phi(t;0)=u_0(t)$ and $\phi(t;1)=u(t)$.

Thus, as q increases from 0 to 1, the solution $\phi(t;q)$ varies from the initial guess $u_0(t)$ to the solution $u(t)$. Defining

$$u_{m(t)}=\frac{1}{m!}\frac{\partial^m\phi(t;q)}{\partial q^m}\bigg|_{q=0} \tag{4}$$

and expanding $\phi(t;q)$ in Taylor series with respect to the embedding parameter q, we have

$$\phi(t;q)=\phi(t;0)+\sum_{m=1}^{+\infty}u_m(t)q^m \tag{5}$$

If the auxiliary linear operator, the initial guess, the auxiliary parameter h and the auxiliary function $H(t)$ are properly chosen, the series Eq.(4) converges at q = 1, one has

$$u(t)=u_0(t)+\sum_{m=1}^{+\infty}u_m(t) \tag{6}$$

Define the vector

$$\boldsymbol{u}_m(t)=\{u_0(t),u_1(t)\cdots,u_m(t)\} \text{ where } n\in N, m\in N. \tag{7}$$

Differentiating the zero-order deformation Eq.(2) m times with respect to q, and finally dividing by m!, we have the mth-order deformation equation

$$\mathscr{L}[u_m(t)-\chi_m u_{m-1}(t)]=hH(t)R_m(\boldsymbol{u}_{m-1}(t)) \tag{8}$$

and

$$\chi_m=\begin{cases}0 & \text{where } m\le 1,\\ 1 & \text{where } m>1.\end{cases} \tag{9}$$

The mth-order deformation equation Eq.(8) is linear and thus can be easily solved, especially by means of symbolic computation software MAPLE or MATHEMATICA.

3 Applying homotopy analysis method to the Degasperis-Procesi (DP) equation

3.1 Mathemaical formulation with the transformation

To verify the validity and the potential of HAM, we apply it to the Degasperis-Procesi equation, which describes the motions of waves in nonlinear fluids. Analytic solutions, loop soliton water waves, for this equation is obtained by use of the present method. The validity and effectiveness of the HAM in solving the nonlinear loop soliton water waves problem are shown.

$$u_{txx}+3\kappa^3u_x+3u_xu_{xx}+uu_{xxx}=0 \tag{10}$$

$$u_{tx}+3\kappa^3u+u_xu_x+uu_{xx}=0 \tag{11}$$

$$\begin{aligned}&\{u_{tx}+3\kappa^3u+u_xu_x+uu_{xx}\}_x=\\&u_{txx}+3\kappa^3u_x+3u_xu_{xx}+uu_{xxx}\end{aligned} \tag{12}$$

we should solve this equation

$$u_{tx}+3\kappa^3u+u_xu_x+uu_{xx}=(u_t+uu_x)_x+3\kappa^3u=c(t) \tag{13}$$

If we take the coordinate transform

$$\begin{aligned}&\mathrm{d}x=(1/\kappa+W_X)\mathrm{d}X+U\mathrm{d}T,\\&\mathrm{d}t=\mathrm{d}T\end{aligned} \tag{14}$$

where

$$W=\int_{-\infty}^{T}U(X,\xi)\mathrm{d}\xi \tag{15}$$

From this coordinate transform, we can easily see that its equivalence form can be written as

$$\begin{aligned}&x=X/\kappa+W+x_0\\&t=T\end{aligned} \tag{16}$$

If we convert the DP equation (13) with x and t phase space into equation with X and T phase space, based on the coordinate transform (16), the chain rule should be applied in the form of

$$\begin{aligned}&\frac{\partial}{\partial X}=(\frac{1}{\kappa}+W_X)\frac{\partial}{\partial x},\\&\frac{\partial}{\partial T}=u\frac{\partial}{\partial x}+\frac{\partial}{\partial t}.\end{aligned} \tag{17}$$

So the equation (13) can be transformed into the X and T phase space in the form of

$$U_{XT}+3\kappa^3(\frac{1}{\kappa}+W_X)U=c(T). \tag{18}$$

from the expression of W, we can rewrite equation (18) in terms of W in the following form:

$$W_{XTT} + 3\kappa^3(\frac{1}{\kappa} + W_X)W_T = c(T) \qquad (19)$$

Setting $c(T)$ as a constant, then taking travelling wave transform $\eta = kX - \omega T$, we get

$$k\omega^2 W''' - 3\kappa^3 k\omega W'^2 - 3\kappa^2 \omega W' = c. \qquad (20)$$

According to the asymptotic theory, we should balance the main terms of this equation in the (Bender CM and Orszag SA , 1978).
Write

$$W = B\exp(\mu\eta), \ \ as \ \ n \to -\infty \qquad (21)$$

where B is a constant. Substituting (21) into (20) and balancing the main term yields

$$\mu = \frac{\sqrt{3}\kappa}{\sqrt{k\omega}}. \qquad (22)$$

Under the transformation

$$\theta = \mu\eta = \sqrt{3}\kappa(\sqrt{\frac{k}{\omega}}y - \sqrt{\frac{\omega}{k}}T) \qquad (23)$$

equation (20) is transformed into

$$W_{\theta\theta\theta} - \sqrt{3}\kappa\sqrt{\frac{k}{\omega}}W_\theta{}^2 - W_\theta = \frac{c}{3\sqrt{3}\kappa^3}\sqrt{\frac{k}{\omega}} \qquad (24)$$

i.e.

$$W_{\theta\theta\theta\theta} - 2\sqrt{3}\kappa\sqrt{\frac{k}{\omega}}W_\theta W_{\theta\theta} - W_{\theta\theta} = 0 \qquad (25)$$

The boundary conditions of equation (25) are given below. Due to the definition (15), we have

$$W(-\infty) = 0 \quad W'(-\infty) = 0 \qquad (26)$$

Considering the symmetry of U(X,T) in X - T space and the continuation of its 1st-order derivative, we have

$$W'(\theta) = W'(-\theta), \quad W''(\theta) = 0 \qquad (27)$$

which means that

$$W(0) = A/2, W(+\infty) = A, W(-\infty) = 0, W''(0) = 0. \qquad (28)$$

3.2 Approximation of loop solution with the homotopy analysis method

Then, we apply the homotopy analysis method to obtain $W(h)$ on $h > 0$, because W(h) on $h < 0$ can be obtained from (18) by the symmetry.
Under the transformation

$$W(\theta) = A + \frac{A}{2}g(\theta) \qquad (29)$$

Eq. (25) becomes

$$g'''' - 2\nu g' g'' - g'' = 0, \qquad (30)$$

where $\nu = A\sqrt{3}\kappa\sqrt{\frac{k}{\omega}}g'$ is the derivative with respect to θ , subject to the boundary conditions

$$g(0) = 1, g''(0) = 0, g(+\infty) = 0, g(-\infty) = -2. \qquad (31)$$

Under the rule of solution expression and with the aid of the governing Eq.(30), we choose the initial approximation

$$g_0(\theta) = -\frac{4}{3}e^{(-\theta)} + \frac{1}{3}e^{(-2\theta)}. \qquad (32)$$

And the auxiliary linear operator

$$\mathscr{L}[\phi(\theta;q)] = \frac{\partial^4\phi(\theta;q)}{\partial\theta^4} - \frac{\partial^2\phi(\theta;q)}{\partial\theta^2} \qquad (33)$$

possesses the property

$$\mathscr{L}(C_1 + C_2 x + C_3 e^{\theta} + C_4 e^{-\theta}) = 0 \qquad (34)$$

where C_1 , C_2 , C_3 , C_4 are integral constants to be determined by initial condition. Furthermore, Eq.(30) defines the nonlinear operator

$$\mathscr{N}[\phi(\theta;q)] = \frac{\partial^4\phi(\theta;q)}{\partial\theta^4} + 2\Gamma(q)\frac{\partial\phi(\theta;q)}{\partial\theta}\frac{\partial^2\phi(\theta;q)}{\partial\theta^2} - \frac{\partial^2\phi(\theta;q)}{\partial\theta^2} \qquad (35)$$

Let $q \in [0,1]$ denote an embedding parameter, $h \neq 0$ an auxiliary parameter. Using above definitions, we construct the zero-order deformation equation

$$(1-q)\mathscr{L}[\phi(\theta;q) - g_0(\theta)] = qh\mathscr{N}[\phi(\theta;q)] \qquad (36)$$

with the initial condition

$$\phi(0,q) = 1, \quad \phi''(0,q) = 0, \quad \phi(+\infty,q) = 0 \qquad (37)$$

We can expand $\phi(\theta,q)$ and $\Gamma(q)$ in power series of q as follows:

$$\phi(\theta,q) = g_0(\theta) + \sum_{i=1}^{M} g_i(\theta)q^i \qquad (38)$$

$$\Gamma(q) = \Gamma_0 + \sum_{i=1}^{m}\Gamma_i q^i \qquad (39)$$

Obviously, when $q = 0$ and $q = 1$,

$$\phi(\theta,0) = g_0(\theta), \quad \phi(\theta,1) = g(\theta), \qquad (40)$$

$$\Gamma(1) = \nu. \qquad (41)$$

According to (36) and (37), we get the mth-order deformation equation

$$\mathscr{L}[g_m(\theta) - \chi_m g_{m-1}(\theta)] = hR_m(g_{m-1}(\theta)) \qquad (42)$$

with initial condition

$$g_m(0)=1, g_m''(0)=0, g_m(+\infty)=0 \tag{43}$$

where

$$R_m(g_{m-1}(\theta))=\frac{\partial^4 g_{m-1}(\theta)}{\partial\theta^4}+\sum_{k=0}^{m-1}\left(\sum_{j=0}^{k}\frac{\partial^2 g_j(\theta)}{\partial\theta^2}\frac{\partial g_{k-j}(\theta)}{\partial\theta}\right)\Gamma_{m-1-k}-\frac{\partial^2 g_{m-1}(\theta)}{\partial\theta^2} \tag{44}$$

and

$$\chi_m=\begin{cases}0 & \text{where } m\le 1,\\ 1 & \text{where } m>1.\end{cases} \tag{45}$$

It should be emphasized that $u_{n,m}(t),(m\ge 1)$ is governed by the linear equation (42) with the linear initial conditions (43). $R_m(g_{m-1}(\theta))$ is dependent upon g_{m-1} and Γ_{m-1} that contain the unknown Γ_{m-1}. The solution of Eq. (44) can be expressed by

$$g_m(\theta)=g^*(\theta)+C_1+C_2x+C_3\mathrm{e}^{\theta}+C_4\mathrm{e}^{-\theta} \tag{46}$$

where C_1, C_2, C_3 and C_4 are the integral constants, $g^*(\theta)$ is a special solution of Eq. (42) and it contains the unknown Γ_{m-1}. Due to the boundary condition (43) at infinity, C_1 and C_2 must be zero. The unknown Γ_{m-1} and the constant C_3 are determined by the two boundary conditions (43) at $\theta=0$.
We get all the solutions as follows:

$$\begin{aligned}g_1(\theta)&=-\frac{38}{165}\mathrm{e}^{-\theta}-\frac{2}{11}\mathrm{e}^{-3\theta}+\frac{13}{33}\mathrm{e}^{-2\theta}+\frac{1}{55}\mathrm{e}^{-4\theta},\\ g_2(\theta)&=-\frac{164826}{1164625}\mathrm{e}^{-\theta}-\frac{8751}{33275}\mathrm{e}^{-3\theta}+\frac{17328}{166375}\mathrm{e}^{-4\theta}-\\ &\frac{57}{3025}\mathrm{e}^{-5\theta}+\frac{10584}{33275}\mathrm{e}^{-2\theta}+\frac{24}{21175}\mathrm{e}^{-6\theta}\cdots\cdots.\end{aligned} \tag{47}$$

Then the solution expression can be written in an accurate form:

$$g(\theta)=g_0(\theta)+g_1(\theta)+g_2(\theta)+\cdots\cdots \tag{48}$$

$$\nu=\Gamma_0+\Gamma_1+\Gamma_2+\cdots\cdots \tag{49}$$

Thus, HAM provides us with a family of solution expression in the auxiliary parameter *h*. The convergence region of solution series depend upon the value of *h*.

In general, the approximate solutions with order M terms can be written in following

$$\begin{aligned}g(\theta)&\approx\sum_{m=0}^{M}g_m(\theta)=\sum_{k=1}^{2M+2}\beta_k\exp(-k\theta)\\ \nu&\approx\sum_{m=0}^{M}\Gamma_m\end{aligned} \tag{50}$$

where β_k is coefficients which can be given by symbol computation system MAPLE.

We have converted the DP equation expressed by $u(x;\ t)$ into equation (30) expressed by $W(\theta)$ and their boundary conditions, respectively. Now, we need to review the concrete transforms between solutions to DP equation and solutions to equation (30).

$$u(x,t)=W_T(\theta)=-\sqrt{3}\kappa\sqrt{\frac{\omega}{k}}W_\theta(\theta)=-\sqrt{3}\kappa\sqrt{\frac{\omega}{k}}\frac{A}{2}g'(\theta), \tag{51}$$

$$\begin{aligned}&x-\frac{\omega}{kk}t=\frac{y}{k}+W+x_0-\frac{\omega}{kk}T=\\ &A+\frac{A}{2}g(\theta)+x_0+\frac{1}{\sqrt{3}k^2}\sqrt{\frac{\omega}{k}}\theta\end{aligned} \tag{52}$$

where $A=\frac{\nu}{\sqrt{3}\kappa}\sqrt{\frac{\omega}{k}}$ and x_0 is a constant. For the symmetry in x-t space, we have

$$x_0=-\frac{A}{2} \tag{53}$$

then, the *M*th-order approximation solution to DP equation can be expressed by

$$u(x,t)=\frac{\nu\omega}{2k}\sum_{k=1}^{2M+2}k\beta_k\exp(-k\theta) \tag{54}$$

where θ satisfies

$$x-\frac{\omega}{k\kappa}t=\frac{A}{2}g(\theta)+\frac{1}{\sqrt{3}\kappa^2}\sqrt{\frac{\omega}{k}}\theta. \tag{55}$$

4 Result analysis

In this section, we verify our analytic solutions with the exact solutions (Matsuno Y, 2006).

$$\begin{aligned}&u(y,t)=\frac{9\kappa}{2k^2}\sec h^2(\frac{\xi}{2}),\\ &x=\frac{y}{\kappa}-\frac{3}{\kappa k}\tanh(\frac{\xi}{2})+d,\xi=k(y-\frac{3\kappa^2}{k^2}t+y_{10})\end{aligned} \tag{56}$$

Note that our solution series contains the parameter *h*, which provides us with a simple way to adjust and control the convergence of the solution series, in general, by means of the so-called *h*-curve, i.e. a curve of a versus *h*. As pointed by Liao (Liao SJ, 1992), the valid region of *h* is a horizontal line segment. Thus, the valid region of *h* in this case is -2 < *h* < 0, as shown in Fig.1. Obviously, our solution series (50) converges to the exact value ν = 3. Of course, the accuracy can be improved by computing more terms of the approximate solution. For example, when *h* =-1, our analytic solution converges. The analytic solution of (30) is presented by Fig.2. Our approximate loop soliton solution of the Degasperis-Procesi equation (1) is expressed by a series of exponential functions (54) and (55). Compared with the exact solution, our 12th-order approximation when h =-1 as is shown in Fig.3. Obviously, our analytic approximation agrees well with the exact one. This verifies the validity and effectiveness of the homotopy analysis method to loop soliton water waves problem.

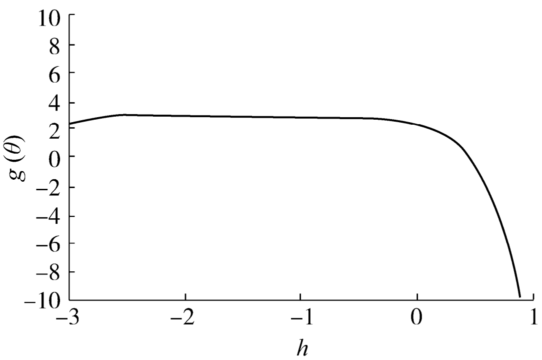


Fig.1 Solution of V of 12th-order approximation under different *h*

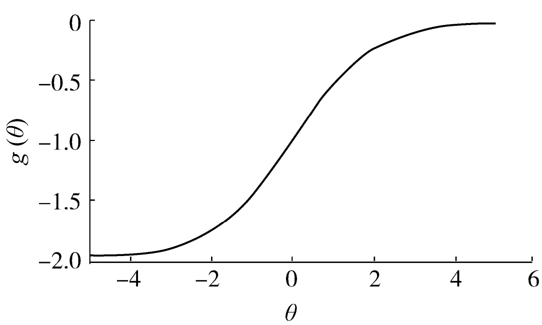


Fig.2 The 12th-order analytic approximation of (31)

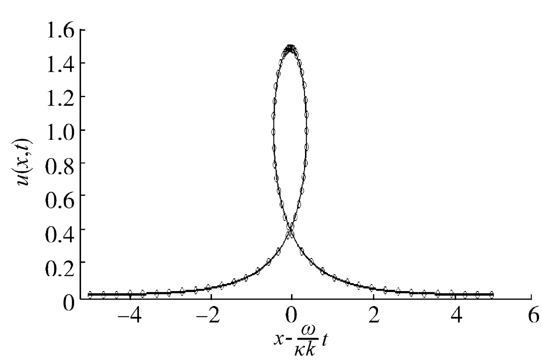


Fig.3 Comparison of the exact solution with the loop soliton solution of HAM approximation, when *h*=-1. Circle symbols: exact solution; solid line: loop soliton solution of HAM approximation

5 Conclusions

We successfully applied the homotopy analysis method to solve loop soliton water waves. We obtain the explicit analytic solutions of loop soliton governing the propagation of short waves. By means of the transformation of independent variables, an analysis of one-loop soliton solution expressed by a series of exponential functions is obtained, which agrees well with the exact solution. We have shown that short-wave models exhibit loop solitons types of solutions. This proposed method provides us with a new analytic way to solve loop soliton wave problems. So, the HAM has great flexibility and potential for complicated nonlinear problems. Application of the results presented here to the short-wave dynamics in real fluid systems will be an interesting topic in the future research.

References

Abbasbandy S (2008). Soliton solutions for the Fitzhugh-Nagumo equation with the homotopy analysis method. *Applied Mathematical Modelling*, **32**, 2706-2714.

Abbasbandy S (2010). Homotopy analysis method for the Kawahara equation. *Nonlinear Analysis: Real World Applications*, **11**, 307-312.

Bender CM, Orszag SA (1978). *Advanced mathematical methods for scientists and engineers*, Mcgraw-hill, NewYork.

Camassa R and Holm DD (1993). An integrable shallow water equation with peaked solitons, Phys. Rev. Lett, **71**(11), 1661-1664.

Constantin A and Lannes D (2009). The hydrodynamical relevance of the Camassa-Holm and Degasperis-Procesi Equations. *Arch. Rat. Mech. Anal*, **192**, 165-186.

Degasperis A and Procesi M (1999). *A degasperis, g. gaeta (eds.), asymptotic integrability*. Symmetry and perturbation theory. World Scientific Singapore, 23-37.

Dinarvand S and Rashidi MM (2010). A reliable treatment of a homotopy analysis method for two-dimensional viscous flow in a rectangular domain bounded by two moving porous walls, Nonlinear Analysis: Real World Applications, **111**, 502-1512.

Kolev B (2009). Some geometric investigations on the Degasperis -Procesi shallow water equation. *Wave Motion*, **46**, 412-419.

Liao SJ (1992). Proposed homotopy analysis techniques for the solution of nonlinear problems, Ph.D. Thesis, Shanghai Jiao Tong University.

Liao SJ and Campo A (2002). Analytic solutions of the temperature distribution in Blasius viscous flow problems. *J. Fluid Mech* , **453,** 411-425.

Liao SJ (2003a). Beyond perturbation: introduction to the homotopy analysis method. CRC Press, Boca Raton, Chapman Hall.

Liao SJ (2003b). On the analytic solution of magnetohydrodynamic flows of non-Newtonian fluids over a stretching sheet. *J. Fluid Mech*, **488**, 189-212.

Liao SJ (2010). An optimal homotopy-analysis approach for strongly nonlinear differential equations, *Commun. Nonlinear Sci. Numer. Simulat*, **15**, 2003-2016.

Matsuno Y (2006). Cusp and loop soliton solutions of short-wave models for the Camassa-Holm and Degasperis-Procesi equations. *Physics Letters A*, **359**, 451-457.

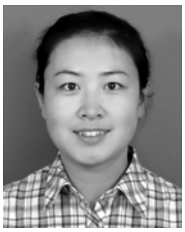
Molabahrami A and Khania F (2009). The homotopy analysis method to solve the Burgers-Huxley equation. *Nonlinear Analysis: Real World Applications*, **10**, 589-600.

Niu Z and Wang C (2010). One-step optimal homotopy analysis method for nonlinear differential equations. *Commun. Nonlinear Sci. Numer. Simulat.*, **15**, 2026-2036.

Rashidi MM and Dinarvand S (2009). Purely analytic approximate solutions for steady three-dimensional problem of condensation Film on inclined rotating disk by homotopy analysis method. *Nonlinear Analysis: Real World Applications*, **10**, 2346-2356.

Sajid M and Hayat T (2008). The application of homotopy analysis method to thin film flows of a third order fluid. *Chaos, Solitons and Fractals*, **38**, 506-515.

Xu H (2005). An explicit analytic solution for convective heat transfer in an electrically conducting fluid at a stretching surface with uniform free stream. *International Journal of Engineering Science*, **43** ,859-874.



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