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Pressure Drop Fluctuations in Periodically Fluctuating Pipe Flow

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Abstract: Experiments were conducted to study characteristics of flow when flow is fluctuating. The experimental results showed a phase difference between the flow rate and the pressure drop fluctuations. This phase difference between the fluctuating flow rate and pressure drop was analyzed for laminar flow. Analysis showed that the phase difference changes with the period of the flow fluctuation, the pipe radius, the density and the dynamic viscosity of the liquid. Fluctuating pipe flow was then numerically simulated. Results of the numerical simulation were compared with theoretical values and experimental results. It was shown that, when the flow rate fluctuates with time as a sine wave, the pressure drop fluctuates with the same periodicity, and there is a phase difference between them.
Keywords: flow fluctuation; pressure drop; flow rate; phase difference
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1 Introduction

Nuclear power ships would be influenced by ocean conditions. They would roll, incline or heave. Ocean conditions could make the coolant flow of the system fluctuate and would influence the thermal hydraulic characteristics of the equipments and system (Ishida *et al*., 1990; Gao *et al*., 1999; Ishida and Yoritsune, 2002; Murata *et al*., 2002). Experiments and theoretical analyses both proved this point (Pang *et al*., 1995; Pendyala *et al*., 2008a; Pendyala *et al*., 2008b; Tan *et al*., 2009).

When the flow is fluctuating, what is the pressure drop of pipe flow like? Are the conventional flow friction factor correlations still applicable? These questions need the study on the fluctuating flow. The study under the conditions of fluctuating flow has been conducted for years. Gerrard (1971) reported that a tendency for the flow laminarization was observed. Similar phenomenon was also investigated by Hino *et al*. (1976), Ohmi and Iguchi (1980), Iguchi and Ohmi (1984), Zhao and Cheng (1996), Barker *et al*. (2000), He and Jackson (2009).

In order to have the investigation means of the flow fluctuation under ocean conditions, a simplified experiment was performed on a specially designed test loop in present study. Under ocean conditions, the loop is in motion. While in present study, the experimental loop is at rest, only the flow is fluctuating. The present experiments are helpful to understand more about the fluctuating flow and to obtain the basic experimental data for further comparison. Theoretical analysis and numerical simulation were also carried out.

2 Flow fluctuation experiments

2.1 Experimental loop

Experiments have been performed on a stationary loop. The experimental loop consists of a water tank, a pump, and valves. The test section is a straight circular pipe. An electromagnetic flowmeter, a thermometer and a differential pressure transducer are used to measure the flow rate, fluid temperature and pressure drop across the test section respectively. The parameters are collected by the data acquisition system.

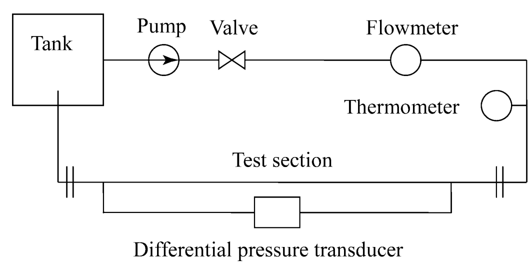

Fig.1 Sketch of experimental loop

In order to check out the reliability of the experimental loop and data acquisition system, pre-experiments were carried out in the condition of steady state. The pressure drop was measured by the differential pressure transducer and the U-tube manometer. The friction factor results got by the two different means agreed well, and they also agreed with the results calculated by conventional

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correlations (Finnemore and Franzini, 2003). This indicates that the experimental loop and data acquisition system are both reliable.

According to the relationship of the characteristics of pump and pipeline, flow fluctuation could be obtained by changing the characteristics of the pump or the pipeline. Both methods were used to change the flow rate in the experiments.

2.2 Experimental results

As the flow in the pipe fluctuated periodically, the pressure drop in the experiment pipe fluctuated, too, as shown in Fig.2.

Fig.2 shows that the periods of the two curves (Δp-t and Q-t curves) are the same. In the meantime, it should be noticed that there is a phase difference between the flow rate and pressure drop fluctuating curves. This could be seen also in Fig.3 and Fig.4, which show the Δp-t and Q-t curves in a short time for laminar flow and turbulent flow respectively.

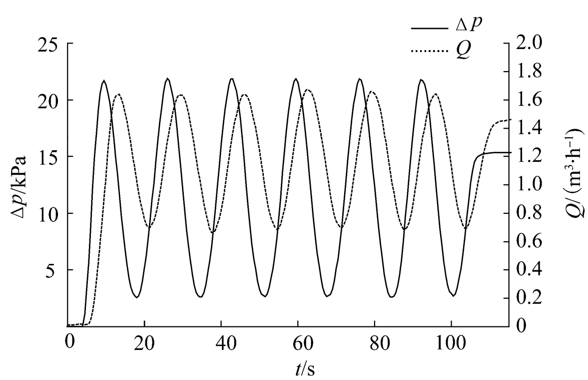

Fig.2 Flow rate and pressure drop fluctuation curves

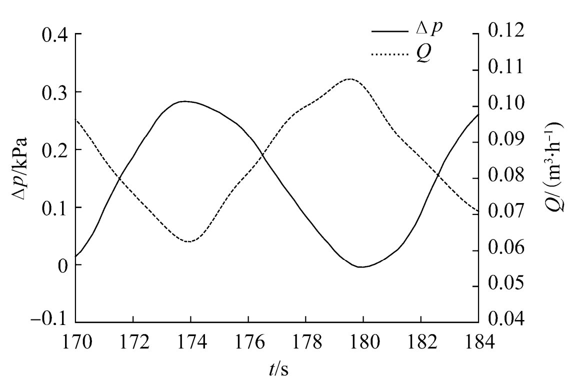

Fig.3 Phase difference in laminar flow

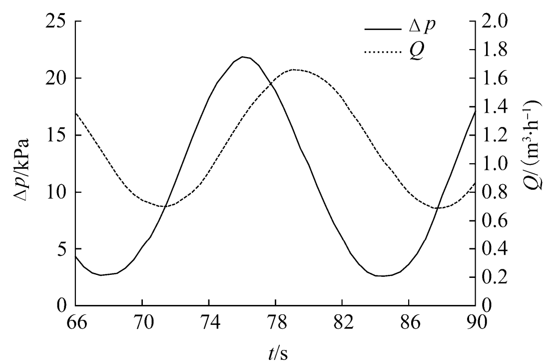

Fig.4 Phase difference in turbulent flow

3 Theoretical analysis of laminar flow fluctuation

In this paper, the fluctuating pressure drop in fluctuating laminar pipe flow is derived. For turbulent flow, the problem is too complicated, it is still under consideration.

In this part, a fluctuating laminar flow is considered. As shown in Fig.5, on any cross-section of the pipe, the value of ($p+\rho gh$) is constant. So the velocity profile of the flow is symmetrical.

The length of the pipe is l, the radius is r, the volume flow rate changes with time periodically. Analyzing the forces on the fluid element, a force equation can be written as

$$\pi r^2 p - \pi r^2 \left(p + \frac{\partial p}{\partial l} \mathrm{d}l\right) - 2\pi r \mathrm{d}l \tau - \pi r^2 \mathrm{d}l \rho g \sin\theta = \pi r^2 \rho \, \mathrm{d}l \frac{\mathrm{d}v}{\mathrm{d}t} \tag{1}$$

where v is the cross-section averaged velocity.

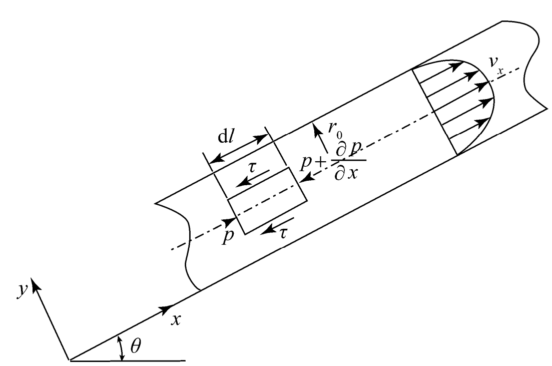

Fig.5 Laminar flow in pipe

For the experiment, the pipe is horizontal, $\theta = 0$, Eq.(1) becomes

$$r\rho \frac{\mathrm{d}v}{\mathrm{d}t} + r\frac{\partial p}{\partial l} + 2\tau = 0 \tag{2}$$

For laminar flow, $\tau = -\mu \dfrac{\mathrm{d}v_x}{\mathrm{d}r}$, where v_x is the local velocity in the axial direction. Eq.(2) can be expressed as

$$r\rho \frac{\mathrm{d}v}{\mathrm{d}t} + r\frac{\partial p}{\partial l} - 2\mu \frac{\mathrm{d}v_x}{\mathrm{d}r} = 0 \tag{3}$$

Integrating Eq.(3), we obtain

$$v_x = \frac{r^2}{4\mu}\left(\rho \frac{\mathrm{d}v}{\mathrm{d}t} + \frac{\partial p}{\partial l}\right) + C \tag{4}$$

The boundary condition is $v_x = 0$ when $r = r_0$, we obtain

$$C = -\frac{r_0^2}{4\mu}\left(\rho \frac{\mathrm{d}v}{\mathrm{d}t} + \frac{\partial p}{\partial l}\right)$$

The solution of Eq.(3) is

$$v_x = \frac{r^2 - r_0^2}{4\mu}\left(\rho \frac{\mathrm{d}v}{\mathrm{d}t} + \frac{\partial p}{\partial l}\right) \tag{5}$$

For the circular cross-section pipe, $Q = \int_0^{r_0} 2\pi r v_x \mathrm{d}r$, where v_x can be given by Eq.(5).

$$Q = \int_0^{r_0} 2\pi r v_x \mathrm{d}r = -\frac{\pi r_0^4}{8\mu}\left(\rho \frac{\mathrm{d}v}{\mathrm{d}t} + \frac{\partial p}{\partial l}\right) \tag{6}$$

We know that $\dfrac{\partial p}{\partial l} = -\dfrac{\Delta p}{l}$, where Δp is the pressure drop along l.

$$Q = -\frac{\pi r_0^4}{8\mu}\left(\rho \frac{\mathrm{d}v}{\mathrm{d}t} - \frac{\Delta p}{l}\right) \tag{7}$$

The volume flow rate can also be written as $Q = \pi r_0^2 v$, so Eq.(7) becomes

$$\Delta p = \frac{8\mu v l}{r_0^2} + \rho l \frac{\mathrm{d}v}{\mathrm{d}t} \tag{8}$$

In the experiment, the flow fluctuation is controlled to follow sine law. Assume the fluctuating flow rate to be

$$Q = A\sin\frac{2\pi t}{T} + B \tag{9}$$

The flow rate amplitude of the flow fluctuation is A, the average flow rate is B, the fluctuation period is T. For example, the flow shown in Fig.3 can be fitted as

$$Q = 0.022\sin\frac{2\pi t}{11.1} + 0.084 \quad (\mathrm{m^3/h})$$

The flow shown in Fig.4 can be fitted as

$$Q = 0.484\sin\frac{2\pi t}{16.5} + 1.172 \quad (\mathrm{m^3/h})$$

Therefore, from Eq.(7), the area-averaged velocity and acceleration of the flow are

$$v = \frac{1}{\pi r_0^2}\left(A\sin\frac{2\pi t}{T} + B\right) \tag{10}$$

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{2A}{r_0^2 T}\cos\frac{2\pi t}{T} \tag{11}$$

Substitute velocity v and acceleration a into Eq.(8) to yield

$$\Delta p = \frac{8\mu l A}{\pi r_0^4}\sin\frac{2\pi t}{T} + \frac{2\rho l A}{r_0^2 T}\cos\frac{2\pi t}{T} + \frac{8\mu l B}{\pi r_0^4} \tag{12}$$

This shows that the pressure drop is also a harmonic function of time t when the flow fluctuates in terms of harmonic law such as the sine law. Next, derive the phase difference between the flow rate $Q(t)$ and the pressure drop $\Delta p(t)$.

According to trigonometric function, Eq.(12) can be written in the form

$$\Delta p = \sqrt{M^2 + N^2}\sin\left(\frac{2\pi t}{T} + \varphi\right) + C \tag{13}$$

where $M = \dfrac{8\mu l A}{\pi r_0^4}$, $N = \dfrac{2\rho l A}{r_0^2 T}$, $C = \dfrac{8\mu l B}{\pi r_0^4}$,

$$\varphi = \arctan\frac{\rho \pi r_0^2}{4\mu T} \tag{14}$$

Eq.(13) indicates that the pressure drop $\Delta p(t)$ fluctuates with the same period as the flow rate $Q(t)$, but with a different phase from $Q(t)$. There is a phase difference φ between the fluctuating flow rate $Q(t)$ and pressure drop $\Delta p(t)$ in laminar flow.

Eq.(14), expression of φ indicates that φ only depends on the values of r_0, T, ρ and μ, and it has nothing to do with the amplitude A and the averaged value B of the fluctuating flow rate $Q(t)$. The phase difference φ increases with the cross section area or the radius r_0 of the pipe. The longer the flow fluctuation period T, the smaller the phase difference φ. The bigger the viscosity $\dfrac{\mu}{\rho}$, the smaller the phase difference φ.

Fig.3 shows an experiment of fluctuating laminar flow with period T=11.1 s, the pipe radius r_0 =0.008 m, the water temperature 16℃, $\frac{\mu}{\rho}$ =1.109 2×10^{-6} m^2/s. Use Eq.(14) to calculate the phase difference,

$$\varphi = 1.33 \ \text{(rad)}$$

so the phase difference in second is

$$\frac{\varphi}{2\pi}\cdot T = \frac{1.33}{2\pi}\times 11.1 = 2.36\ \text{(s)}$$

This theoretical analysis shows that a phase difference does exist in laminar flow. Compared with the experimental result (see Fig.3), the experimental phase difference is 5.4 s. The two values 2.36 s and 5.4 s do not agree well.

4 Numerical simulation of fluctuating flow

The fluctuating pipe flow was numerically simulated. For the flow shown in Fig.3, the pressure drop per unit length in fully developed region was gotten, as shown in Fig.6. In this flow, the *Re* number fluctuates from 1 236 to 2 112, the flow is laminar. Fig.6 shows that as the flow in the pipe fluctuates periodically, the pressure drop in the pipe fluctuates with the same period. There is a phase difference between the two curves.

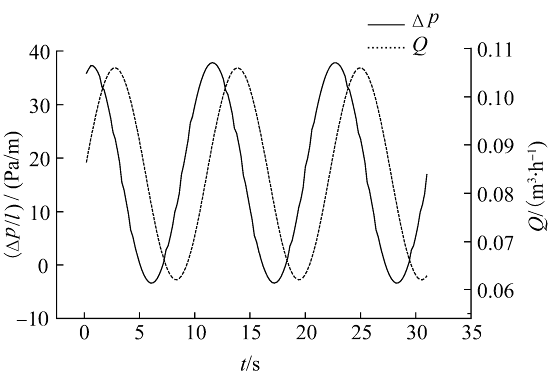

Fig.6 Numerical simulation results of laminar flow fluctuation

By Eq.(13), we get

$$\Delta p / l = 17.69 \sin(0.566\, t + 1.33) + 16.05 \quad \text{(Pa/m)} \qquad (15)$$

Fig.7 shows the pressure drops obtained by experiment, theoretical derivation Eq.(15) and CFD numerical simulation in the condition of Fig.3. The theoretical results and CFD results agree well, however, theoretical and CFD results are different from the experimental results in both phase and value.

For turbulent flow as shown in Fig.4, the *Re* number fluctuates from 13 750 to 32 881. The CFD results are drawn in Fig.8. Compared with experimental results, the CFD simulation results give a phase difference of 0.5 s, while the experimental results give 3.4 s. The pressure drop value by CFD simulation is about 1 kPa/m lower than the experimental value at the crests.

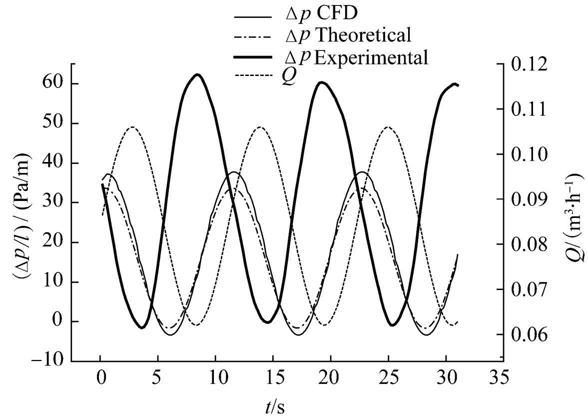

Fig.7 Numerical simulation results of laminar flow fluctuation and comparison with experimental results, theoretical analysis results

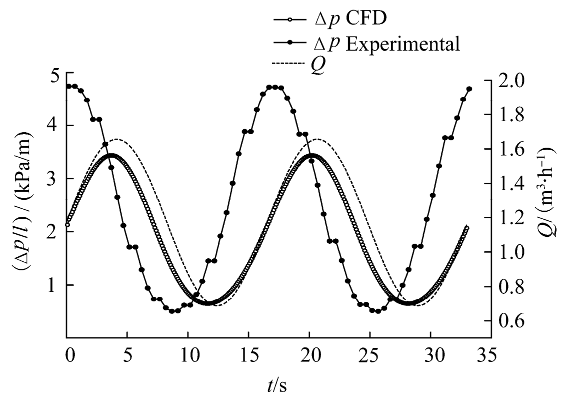

Fig.8 Numerical simulation results of turbulent flow fluctuation and comparison with experimental results

5 Discussion of results

Table 1 compares the phase differences obtained by three different methods for two flows shown in Fig.3 and Fig.4. According to the instructions of the electromagnetic flowmeter and the differential pressure transducer, both instruments have a time constant of 0.2 s. Therefore, the equal time constants of the two instruments should not induce phase difference. However, the measuring system consists of many components. The time constants of these components are unknown.

A special test was conducted in which the flow started up suddenly. The pump started up suddenly when the flow rate was zero. Then, the flow rate attained a steady value. In the meantime, the pressure drop of the test section increased suddenly from zero to a certain value. In this transition process, both the flow rate and pressure drop need time to attain the steady states although the transition time is very short. The test shows that there exists a time difference between them, about 1.2 s. This time difference 1.2 s subtracted from the experimental phase differences gives new results, also shown in Table 1.

Table 1 Comparison of phase differences between pressure drop and flow rate fluctuations

Flow	Phase difference/s			
	Exp.	$\text{Exp.}_{\text{-m}}$*	Theoretical	CFD
Laminar (Fig.3)	5.4	4.2	2.36	2.3
Turbulent (Fig.4)	3.4	2.2	-	0.5

* The $\text{Exp.}_{\text{-m}}$ means the modified experimental result, which is the experimental phase difference minus 1.2 s.

By now, it could be seen that the phase difference could be induced by many factors. On the one hand, the measuring system induces phase difference; on the other hand, the phase difference may be the inherent characteristics of the experimental system. The theoretical analysis and numerical simulation show that in the flow fluctuation condition, phase difference between flow rate and pressure drop is a real physical existance, not only due to the measuring system. For further study of the other characteristics of the flow fluctuation, such as the friction factor, the phase difference components must be identified, and the value of every component is also indispensable. The phase difference due to the measuring system should be removed in the data processing. Next, we will try to measure the inherent phase difference of the measuring system, and the real time constants of the measuring instruments. It is not easy to do.

This paper places emphasis on the phase difference between the flow rate and pressure drop. It could be seen from the figures that the values of the calculated and experimental pressure drops also have difference. Under the same conditions, the experimental pressure drop is bigger than the CFD simulation value. One reason may be the friction factor under flow fluctuation condition really increases. This trend is obtained from data processing, but further systematic experiments are needed to verify and quantify the trend. The possible change of the friction factor is under study. Preliminary analysis is that the flow zone may change due to flow fluctuation, for example, the hydraulically-smooth-pipe flow may change to rough-pipe flow (zone of complete turbulence), i.e. the surface roughness projections originally covered by the viscous sublayer now has pierced through the viscous sublayer. These analyses need further verification by experiments.

6 Conclusions

Experiments of fluctuating pipe flow show that when the flow rate fluctuates periodically, the pressure drop fluctuates periodically with the same period, but there is a phase difference between the fluctuating flow rate and pressure drop. This phase difference is also proved by theoretical analysis and numerical simulation.

1) The theoretical analysis shows that for laminar flow, the phase difference φ depends on r_0, T, ρ and μ. The turbulence flow is too complex to get an expression of φ for the moment.

2) Although the theoretical analysis can explain the phase difference, the theoretical result and CFD result do not agree well with experimental result of φ up to now. It is necessary to find the other reasons for the phase difference. Quick response sensors, transducers and measuring system should be selected in the experiment. The shorter the response time, the better the measurement.

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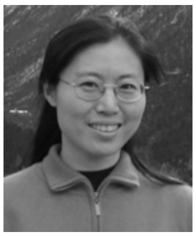

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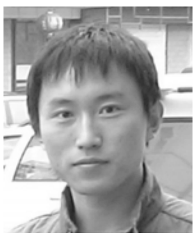

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