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The Effect of Hydrostatic Pressure Fields on the Dispersion Characteristics of Fluid-shell Coupled System

Zhi-zhong Liu*, Tian-yun Li, Xiang Zhu and Jun-jie Zhang

School of Naval Architecture and Ocean Engineering, Huazhong University of Science and Technology, Wuhan 430074, P.R. China

Abstract: The effect of hydrostatic pressure on the vibration dispersion characteristics of fluid-shell coupled structures was studied. Both fluid-loaded cylindrical shells and fluid-filled cylindrical shells were considered. Numerical analysis was applied to solve the dispersion equations for shells filled with or loaded with fluid at various hydrostatic pressures. The results for external pressure showed that non-dimensional axial wave numbers are nearly independent when the pressure is below the critical level. The influence of internal pressure on wave numbers was found significant for the real branch s=1 and the complex branches of dispersion curves. The presence of internal pressure increased the cut on frequencies for the branch s=1 for high order wave modes.

Keywords: hydrostatic pressure; dispersion; fluid-shell coupled system; wave propagation
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1 Introduction

Circular cylindrical shells are commonly used as primary structural components in aerospace and naval structures. These cylinders (e.g. submersible vehicles or high-pressure pipelines) are typically either surrounded by fluid under high pressure or are filled with it. The presence of a static prestress caused by hydrostatic pressure modifies structural response characteristics. These include the natural frequencies of vibration (Zhang *et al*., 2004) and the acoustic response (Keltie, 1983).

The vibration of shells not only causes noise pollution, but also affects the functionality of any devices mounted on the piping system. Hence, there was an obvious need to investigate the dynamic behavior of these structures, which can be effectively studied by considering wave propagation characteristics in an infinite cylinder. Wave propagation through cylindrical shells in a vacuum has been widely studied (Fuller,1981) including purely real, imaginary and complex branches, which can be easily computed.

However, when the shell is filled with fluid or immersed in fluid, considerable difficulty arises when calculatiing Bessel or Hankel functions. The propagation of waves through fluid contained in a cylindrical shell was researched by Lin *et al*. (1956) at first. He calculated the dispersion curves of axisymmetric waves in shells in the following cases: (1) liquid in a rigid tube; (2) liquid in an elastic tube; (3) liquid in an infinitely flexible tube, i.e. a cylinder immersed in fluid. His study is restricted to waves which have axially symmetric (circumferential mode order n=0) and purely real wave numbers. Kummer (1972) studied the dispersion of axially symmetric waves in empty and fluid-filled cylindrical shells of various wall-thickness, and purely real, imaginary and complex branches were also obtained in his study. Later researchers such as Chen and Rosenberg(1974) and Merkulov *et al*. (1979) computed dispersion curves for antisymmetric and non-symmetric wave propagation. Fuller and Fahy (1982) investigated the dispersion characteristics of free wave propagation in an infinite fluid-filled cylindrical shell. The method of iteration on the complex plane was applied to solve the dispersion equation. Their work was concerned with the solution and detailed physical interpretation of the dispersion equation for the coupled system. Both dispersion behavior and energy distribution of free waves were studied. The frequency characteristics of the acoustic wave transmission in a medium with mean flow are considered by Tsuji *et al*.(2002). Effect of the velocity of an internal flow on the dispersion characteristics and energy distribution in an infinite cylindrical shell has been examined by Brevart and Fuller (1993). Chen *et al*.(2000) has discussed the dispersion results of low and high order circumferential mode waves in cylindrical shells filled with fluid, and analyzed the fluid load item which reflects the interaction between shell wall and fluid. The forced vibration in thin pipe has been studied furthermore by Xu and Zhang (1998, 2000). Recently, an approach based on FEM (Maess *et al*., 2006; Mace *et al*., 2005) which allows periodic wave guides with any cross section to be analyzed has been applied to compute the dispersion curves for periodic structures.

The case of an infinite thin cylindrical shell loaded by fluid on the outside has been considered by Scott (1988) who computed the complex roots of the dispersion equation.

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***Corresponding author Email:** hichrisliu@163.com

However, many simplifications were introduced in his study. Schenck and Benthien (1995) have made a thorough study on the calculation of dispersion relation for this coupled system. The results were compared with those obtained by Scott (1988) and good agreement was obtained. Theoretical and experimental investigations of axisymmetric wave propagation in coupled fluid-pipe system have been presented by Sinha *et al*. (1992) and Plona *et al*. (1992). Guo (1994) has derived approximate analytical solution for the complex wave numbers and compared them with exact numerical solutions, which showed very satisfactory agreement between the two cases.

Many works on wave propagation in a prestressed tube have been investigated for large blood vessels (Demiray, 2002), which can be treated as a thin walled and prestressed elastic cylindrical shell filled with fluid. The effects of initial deformation and the viscosity parameter of fluid on the propagation characteristics were discussed. The effect of hydrostatic pressure fields on the structure and acoustic response of cylindrical shells immersed in water have been examined in the literature (Keltie, 1983).

In this study, the theory of thin shells theory and wave propagation approach are used to establish the model for free wave propagating in prestressed cylindrical shells filled with fluid and loaded with fluid. The effect of initial hydrostatic pressure on the dispersion characteristics is investigated.

2 The governing equations

In this paper, two models are investigated. The two models consist of an infinite circular cylindrical shell, one immersed in an acoustic medium, subject to a uniform external hydrostatic pressure field, and the other filled with the acoustic medium, subject to a uniform internal hydrostatic pressure field. The dynamic problem of the coupled system is formulated in a cylindrical polar coordinate system (x , θ , r).The coordinate axis x is chosen to coincide with the cylindrical shell centerline, while the coordinate axes r and θ respectively taken along the radial and circumferential directions as shown in Fig.1. The displacements of shell are defined by u, v, w in the x , θ , r directions respectively.

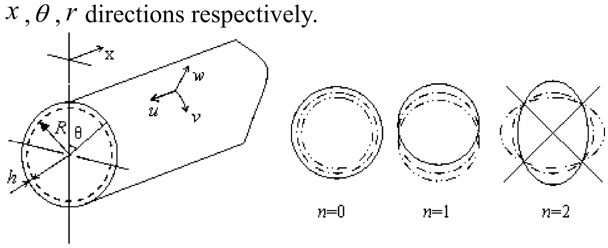


Fig.1 Coordinate system and circumferential mode shapes

2.1 Shell equations

It is assumed that the thin cylindrical shell material is isotropic and elastic, and that the shell thickness, h , is small compared with the shell mean radius, R. The shell equations of motion are taken from Flügge's thin shell theory.

The hydrostatic pressure, p_0, may be divided into two parts: a uniform normal pressure on the shell wall and an axial compression applied at the two edges.
The normal pressure produces the hoop force

$$N_\theta^0 = Rp_0$$

and the axial load produces the longitudinal force

$$N_x^0 = Rp_0 / 2$$

According to the equilibrium conditions of the shell element, three equations can be obtained (Flügge,1973):

$$\frac{\partial N_x}{\partial x} + \frac{1}{R}\frac{\partial N_{\theta x}}{\partial \theta} + N_\theta^0\left(\frac{1}{R^2}\frac{\partial^2 u}{\partial \theta^2} - \frac{1}{R}\frac{\partial w}{\partial x}\right) + N_x^0\frac{\partial^2 u}{\partial x^2} = \rho_s h\frac{\partial^2 u}{\partial t^2} \tag{1a}$$

$$\frac{1}{R}\frac{\partial N_\theta}{\partial \theta} + \frac{\partial N_{x\theta}}{\partial x} - \frac{1}{R^2}\frac{\partial M_\theta}{\partial \theta} - \frac{1}{R}\frac{\partial M_{x\theta}}{\partial x} + N_\theta^0\frac{1}{R^2}\left(\frac{\partial^2 v}{\partial \theta^2} + \frac{1}{R}\frac{\partial w}{\partial \theta}\right) + N_x^0\frac{\partial^2 v}{\partial x^2} = \rho_s h\frac{\partial^2 v}{\partial t^2} \tag{1b}$$

$$\frac{1}{R^2}\frac{\partial^2 M_\theta}{\partial \theta^2} + \frac{1}{R}\frac{\partial^2 M_{x\theta}}{\partial x \partial \theta} + \frac{1}{R}\frac{\partial^2 M_{\theta x}}{\partial x \partial \theta} + \frac{\partial^2 M_x}{\partial x^2} + \frac{1}{R}N_\theta - N_\theta^0\left(\frac{1}{R}\frac{\partial u}{\partial x} - \frac{1}{R^2}\frac{\partial v}{\partial \theta} + \frac{1}{R^2}\frac{\partial^2 w}{\partial \theta^2}\right) - N_x^0\frac{\partial^2 w}{\partial x^2} = \rho_s h\frac{\partial^2 w}{\partial t^2} \tag{1c}$$

where, N_x , N_θ , $N_{x\theta}$ and $N_{\theta x}$ are force resultants; M_x , M_θ , $M_{x\theta}$ and $M_{\theta x}$ are moment resultants as shown in Fig.2. In this figure, Q_x and Q_θ are loads acting on a shell element; ρ_s is density of the shell material.

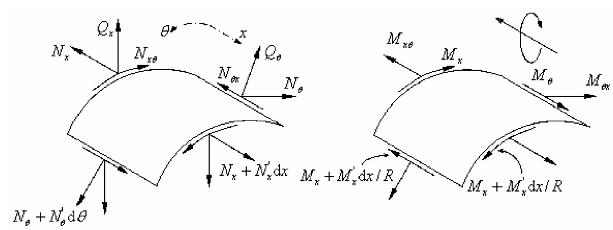


Fig.2 Force and moment resultants and loads acting on a shell element

According to Hook's law, Eq.(1) can be written in the form using displacements description as

$$(1+T_1)u_{xx} + (T_2 + \frac{1-\mu}{2})u_{\theta\theta} + \frac{1-\mu}{2}v + (\mu - T_2)w_x +$$

$$K(\frac{1-\mu}{2}u_{\theta\theta} - w_{xxx} + \frac{1-\mu}{2}w_{x\theta\theta}) - \frac{\rho_s R^2(1-\mu^2)}{E}u_{tt} = 0$$

$$\frac{1+\mu}{2}u_{x\theta} + (T_1 + \frac{1-\mu}{2})v_{xx} + (1+T_2)v_{\theta\theta} + (1+T_2)w_\theta +$$

$$K[\frac{3(1-\mu)}{2}v_{xx}-\frac{3-\mu}{2}w_{xx\theta}-\frac{\rho_s R^2(1-\mu^2)}{E}v_{tt}=0 \quad (2)$$

$$(\mu-T_2)u_x-Ku_{xxx}+K\frac{1-\mu}{2}u_{x\theta\theta}+(1+T_2)v_\theta-K\frac{3-\mu}{2}v_{xx\theta}+$$

$$(1+K)w+Kw_{xxxx}+2Kw_{xx\theta\theta}+Kw_{\theta\theta\theta\theta}+(2K-T_2)w_{\theta\theta}-$$

$$T_1 w_{xx}+\frac{\rho_s R^2(1-\mu^2)}{E}w_{tt}=-\frac{R^2(1-\mu^2)}{Eh}P$$

where, $()_x=R\frac{\partial()}{\partial x}, ()_\theta=\frac{\partial()}{\partial\theta}, ()_t=\frac{\partial()}{\partial t}$; E and μ are the Young's modulus and Poisson's ratio of the shell material respectively; K is a parameter proportional to the shell bending stiffness, $K=h^2/12R^2$; P_f is the acoustic pressure acting on the shell surface; T_1 , T_2 are the terms containing the effects of the hydrostatic pressure field defined by $T_1=(R/2Eh)(1-\mu^2)p_0$ and $T_2=(R/Eh)(1-\mu^2)p_0$.

The effects of the fluid pressure are modeled by including static prestressed terms, which consist of an axial prestressed component and a hoop prestressed component. These stresses are assumed to exist prior to an independent dynamic stresses arising from vibration (Keltie, 1983).

2.2 Fluid acoustic equations

The fluid is assumed to be non-viscous, isotropic and irrotational which satisfies the acoustic wave equation. The equation of motion of the fluid can be written by Helmholtz equation in the cylindrical coordinate system (x , θ , r) as follows:

$$\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial P_f}{\partial r})+\frac{1}{r^2}\frac{\partial^2 P_f}{\partial\theta^2}-\frac{1}{C_f^2}\frac{\partial^2 P_f}{\partial t^2}=0$$

where C_f is the sound speed in the fluid. x , θ and r coordinates are the same as those of the shell.

Applying variables separation method to solve the acoustic wave equation, the associated form of the pressure field is expressed as

$$P_f=\sum_{n=0}^{\infty}\sum_{s=1}^{\infty}P_{ns}\cos(n\theta)Z_n(k_s^r r)\exp(\mathrm{i}\omega t-ik_{ns}x) \quad (3)$$

where n is the circumferential mode number and subscript s denotes a particular branch of the dispersion curve; P_{ns} is the fluid acoustic pressure amplitude of every n and s; ω is the frequency; K_s^r and k_{ns} are the radial and axial wave numbers respectively, and their relation is

$$(k_s^r)^2=k_f^2-k_{ns}^2$$

where, k_f is the free wave number, $k_f=\omega/C_f$. The Bessel function $Z_n(k_s^r r)$ is chosen by

$$Z_n(k_s^r r)=J_n(k_s^r r) \qquad \text{for fluid-filled shell}$$

$$Z_n(k_s^r r)=H_n^2(k_s^r r) \qquad \text{for immersed shell,}$$

where $J_n()$ is Bessel function of order n, and $H_n^2()$ is Hankel function of the second kind and order n.

3 Wave propagation approach

The displacement components of the shell are expressed in a travelling wave form as

$$u=\sum_{n=0}^{\infty}\sum_{s=1}^{\infty}U_{ns}\cos(n\theta)\exp(i\omega t-ik_{ns}x) \quad (4a)$$

$$v=\sum_{n=0}^{\infty}\sum_{s=1}^{\infty}V_{ns}\sin(n\theta)\exp(i\omega t-ik_{ns}x) \quad (4b)$$

$$w=\sum_{n=0}^{\infty}\sum_{s=1}^{\infty}W_{ns}\cos(n\theta)\exp(i\omega t-ik_{ns}x) \quad (4c)$$

Where U_{ns} , V_{ns} and W_{ns} are the displacement amplitudes in the x , θ and r directions respectively.

As usual, the fluid velocity is continuous across the fluid-shell boundary, leading to the boundary condition

$$-\frac{1}{i\omega\rho_f}\frac{\partial P_f}{\partial r}\bigg|_{r=R}=\frac{\partial w}{\partial t}\bigg|_{r=R} \quad (5)$$

where, ρ_f is density of the fluid.

Substituting Eqs.(3) and (4) into Eq.(5), the fluid acoustic pressure amplitude P_{ns} is obtained as

$$P_{ns}=[\omega^2\rho_f/(k_s^r Z_n^{'}(k_s^r R))]W_{ns} \quad (6)$$

Substitution of Eqs.(3), (4) and (6) into Eq.(2) results in the equations of motion in the coupled system

$$[L_{3\times3}][U_{ns}\ V_{ns}\ W_{ns}]^{\mathrm{T}}=[0\ 0\ 0]^{\mathrm{T}}$$

where, $[L_{3\times3}]=\begin{bmatrix}L_{11} & L_{12} & L_{13}\\ L_{12} & L_{22} & L_{23}\\ L_{13} & L_{23} & L_{33}\end{bmatrix}$ is a symmetric matrix, and the elements are given by

$$L_{11}=\Omega^2-(1+T_1)\lambda^2-[T_2+(1+K)(1-\mu)/2]n^2,$$

$$L_{12}=-i\lambda(1+\mu)/2,$$

$$L_{13}=-i[(\mu-T_2)\lambda+K\lambda^3-K(1-\mu)\lambda n^2/2],$$

$L_{22} = [T_1 + (1+3K)(1-\mu)/2]\lambda^2 + (1+T_2)n^2 - \Omega^2$,

$L_{23} = (1+T_2)n + Kn\lambda^2(3-\mu)/2$,

$L_{33} = 1 + K + K\lambda^4 + 2Kn\lambda^2 + Kn^4 - (2K - T_2)n^2 +$

$$T_1\lambda^2 - \Omega^2 + FL$$

where Ω is non-dimensional frequency, $\Omega = \omega\sqrt{\rho_s R^2(1-\mu^2)/E}$; λ is non-dimensional axial wave number, $\lambda = k_{ns}R$; FL is fluid-loading term, and is given by

$$FL = -\Omega^2 \frac{\rho_f}{\rho_s}\frac{R}{h}\frac{J_n(k_s^r R)}{(k_s^r R)J_n'(k_s^r R)} \quad \text{for fluid-filled shell;}$$

$$FL = \Omega^2 \frac{\rho_f}{\rho_s}\frac{R}{h}\frac{H_n^2(k_s^r R)}{(k_s^r R)(H_n^2)'(k_s^r R)} \quad \text{for immersed shell.}$$

The system is solved using the condition that the three amplitudes U_{ns}, V_{ns} and W_{ns} do not vanish at the same time. This condition requires the determinant of the coefficient matrix being equal to zero, providing the characteristic equation of this coupled system,

$$|L| = 0 \tag{7}$$

Expansion of determinant in Eq.(7) gives the characteristic equation with another form, expressed as

$$P_1(\lambda) + P_2(\lambda)FL = 0 \tag{8}$$

where both $P_1(\lambda)$ and $P_2(\lambda)$ are polynomials of λ . For the sake of brevity, the coefficients are not given here. This dispersion relation is a transcendental function regarding the non-dimensional frequencies Ω , axial wave number λ and circumferential mode order *n*. Numerical methods are used to obtain λ for given Ω and *n*.

4 Numerical analysis

Due to the presence of the fluid-loading term FL, the dispersion equation of the coupled system, Eq.(8), is a nonlinear equation on complex plane. Numerical methods have to be employed to find the desired eigenvalues λ . Cylindrical shell waves can be seen to have three forms of roots: purely real roots which correspond to propagating axial, torsional and flexural type motion, purely imaginary roots which is asymptotic to the same value as a plate bending near field and complex roots which when paired together correspond to an attenuated, near-field standing wave (Fuller,1981). In this paper, a winding-number integral method (Brazier and Scot, 1991; Ivansson and Karasalo, 1993) is used to find roots on the complex λ -plane.

Dispersion curves are obtained for different shells made of steel or aluminum immersed in water or filled with water, the properties of which are given in Table 1.

Table 1 Material properties

Material	Young's modulus / (N·m^{-2})	Poisson's ratio	Density /(kg·m^{-3})
Steel	1.92×10^{11}	0.3	7850
Aluminum	7.0×10^{10}	0.34	2700
Water	-	-	1000

4.1 Cylindrical shell immersed in water

When the external hydrostatic pressure exceeds certain value, the shell may lose stability. Therefore, the investigation of the free vibration of cylindrical shell under hydrostatic pressure is meaningful when the hydrostatic pressure is below the critical pressure q_{cr} . The critical pressure q_{cr} of an infinite long cylindrical shell can be obtained from the formula (Pinna and Ronalds, 2000) as

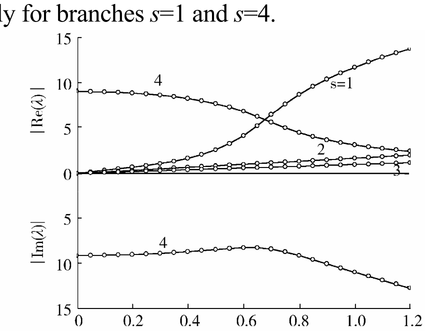
$$q_{cr} = \frac{E}{4(1-\mu^2)}\left(\frac{h}{R}\right) \tag{9}$$

The critical pressure q_{cr} and external hydrostatic pressure p_0 (70% of the critical pressure) of different shells are obtained from Eq. (9) and shown in Table 2.

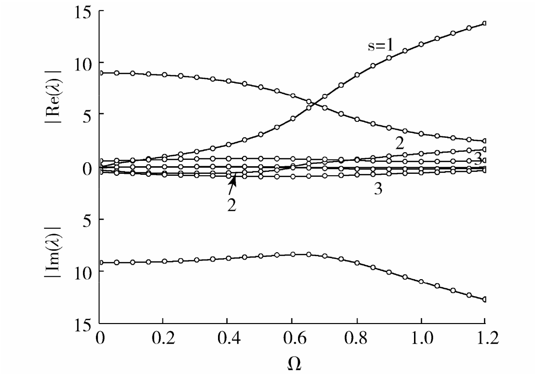
Table 2 The critical pressure q_{cr} and external hydrostatic pressure p_0 (70% of the critical pressure) of different shells

Shell parameters	*h*/*R*=0.02 Steel	*h*/*R*=0.005 Steel	*h*/*R*=0.02 Aluminum
q_{cr}/p_0 / kPa	422/295	6.6/4.6	158/111

Figs.3-5 show the non-dimensional axial wave number λ plotted against non-dimensional frequency Ω of circumferential mode orders *n*=0 and 1 for a steel shell with thickness *h/R*=0.02, a steel shell with thickness *h/R*=0.005 and an aluminum shell with thickness *h/R*=0.02 respectively. The results are given for the cases both with and without external hydrostatic pressure. The waves shown in Fig.3(a) are the flexural wave (*s*=1), decoupled torsional wave (*s*=2) and the compressional wave (*s*=3) which has a very small imaginary part. It can be seen from Fig.3 and Fig.4 that the wave numbers increase as thickness-radius ratios increase especially for branches *s*=1 and *s*=4.



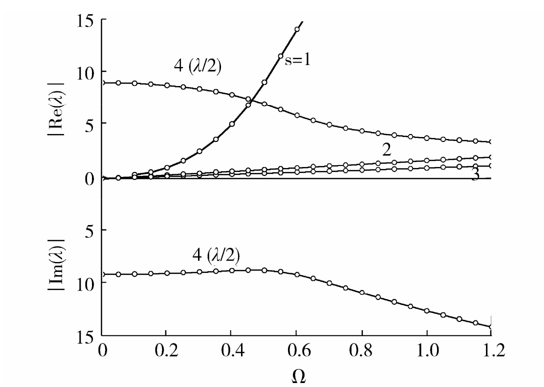
(a) *n*=0



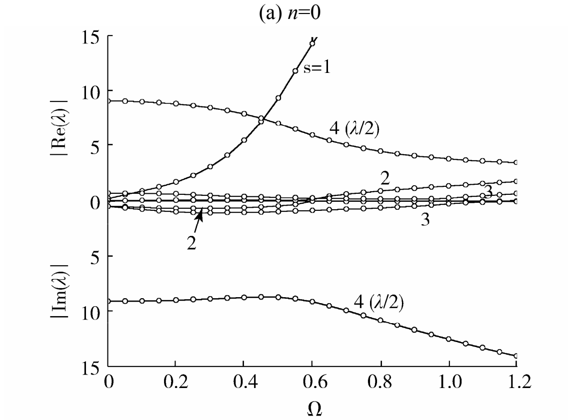
(b) n=1

—p_0=0 kPa; ○ ○p_0=295 kPa.

Fig.3 Dispersion curves for an immersed steel shell (thickness h/R=0.02) with external hydrostatic pressure



(a) n=0

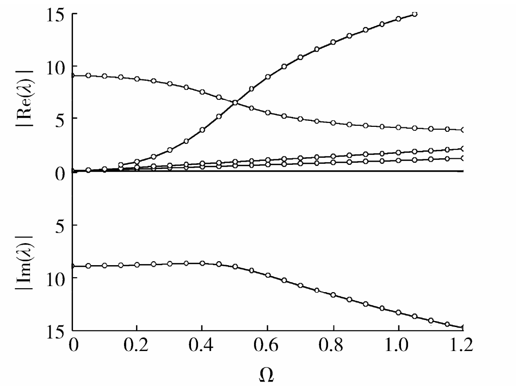


(b) n=1

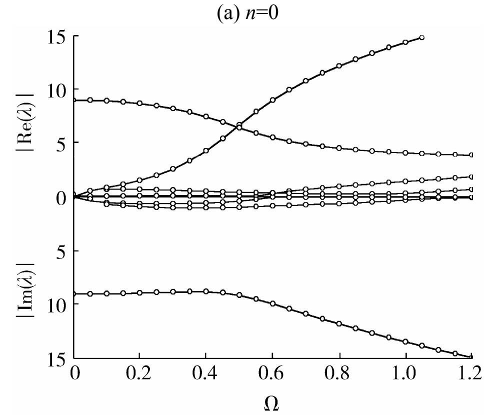
—p_0=0 kPa; ○ ○p_0=4.6 kPa.

Fig.4 Dispersion curves for an immersed steel shell (thickness h/R=0.005) with external hydrostatic pressure

For a water-loaded shell, the external hydrostatic pressure is shown to have little influence on the axial wave number in the frequency range Ω=0~1.2 for given modes. The external hydrostatic pressure which is restricted to the critical pressure is too low to make influence on free wave propagating.



(a) n=0



(b) n=1

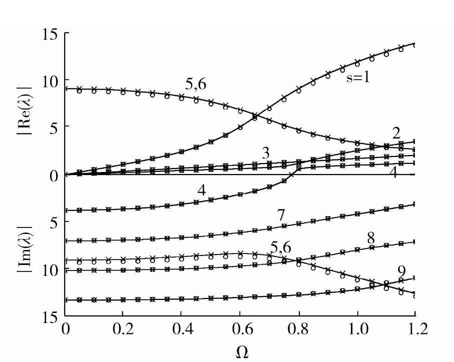
—p_0=0 kPa; ○ ○p_0=111 kPa

Fig.5 Dispersion curves for an immersed aluminum shell (thickness h/R=0.02) with external hydrostatic pressure

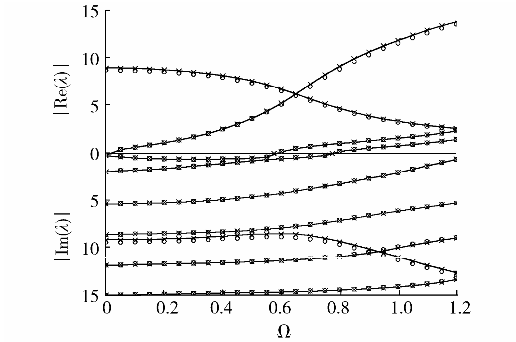
However, a submerged cylindrical shell reinforced by rings and bulkheads is used practically, as its critical pressure is high. The dispersion characteristics of submerged stiffened cylindrical shells will be studied in the further work.

4.2 Cylindrical shell filled with water

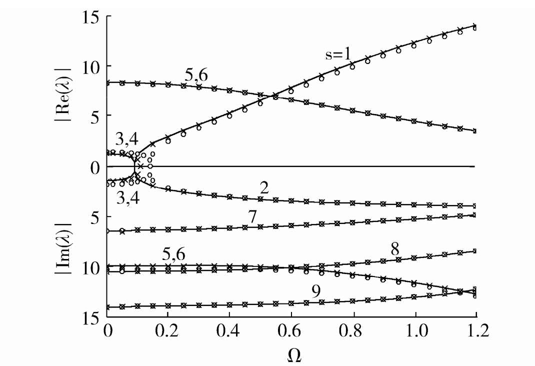
The non-dimensional axial wave number λ against non-dimensional frequency Ω of circumferential mode orders n=0, 1 and 5 are plotted in Figs.6-8. The results are given for the shell filled with water with internal hydrostatic pressure p_0=1 MPa and 5 MPa and the shell without internal pressure with p_0=0 MPa. A complete discussion of wave modes in fluid filled shells without considering the effect of hydrostatic pressure can be found in Fuller(1982).



(a) n=0



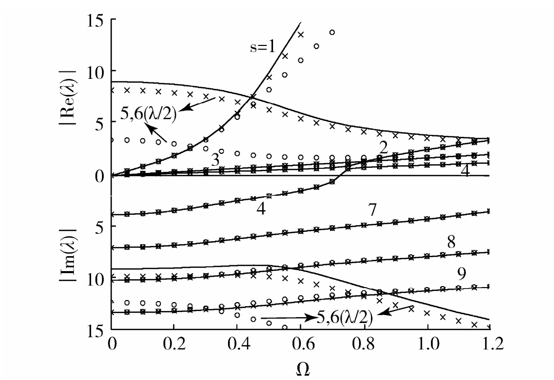
(b) n=1



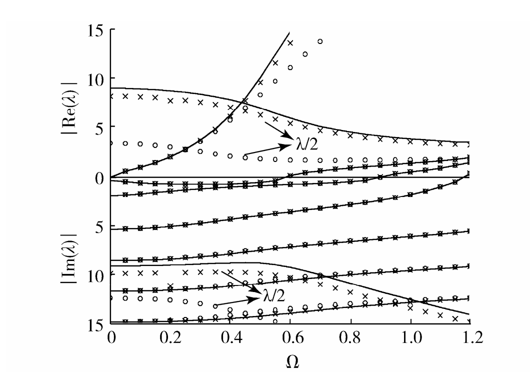
(c) n=5

—p_0=0MPa; × ×p_0=1 MPa;○ ○p_0=5 MPa.

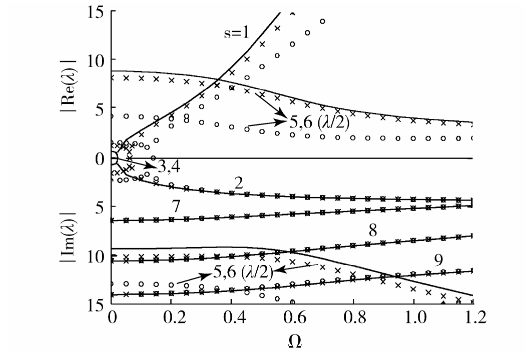
Fig.6 Dispersion curves for a water-filled steel shell (thickness h/R=0.02) with internal hydrostatic pressure



(a)n=0



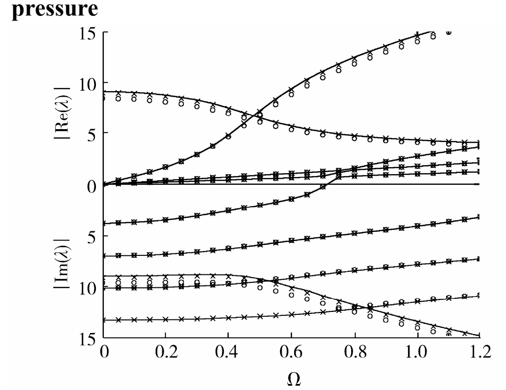
(b)n=1



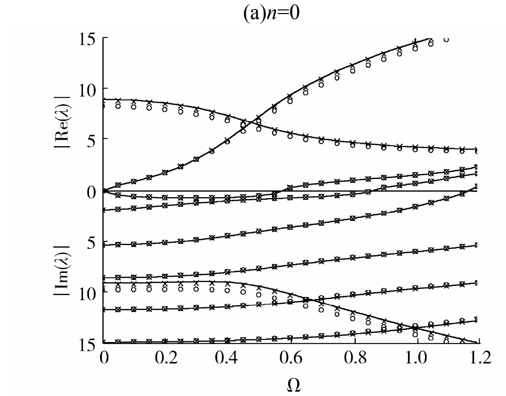
(c)n=5

—p_0=0MPa; × ×p_0=1MPa;○ ○p_0=5MPa.

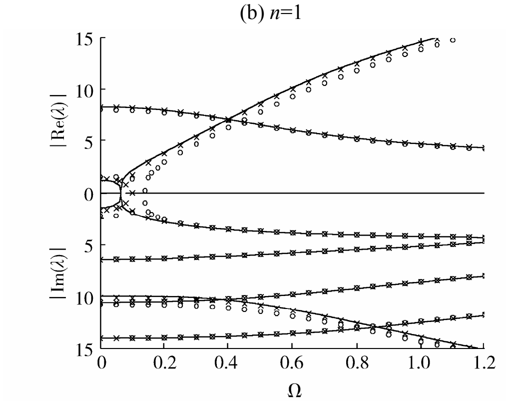
Fig.7 Dispersion curves for a water-filled steel shell (thickness h/R=0.005) with internal hydrostatic pressure



(a)n=0



(b) n=1



(c)n=5

—p_0=0MPa;× ×p_0=1MPa;○ ○p_0=5 MPa.

Fig.8 Dispersion curves for a water-filled aluminum shell (thickness h/R=0.02) with internal hydrostatic pressure

The waves in a fluid-filled steel shell with thickness h/R=0.02 have been shown in Fig.6. For n=0 and 1, there appears only one pair of complex branch. As shown in the figure, the dispersion curves of n=0, 1 with internal pressure p_0 =1MPa (× ×) and those without internal pressure are coincident. Nevertheless, when the internal pressure p_0 increases to 5MPa (○○), the pressure has nearly no influence on propagating waves except the first branch (s=1) and the complex branch (s=5). Due to the internal pressure, the wave numbers of s=1 and the real part of the complex λ decrease while the imaginary part increases in absolute magnitude. It can be seen that the effect of internal pressure on wave numbers for n=0, 1 above Ω =0.6 is more significant than that at lower frequencies for s=1.

In the case of the n=5 mode as shown in Fig.6(c), waves of the branch (s=1) have non-zero cut on frequencies and the presence of internal pressure has increased the cut on frequencies, below which there appears two pairs of complex branches (s=2 and s=5). The internal pressure exhibits the same effect on the real branch (s=1) and the complex branch (s=5) for n=5 mode as seen for n=0 and 1 mode. In particular, the other complex branch (s=2) is found to increase in absolute magnitude due to the internal pressure. It can be also seen that the effect of hydrostatic pressure increases as the pressure increases in absolute magnitude.

Waves in a fluid-filled steel shell with a much smaller thickness h/R=0.005 than previous case are shown in Fig.7. With decreasing of shell thickness, the complex branches (s=5) and the real branches (s=1) are found to increase in absolute magnitude. As shown in Fig.7, the internal pressure has shown the effects on waves similar to those with thickness h/R=0.02 but more obvious. It can be found that the wave numbers of purely imaginary branch slightly decrease due to the internal pressure. The effects of pressurization on dispersion curves of a fluid-filled aluminum shell, plotted in Fig.8, are seen to be the same as those of a steel shell.

5 Conclusions

In this paper, the characteristics of dispersion in both fluid-filled shell and fluid-loaded shell taking hydrostatic pressure into account have been presented and numerical analysis has been investigated. The effect of the presence of internal pressure or external pressure which is modeled by including static prestressed terms in the shell equations of motion is then examined. It is observed that external pressure has almost no effect on the dispersion characteristics of the shell-fluid structure when external pressure is small and the influence of internal pressure is significant for the waves of the real branch (s=1) and the complex branches.

Of the real branch (s=1), the wave number has decreased due to the internal pressure. For low order wave mode, there exists only one pair of complex branches, of which the real part decreases and the imaginary part increases in absolute magnitude because of the internal pressure. Below the cut on frequency of s=1 for high wave mode, there appears another complex branch, of which both the real part and imaginary part increase when considering internal hydrostatic pressure. The internal pressure has increased the cut on frequencies of the real branch (s=1) for high wave modes. The effects of the internal pressure on dispersion characteristics are more obvious with increasing of the pressure.

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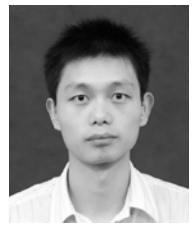

Zhi-Zhong Liu was born in 1980. He is a doctorate student at the School of Naval Architecture and Ocean Engineering, Huazhong University of Science and Technology, majoring in vibration and noise control of ship structure.