J. Marine. Sci. Appl. (2010) 9: 99-104
DOI: 10.1007/s11804-010-9030-z

Application of Empirical Mode Energy to the Analysis of Fluctuating Signals

Yang Li*, Si-chun Li, Sheng-chun Piao and Shi-jun Sun

National Laboratory of Underwater Acoustic Technology, Harbin Engineering University, Harbin 150001, China

Abstract: After an aerial object enters the water, physical changes to sounds in the water caused by the accompanying bubbles are quite complex. As a result, traditional signal analyzing methods cannot identify the real physical object. In view of this situation, a novel method for analyzing the sounds caused by an aerial object's entry into water was proposed. This method analyzes the vibrational mode of the bubbles by using empitical mode decomposition. Experimental results showed that this method can efficiently remove noise and extract the broadband pulse signal and low-frequency fluctuating signal, producing an accurate resolution of entry time and frequency. This shows the improved performance of the proposed method.
Keywords: empirical mode decomposition; energy feature extraction; fluctuant signal analysis
Article ID: 1671-9433(2010)01-0099-06

1 Introduction

It is well known that torpedo is an important naval battle weapon, which has played a vital role in the previous war. Therefore, how to effectively resist various torpedo attacks becomes the focus of national underwater confrontation, which calls for the early detection to aerial torpedo's water entry signal, and an accurate identification to torpedo, so as to reach early warning purpose. Thus, it is of great significance to the research of aerial torpedo's water entry signals.

After the aerial torpedo has been launched by aircraft or by the rocket to help fly, it enters the water upon its own weight and speed, and the help of its own stable equipment. As an instant impact extrusion to a torpedo, seawater can form water entry impact noise, which called splash sound. From Bao and Tang (1981), it can be seen the splash sound of the typical aerial torpedoes is composed of three parts: water entry pulse signal, "quiet" interval and attenuated fluctuant signal of bubbles.

At present, there are many methods to detect and identify water entry signals such as short-time Fourier transform, short-related method, wavelet analysis and so on (Sun *et al*., 2005). However, fluctuant signal is formed by the bubbles' compression and expansion, with obvious non-linear, non-stationary features. As traditional methods such as Fourier transform, short-time Fourier transform, Wigner-Ville distribution and wavelet analysis have afore-fixed primary function, there are such problems as leak, non-adaptive, and limitation by Heisenberg principle, so their time and frequency resolution is limited. Therefore, none of these methods can reveal the fluctuant signal's nonlinear and non-stationary nature, nor can they accurately show the modes bubbles oscillate.

This paper adopts the Hilbert-Huang transform to analyze the fluctuant signal. The signal is restricted according to the energy feature of the signal, then we can get the primary oscillating modes of the bubbles after the torpedo has entered water. This research provides a new method to study fluctuant signal of torpedo's water entry, and lays a foundation for the research of torpedo countermeasure.

2 Hilbert-Huang transform

2.1 Concepts

Hilbert-Huang transform (HHT) is a new method for signal analysis developed by Huang *et al*. (1998). The main conceptual innovation of HHT is the raise of intrinsic mode function (IMF) and the introduction of empirical mode decomposition (EMD). It decomposes signal into a series of IMF (generally finite number), and then with the Hilbert transform, the intrinsic mode functions yield meaningful instantaneous frequencies as the functions of time. The final presentation of the results is an energy-frequency-time distribution named Hilbert spectrum. HHT is based on the local characteristic and adaptive, and therefore, it is efficient. HHT is particularly suitable for analyzing nonlinear and non-stationary signals whose frequencies change as time goes.

In order to make the instantaneous frequency meaningful, functions have to satisfy two conditions: First, they are symmetric with respect to the local mean. Second, they have the same number of zero crossings and extrema. Accordingly Huang identified IMF as follows (Huang *et al*., 1998): 1) In the whole data set, the number of extrema and the number of zero-crossings must either equal or differ at most by one; 2) At

Received date: 2009-05-21.
***Corresponding author Email:** liyang2007@hrbeu.edu.cn

any point, the mean value of the envelope defined by the local maxima and the envelope defined by the local minima is zero.

Essentially, EMD is to stabilize a signal (or its differential coefficient, depending on the required accuracy of the decomposition). As a result, the signal is decomposed into undulations or tendencies of different scales, and then a series of different characteristic scaled data sequences are got, each sequence is called an IMF component. The approaches of EMD (Li *et al.*, 2005; Oonincx and Hermand, 2004) mean that if the primary datum doesn't satisfy the definition of IMF, it must be stabilized.

Find the extrema of the data $x(t)$, and then get the upper and lower envelopes by using interpolation. Calculate the mean value of the two envelopes, recorded as m_1. The difference between $x(t)$ and m_1 is h_1:

$$h_1 = x(t) - m_1 \tag{1}$$

If h_1 doesn't satisfy the definition of IMF, then see h_1 as a new data set, fit its upper and lower envelopes whose mean value is m_{11}, and then calculate the difference between h_1 and m_{11}, recorded as h_{11}:

$$h_{11} = h_1 - m_{11} \tag{2}$$

Repeat the above operations for i times, until h_{1i} satisfies the definition of IMF. So

$$imf_1 = h_{1i} \tag{3}$$

Remove imf_1 from the data:

$$r_1 = x(t) - imf_1 \tag{4}$$

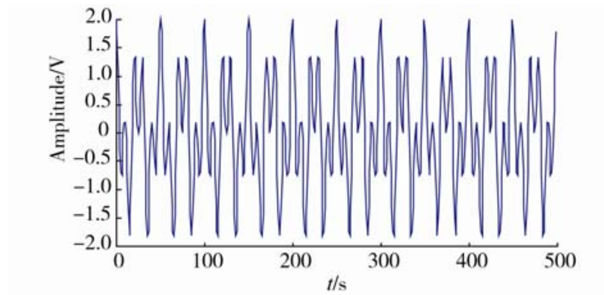
If the remaining part r_1 still contains longer period components, see it as new data and repeat the above steps. When the residue becomes a constant or a monotonic function or a function which has only one extremum, we can finish the screening. Then $r(t)$ is called remainder term, and it is the trend of the original signal. The latter IMF has lower frequency feature than the former. Since EMD is complete, the time sequence can be presented as the sum of n intrinsic mode functions and a residue:

$$x(t) = \sum_{i=1}^{n} imf_i(t) + r(t) \tag{5}$$

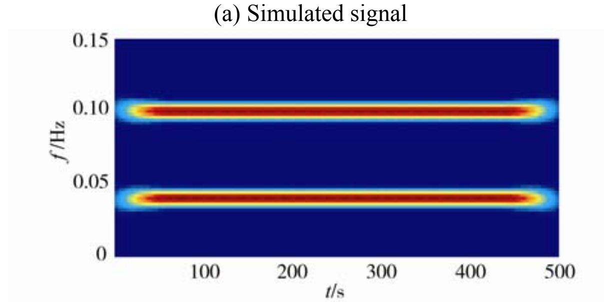
Simulated signal is analyzed by using short-time Fourier transform (STFT), Wigner-Ville distribution (WVD), wavelet transform and Hilbert-Huang transform (Fig.1), from which it can be seen that HHT has a high resolution not only in time domain but also in frequency domain, and HHT can also avoid the problem of severe cross terms as Wigner-Ville transform. The data model is:

$$s(t) = \cos(2\pi f_1 t) + \cos(2\pi f_2 t)\,,\quad t \in [0,499] \tag{6}$$

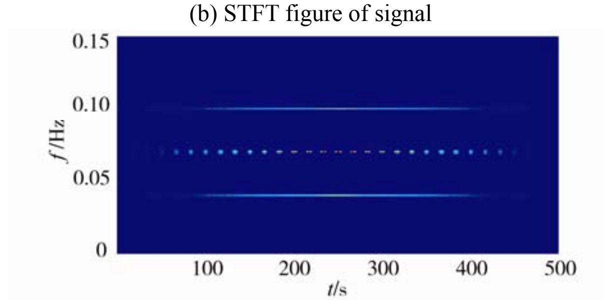
where f_1=0.04 Hz, f_2=0.1 Hz, and the sampling frequency is 1Hz.



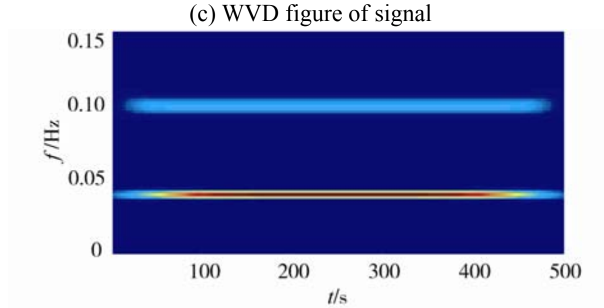
(a) Simulated signal



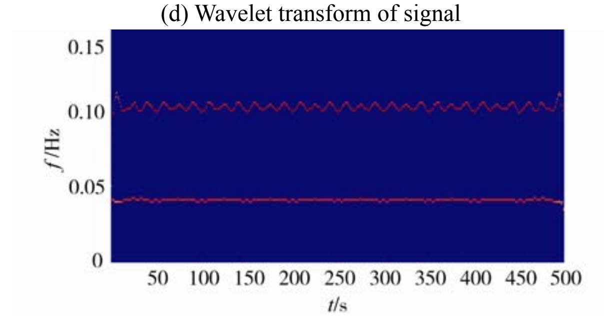
(b) STFT figure of signal



(c) WVD figure of signal



(d) Wavelet transform of signal



(e) HHT of signal

Fig.1 Simulated signal and its time-frequency analysis

2.2 Curve fitting method

Beacause signal's two envelope curves are fitted by using its extreme points in the EMD method, how to fit the envelope curves with the extreme points is important, which plays an important role in EMD method and may affect the calculation of instantaneous frequency of signal. But how to fit the envelope curves accurately has been an unsolved problem so far, cubic spline interpolation algorithm is the most commonly used. It is proved that cubic spline interpolation algorithm is a ralatively good method to fit curves,and in some circumstances the analysis has quite satisfactory results.

From Zhong *et al*.(2005), we know that cubic spline interpolation is easy to lead to "overshoot" effect, which is resolved by using subsection power function method. From Yang *et al*.(2004), we know that although EMD based on high-order spline interpolation can effectively improve the the precision of the time-frequency analysis, it costs much more time than cubic spline interpolation, and with the order higher, the precision may not be improved any more. So cubic spline interpolation algorithm is adopted in this paper.

Cubic polynomial is needed to approximate curves between each two points of the data. Although two points can determine a line, curve between two points can be approximated by infinite cubic polynomials, so in order to determin the cubic polynoimal, we limit that the first-order derivative of the polynoimal is equal to the scecond-order derivative of the polynoimal at interpolation points.

2.3 Boundry problem processing

Boundary points of signal are needed to be implemented in EMD. Why is this needed to do? One interpretation is (Huang *et al*., 2003; Deng *et al*., 2001; Zhang *et al*., 2003), because we don't know the signal beyound the interval which is given beforehand, we only make use of extreme points of the signal within the given section to fit the envelop curves, which are seriously anamorphic compared with the real envelp curves. At first, the distortion only lies in the beginning section and the ending section of the signal, but as the decomposition is going, the error propagates toward the inner of the data, which leads to a wrong result finally. So it is necessary to extend the boundary points according to some criterion. Another interpretation is, because cubic spline interpolation needs two adjacent points, in order to make sure that the envelope curves reach the end of the signal, it is necessary to extend the signal. We make sure that both of the upper and the lower envelope curves are acrossed with the end points of the signal in order to keep the total number of data invariable. Since the two envelope curves are fitted by maximum points and minimum points, maximum points and minimum points can be extended instead of signal itself. On both side of the signal, two maximum points and minimum points are extended. In summary, boundary points are extended in order to make full use of the certain message of signal to predict uncertain part of the signal, which is used to reduct the analysis error.

There are many methods to extend the data. The envelop extreme extending method (Huang *et al*.,2003) is adopted in this paper. Suppose a discrete siganl

$$t \in [t(1), t(2), \cdots, t(n)] = [t_1, t_2, \cdots, t_n] \tag{7}$$

$$x(t) \in [x(t_1), x(t_2), \cdots, x(t_n)] = [x_1, x_2, \cdots, x_n] \tag{8}$$

In which the sampling time is Δt , there are M maxmum and N minimum in $x(t)$, the according subscript of the sequence is (I_m, I_n):

$$I_m = [I_m(1), I_m(2), \cdots, I_m(M)] \tag{9}$$

$$I_n = [I_n(1), I_n(2), \cdots, I_n(N)] \tag{10}$$

The time (T_m, T_n) and the value(U, V) are

$$T_m(i) = t_{I_m}, \quad U(i) = x_{I_m}, \quad i = 1, \cdots, M \tag{11}$$

$$T_n(i) = t_{I_n}, \quad V(i) = x_{I_n}, \quad i = 1, \cdots, N \tag{12}$$

Calculate the number of the points of the first characteristic wave k_1 from the left side

$$k_1 = \begin{cases} I_m(2) - I_m(1), & \text{when } I_m(1) < I_n(1) \\ I_n(2) - I_n(1), & \text{when } I_m(1) > I_n(1) \\ 2\left|I_m(1) - I_n(1)\right|, & \text{when } M = N = 1 \end{cases} \tag{13}$$

The time (T_m, T_n) and the value (U, V) of extending extreme points towards the left are:

$$T_m(0) = T_m(1) - k_1 \Delta t, \quad U(0) = U(1) \tag{14}$$

$$T_m(-1) = T_m(1) - 2k_1 \Delta t, \quad U(-1) = U(1) \tag{15}$$

$$T_n(0) = T_n(1) - k_1 \Delta t, \quad V(0) = V(1) \tag{16}$$

$$T_n(-1) = T_n(1) - 2k_1 \Delta t, \quad V(-1) = V(1) \tag{17}$$

Calculate the number of the points of the first characteristic wave k_2 from the right side

$$k_2 = \begin{cases} I_m(M) - I_m(M-1), & \text{when } I_m(M) > I_n(N) \\ I_n(N) - I_n(N-1), & \text{when } I_m(M) < I_n(N) \\ 2\left|I_m(M) - I_n(N)\right|, & \text{when } M = N = 1 \end{cases} \tag{18}$$

The time (T_m, T_n) and the value (U, V) of extending extreme points towards the right are:

$$T_m(M+1) = T_m(M) - k_2 \Delta t, \quad U(M+1) = U(M) \tag{19}$$

$$T_m(M+2) = T_m(M) - 2k_2 \Delta t, \quad U(M+2) = U(M) \tag{20}$$

$$T_n(N+1) = T_n(N) - k_2 \Delta t, \quad V(N+1) = V(N) \tag{21}$$

$$T_n(N+2) = T_n(N) - 2k_2 \Delta t, \quad V(N+2) = V(N) \tag{22}$$

If we only use the above method to extend the extreme points, there might be "the end swing" phenomenon (Long, 2005). So it is necessary to do some special processing when the value of the end of the data bigger than the first maximum or smaller than the first minimum in order to avoid that the signal is outside of the envelope. That is in this case, the end is used as the extending maximum extreme point or minimum extreme point, which could avoid the end swing phenomenon. Do processing as follows:

$$T_m(0) = t_1, U(0) = x_1,\ \text{when}\ \ x_1 > U(1) \tag{23}$$

$$T_n(0) = t_1, V(0) = x_1,\ \text{when}\ \ x_1 < V(1) \tag{24}$$

$$T_m(M+1) = t_n, U(M+1) = x_n,\ \text{when}\ \ x_n > U(M) \tag{25}$$

$$T_n(N+1) = t_n, V(N+1) = x_n,\ \text{when}\ \ x_n < V(N) \tag{26}$$

2.4 Stop criterion

If we get an IMF, the epicycle screening will stop. Then the IMF is removed from the signal, and the new data is got to be analyzed again. How to judg if a signal is an IMF or not? There are two criteria which are commonly used. One is

$$S_D = \sum_{t=0}^{T} \left[\frac{\left| h_{j(i-1)}(t) - h_{ji}(t) \right|^2}{h_{(j(i-1))}^2(t)} \right] \tag{27}$$

If S_D is between 0.2 and 0.3, the *j*th screening will stop, then *imf* is got:

$$imf_j = h_{ji}(t) \tag{28}$$

But it is proved that S_D can not always be in the interval, so the stop criterion which is adopted in this paper is the value of S_D , which is smaller than 0.3.

Another criterion is got by the definition of IMF, and it is in the data section, the number of extreme points is equal to the number of zero points or the difference between them is no more than one.

3 Scale Filtering

Usually signal filtering is realized in the frequency domain. We can get signal's frequency-domain representation by using Fourier transform. Use a filter with certain structure and bandwidth to eliminate interferential waves and get the useful signal according to signal's frequency distribution. However, for nonlinear and non-stationary signals, it is difficult to execute frequency-domain filtering, because after filtering, signals lose their intrinsic nonlinear and non-stationary feature, which leads to signal distortion. By use of IMF we can construct a novel filtering called "time-space filtering" (Tan *et al*., 2004) or "scale filtering". This method can sufficiently remain signal's nonlinear and non-stationary feature.

Through EMD, different frequency components are separated from higher frequency to lower frequency. So the low-pass scale filtering of signal containing *n* IMFs is described by

$$x_{lk} = \sum_{i=k}^{n} imf_i(t) + r(t) \tag{29}$$

where the high-pass scale filtering is described by

$$x_{hk} = \sum_{i=1}^{k} imf_i(t) \tag{30}$$

The band-pass scale filtering is described by

$$x_{bk} = \sum_{i=k_1}^{k_2} imf_i(t) \tag{31}$$

On the basis of the idea of scale filtering, select the oscillating modes which are useful for signal processing, and remove the oscillating modes of undesired signal.

4 Energy feature extraction

Based on the thought of reference (Hu *et al*., 2007), calculate the energy of *n* IMFs from EMD, the energy of the *i*th IMF component is

$$E(imf_i(t)) = \frac{1}{N-1} \sum_{i=1}^{N} (imf_i(t))^2 \tag{32}$$

where *N* is the length of IMF component. The total energy of IMFs is

$$E = E(imf_1) + E(imf_2) +, \cdots, + E(imf_n) \tag{33}$$

Then normalize each IMF, the relative energy of the *k*th IMF is

$$e_k = \frac{E(imf_k(t))}{E} \tag{34}$$

So the energy feature vector of the intrinsic mode components can be described by

$$e = e_1, e_2, \cdots, e_n \tag{35}$$

Because each IMF component represents a kind of oscillating mode, we can judge in which oscillating mode(s) the signal oscillates mainly, according to the proportion of each IMF's energy accounting for the total energy. And this may highlight the feature of corresponding physical process.

This paper uses the rules of selecting useful IMF components as follows: 1) Select one IMF component whose energy proportion is larger than 50％; 2) If none of the IMF components is larger than 50％, select the largest two (or three) IMF components, the total proportion of which is larger than 50％.

5 Fluctuant signal analysis of aerial object's water entry

In order to verify the validity of the above method, the experimental group executed data acquisition on the Songhua Lake in Jilin Province on Nov. 26th, 2007. The signal which was received after a certain aerial object had entered the water is shown in Fig.2 (with reduced sampling rate). The signal was intercepted for 0.5 second, with 10 000 sampling points. From this figure, we can distinguish the water entry pulse signal, "quiet" interval and fluctuant signal. So, this signal is a typical signal of aerial object's water entry. Send the signal to EMD, and then we can get 19 IMFs and a residue. From the IMFs it can be known that the modes are arrayed from smaller extremum to larger extremum, while in the frequency domain they are arrayed from higher frequency to lower frequency.

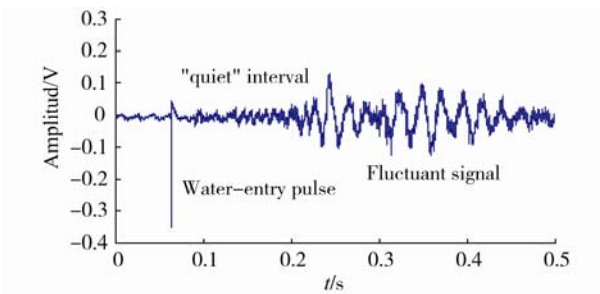


Fig.2 Aerial object's water entry signal

According to the method described in this paper, calculate the energy of each IMF to get energy feature vector ***e***, and plot the distribution of the energy feature vector in a bar figure (Fig.3). It can be seen that the energy of signal mainly distributes in the 10th and 11th IMFs, which indicates that the bubbles oscillate mainly in the mode of the 10th and 11th IMFs after the object has entered water. This phenomenon is shown in IMFs as the two components can describe the fluctuant signal more clearly. So the two modes can be analyzed mainly. Then execute scale filtering using the method in this paper to get the 10th and 11th components, with which the signal is reconstructed (Fig.4).

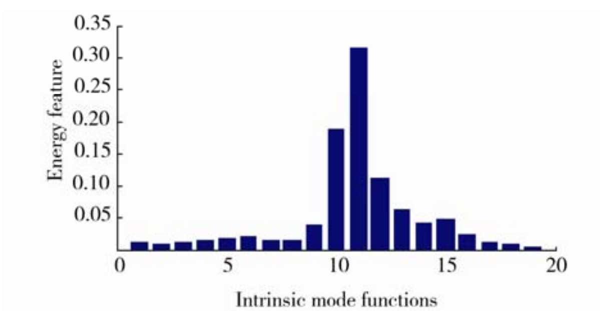


Fig.3 Energy feature vector's bar distribution

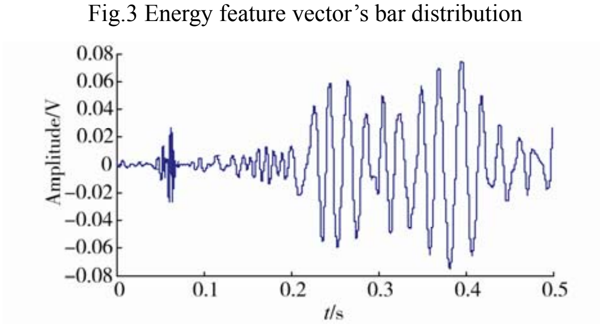


Fig.4 Signal reconstruction based on the 10th and 11th IMFs

Compare Fig.4 with Fig.2, it can be seen that in Fig.4 both the noise and the modes whose energy is small have been removed, which highlights the feature of fluctuant signal. Because the bandwidth of water entry pulse is too wide, the pulse is not entirely contained in the two modes; while the bandwidth of the fluctuant signal is narrow, and mainly in the two modes. So we can take the two modes to Hilbert transform in order to get their instantaneous frequency, then we can get the Hilbert spectrum.

Fig.5 shows the Hilbert spectrum of all the IMFs, while Fig.6 shows the Hilbert spectrum of the signal only containing the 10th and 11th IMFs. Compare the two figures, it can be seen that the energy in Fig.6 is more focused, and Fig.6 can prominently highlight the time-frequency distribution feature. From the picture, it can be seen that the fluctuant signal is a typical kind of nonlinear and non-stationary signal; its oscillating frequencies are mainly distributed from 40 Hz to 60 Hz.

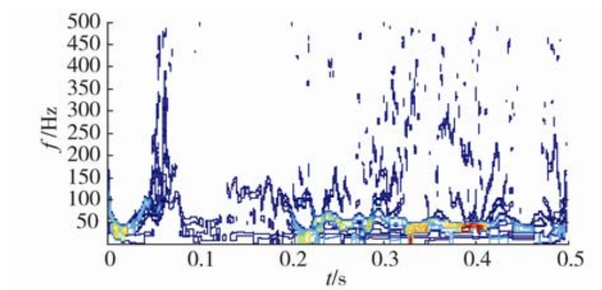


Fig.5 Hilbert spectrum of all the IMF modes

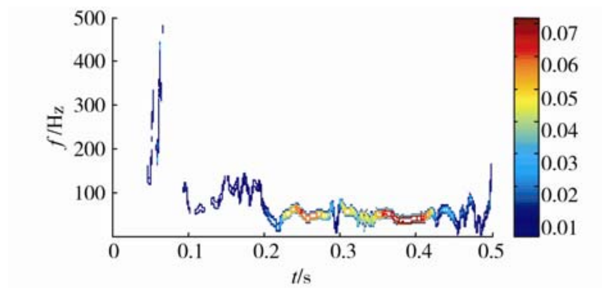


Fig.6 Hilbert spectrum of the 10th and 11th IMF modes

6 Conclusions

This paper adopts empirical mode of feature extraction method to analyze the fluctuant signal received after the aerial object has entered the water. A conclusion can be drawn that this method is of great capacity to analyzing the nonlinear and non-stationary data, and it has high resolution not only in the time domain but also in the frequency domain. It breaks through the shortcomings of traditional signal analytical methods. The authors adopt the idea of scale filtering based on Hilbert-Huang transform and the method of energy feature extraction, which makes the method more self-adaptive, and verify that the method has a strong anti-interference capacity and the energy of its time-frequency distribution is focused. Experimental results have affirmed the effectivity of the method.

References

Bao Xiaoling, Tang Weilin (1981). Investigations of the impact noise. *Journal of Harbin Shipbuilding Engineering Institute*, (1), 27-39. (in Chinese)

Deng Yongjun, Wang Wei, Qian Chengchun, Wang Zhong, Dai Dejun (2001). Boundary problem processing in EMD and hilbert transform. *Chinese Science Bulletin*, **46**(3), 257-263. (in Chinese)

Hu Qiao, Hao Baoan, Lv Linxia, Chen Yalin, Sun Qi (2007). Application of empirical mode energy entropy to target-detection of underwater acoustic signals. *Classification and Identification Technology of Underwater Targets*, **26**(5), 181-183. (in Chinese)

Huang Daji, Zhao Jinping, Su Jilan (2003). Practical implementation of the Hilbert-Huang transform algorithm. *Acta Oceanologica Sinica*, **25**(1), 1-11. (in Chinese)

Huang NE, Shen Zheng, Long SR, Wu MC, Shih HH, Zheng Quanan, Yen NC, Chi CT, Liu HH (1998). The empirical mode decomposition and the Hilbert spectrum for nonlinear non-stationary time series analysis. *Proceeding of the Royal Society of London Series A*, **454**, 903-995.

Li Ying, Wang Bangfeng, Sun Yafei, Shi Yifeng (2005). Analysis and application of empirical mode decomposition and Hilbert spectrum. *Journal of Transducer Technology*, **24**(5), 71-76. (in Chinese)

Long Sisheng, Zhang Tiebao, Long Feng (2005). Causs and solutions of overshoot and undershooot and end swing in Hilbert-Huang transform. *Acta Seismolo Gica Sinica*, **27**(5), 561-568. (in Chinese)

Oonincx PJ, Hermand JP (2004). Empirical mode decomposition of ocean acoustic data with constraint on the frequency range. *Proceedings of the 7th European Conference on Underwater Acoustics*, Delft, The Netherlands.

Sun Peng, Liu Pingxiang, Xu Bailing (2005). The research on the detection of transient signal. *Ship Science and Technology*, **27**(3), 72-76. (in Chinese)

Tan Shanwen, Qin Shuren, Tang Baoping (2004). The filtering character of Hilbert-Huang transform and its application. *Journal of Chongqing University*, **27**(2), 9-12. (in Chinese)

Yang Shixi, Hu Jinsong, Wu Zhaotong, Yan Gongbiao (2004). Study of empirical mode decomposition based on high-order spline interpolation. *Journal of Zhejiang University*, **38**(3), 267-270. (in Chinese)

Zhang Yushan, Liang Jianwen, Hu yuxiang (2003). Application of auto-regressive model to process boundary problem in EMD. *Progress in Natural Science*, **13**(10), 1054-1059. (in Chinese)

Zhong Youming, Jin Tao, Qin Shuren (2005). New envelope algorithm for Hilbert-Huang transform.. *Jounal of Data Acquisition & Processing*, **20**(1), 13-17. (in Chinese)

Yang Li was born in 1982. She graduated from Harbin Engineering University and received the M.S. degree in 2009. She is now a PhD candidate for underwater acoustic engineering at Harbin Engineering University. Her main interests are focused on acoustic signal processing and underwater target recognition.

Si-Chun Li was born in 1963. She is a professor at Harbin Engineering University. She received the PhD degree in underwater acoustic engineering from Harbin Engineering University in 2008. Her current research interests include underwater acoustic signal processing, signal detection, passive target feature extraction and target recognition.