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H_∞ Robust Fault-Tolerant Controller Design for an Autonomous Underwater Vehicle's Navigation Control System

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Abstract: In order to improve the security and reliability for autonomous underwater vehicle (AUV) navigation, an H_∞ robust fault-tolerant controller was designed after analyzing variations in state-feedback gain. Operating conditions and the design method were then analyzed so that the control problem could be expressed as a mathematical optimization problem. This permitted the use of linear matrix inequalities (LMI) to solve for the H_∞ controller for the system. When considering different actuator failures, these conditions were then also mathematically expressed, allowing the H_∞ robust controller to solve for these events and thus be fault-tolerant. Finally, simulation results showed that the H_∞ robust fault-tolerant controller could provide precise AUV navigation control with strong robustness.
Keywords: AUV; navigation control; robust H_∞ fault-tolerant control; gain variations; LMI
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1 Introduction

AUV has been a subject of research and development, particularly in exploring unknown marine environment, completing specific underwater operation and carrying out submarine military missions. Although studies have been made on AUV over the past thirty years, still the AUV technology limitations remain, as in Jantapremjit and Wilson (2007), Wang *et al*. (2006), Repoulias (2007). Due to the complexity of the marine environment, the actuators and sensors, even the included software of AUV have hidden troubles. Therefore, the real-time monitor and the robust fault-tolerant control systems become one of the foundation technologies in the field of AUV research.

The navigation control system of AUV is constructed with host thrusters, auxiliary thrusters, rudders, and electromotor as the actuators, Octans and other sensors as the measurement tache. Thus, the AUV can realize the course maintenance, tracking, avoiding the obstacle and other tailing missions. But the thrusters and the rudders are easily broken down and blocked under the marine environment, where there are cables and seaweeds. Focusing on improving the security and reliability of navigation, and enhancing the accuracy and robustness of navigation control system for AUV, an H_∞ robust fault-tolerant controller is designed by analyzing state-feedback gain variation in this paper.

2 System description

The AUV has a flat contour as shown in Fig.1, which shows the actuators as hydroplane, a pair of rudders (left and right rudder linkage), four host thrusters and four auxiliary thrusters in annular groove. The hydroplane is used to control the obliquity and depth; rudders are used to control the course and tracking; host thrusters are used to control the velocity, also the left and right host thrusters are carried on the differential motion to control the course when the AUV navigates at low speed; auxiliary thrusters can make the AUV revolve, transversely move, as well as control the course of AUV.

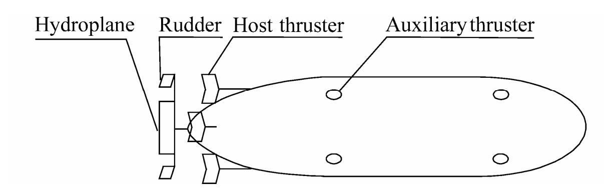

Fig.1 The planform of the AUV

2.1 Fault-tolerant control strategy for navigation control

Aiming at the characteristic of the mechanical installation for actuators, there are three kinds of strategies which may realize the navigation control over the AUV under complicated marine environment at different speeds. They are rudder mode, host thrusters in the differential motion mode and auxiliary thruster mode in Fig.2.

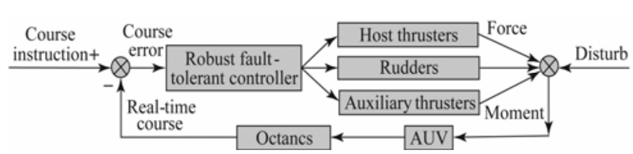

Fig.2 The frame of navigation fault-tolerant control system

2.1.1 Rudder mode

When the AUV navigates over the speed of 2.0 m/s, the rudder linkage can provide perfect rudder effect. Thus, the AUV can realize precise navigation control. At this time,

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auxiliary thrusters are not used.

2.1.2 Host thrusters in differential motion mode

When the AUV navigates at low speed, the rudder effect is very low, so it is not easy to realize precise navigation control in the ocean current situation. Moreover, when the rudders are blocked or the electromotor is fault, host thrusters may be used in differential motion mode to realize navigation control.

2.1.3 Auxiliary thrusters mode

When the host thrusters and the rudders are broken down or blocked under the marine environment, as well as the AUV under the hovering condition, auxiliary thrusters may be carried on to adjust the navigation.

2.2 Modeling for AUV navigation control system

In general, the motion of AUV can be simplified as the horizontal and vertical plane movement campaign when neglecting the rolling movement and the influence of coupling between the two planes. Thus, it is useful and convenient to research the course maintenance and control over the AUV, as in Li *et al*. (2007).

Assume that the navigation speed of the AUV is u_0, the center of gravity is on the origin of the coordinate system, and the influence of vertical movement and roll motion are neglected. As in Xiong *et al*. (2005), the course- keeping control system can be

$$\begin{cases}(m-\frac{1}{2}\rho L^3Y_{\dot v}^{'})\dot v+(-\frac{1}{2}\rho L^4Y_{\dot r}^{'})\dot r= \\ (\frac{1}{2}\rho L^2Y_{uv}'u_0)v+(\frac{1}{2}\rho L^3Y_{ur}'u_0-mu_0)r+Y_{\text{prop}}+(\frac{1}{2}\rho L^2Y_{\delta r}'u_0^2)\delta_r+d_v \\ (-\frac{1}{2}\rho L^4N_{\dot v}')\dot v+(I_z-\frac{1}{2}\rho L^5N_{\dot r}^{'})\dot r=(\frac{1}{2}\rho L^3N_{uv}'u_0)v+(\frac{1}{2}\rho L^4N_{ur}'u_0)r \\ \dot\psi=r+d_\psi\end{cases} \quad (1)$$

where L , m , I_z respectively are length, quality, and moment of inertia along coordinates; u_0 , v are translational motions; r is rotational motions (angular velocity). ψ is the attitude angle; $Y^{'}$, $N^{'}$ are hydrodynamic coefficients; Y_{prop} , N_{prop} are force and moment, respectively; δ_r is the rudder angle; ρ is the density of seawater; d_v , d_r , d_ψ include the error, uncertainty and outside interference, which are produced by linear approximation and bounded toward the system input.

Define the course instruction as a constant ψ_r, and $\dot\psi_r$=0, navigation error is ψ_e, then

$$\dot\psi_e=\frac{\mathrm{d}(\psi_r-\psi)}{\mathrm{d}t}=-\dot\psi=-r-d_\psi \quad (2)$$

Considering the outside disturbance (ocean current and wave), introduce an integral unit to eliminate the accumulated error:

$$\dot\psi_I=\psi_e \quad (3)$$

By the Eqs.(1)~(3), the dynamic model of the AUV navigation control system could be

$$\begin{bmatrix}\dot\psi_I\\ \dot\psi_e\\ \dot v\\ \dot r\end{bmatrix}=\begin{bmatrix}0&1&0&0\\0&0&0&-1\\0&0&a_{11}&a_{12}\\0&0&a_{21}&a_{22}\end{bmatrix}\begin{bmatrix}\psi_I\\ \psi_e\\ v\\ r\end{bmatrix}+\begin{bmatrix}0&0&0\\0&0&0\\b_{11}&b_{12}&b_{13}\\b_{21}&b_{22}&b_{23}\end{bmatrix}\begin{bmatrix}\delta_r\\ Y_{\text{prop}}\\ N_{\text{prop}}\end{bmatrix} \quad (4)$$

where

$$\begin{cases}\begin{bmatrix}a_{11}&a_{12}\\a_{21}&a_{22}\end{bmatrix}=\boldsymbol{H}^{-1}\boldsymbol{P},\begin{bmatrix}b_{11}&b_{12}&b_{13}\\b_{21}&b_{22}&b_{23}\end{bmatrix}=\boldsymbol{H}^{-1}\boldsymbol{Q}\\ \boldsymbol{H}=\begin{bmatrix}m-\frac{1}{2}\rho L^3Y_{\dot v}^{'}&-\frac{1}{2}\rho L^4Y_{\dot r}^{'}\\-\frac{1}{2}\rho L^4N_{\dot v}'&I_z-\frac{1}{2}\rho L^5N_{\dot r}^{'}\end{bmatrix},\\ \boldsymbol{P}=\begin{bmatrix}\frac{1}{2}\rho L^2Y_{uv}'u_0&\frac{1}{2}\rho L^3Y_{ur}'u_0-mu_0\\ \frac{1}{2}\rho L^3N_{uv}'u_0&\frac{1}{2}\rho L^4N_{ur}'u_0\end{bmatrix},\\ \boldsymbol{Q}=\begin{bmatrix}\frac{1}{2}\rho L^2Y_{\delta r}'u_0^2&1&0\\ \frac{1}{2}\rho L^3N_{\delta r}'u_0^2&0&1\end{bmatrix}.\end{cases} \quad (5)$$

3 Design H∞ robust fault-tolerant controller

Consider the descriptor system

$$\begin{cases}\dot{\boldsymbol{x}}=\boldsymbol{A}\boldsymbol{x}+\boldsymbol{B}_1\boldsymbol{w}+\boldsymbol{B}_2\boldsymbol{u}\\ \boldsymbol{z}=\boldsymbol{C}_1\boldsymbol{x}+\boldsymbol{D}_{12}\boldsymbol{u}\end{cases} \quad (6)$$

where, $\boldsymbol{x}=\begin{bmatrix}\psi_I&\psi_e&v&r\end{bmatrix}^{\mathrm{T}}$ is state variable, $\boldsymbol{w}\in R^3$ are exogenous disturbance, $\boldsymbol{u}=\begin{bmatrix}\delta_r&Y_{\text{prop}}&N_{\text{prop}}\end{bmatrix}^{\mathrm{T}}$ is control input, $\boldsymbol{z}\in R^4$ is controlled output. $\boldsymbol{A}$, $\boldsymbol{B}_1$, $\boldsymbol{B}_2$, $\boldsymbol{C}_1$, $\boldsymbol{D}_{12}$ are coefficient matrixes of appropriate dimensions. Design with state-feedback gain variation:

$$\boldsymbol{u}=(\boldsymbol{k}+\Delta\boldsymbol{k})\boldsymbol{x} \quad (7)$$

where $\boldsymbol{k}$ is gain array，$\Delta\boldsymbol{k}$ is gain perturbation, as in Jiang *et al*.(2006), Chen *et al*. (2007).

Consider the form of perturbation:

$$\begin{cases}\Delta\boldsymbol{k}=\begin{bmatrix}k_{ij}\delta_{ki}\end{bmatrix}_{m\times n}=\boldsymbol{\delta_k}\boldsymbol{k}\\ \boldsymbol{\delta_k}=\mathrm{diag}\{\delta_{k1},\cdots,\delta_{km}\}\end{cases} \quad (8)$$

where $\boldsymbol{k}=\begin{bmatrix}k_{ij}\end{bmatrix}_{m\times n}$, δ_{ki} are the uncertainties, which satisfy

$$\left|\delta_{ki}\right| \le \bar{\delta}_{ki} < 1,\quad i = 1,2,\cdots,m \tag{9}$$

$\bar{\delta}_{ki}$ are constants, let $\boldsymbol{\delta} = \mathrm{diag}\left\{\bar{\delta}_{k1}^2,\ \bar{\delta}_{k1}^2,\ \cdots\ ,\bar{\delta}_{km}^2\right\}$. Generally, the perturbation is produced by performance attenuation of controller. Then, Eq.(6) can be transformed into

$$\begin{cases} \dot{\boldsymbol{x}} = \bar{\boldsymbol{A}}\boldsymbol{x} + \boldsymbol{B_1 w} \\ \boldsymbol{z} = \bar{\boldsymbol{C}}\boldsymbol{x} \end{cases} \tag{10}$$

where $\bar{\boldsymbol{A}} = \boldsymbol{A} + \boldsymbol{B}_2(\boldsymbol{k} + \Delta\boldsymbol{k})$, $\bar{\boldsymbol{C}} = \boldsymbol{C}_1 + \boldsymbol{D}_{12}(\boldsymbol{k} + \Delta\boldsymbol{k})$.

3.1 Preliminary knowledge

In Wang and Yang (2002), define $\boldsymbol{\Omega} = \{1,2,\cdots,m\}$, expressing the proper subset of the actuators which are easily broken down; $\bar{\boldsymbol{\Omega}} = \{1,2,\cdots,m\} - \boldsymbol{\Omega} \ne \boldsymbol{\Phi}$ is the complementary set; $\boldsymbol{\Phi}$ is the null subset; $\boldsymbol{L_k}$ is actuators switching matrix. Assume that $\boldsymbol{\Omega} = \{q+1,\cdots,m\}$, thus $\boldsymbol{L_k} = \mathrm{diag}\{\boldsymbol{I_q}\quad 0\}$. It means that q actuators are normally working, the other $m-q$ actuators are to be failed easily. They can be written as

$$\begin{cases} \boldsymbol{B_2} = \begin{bmatrix} \boldsymbol{B}_{2\bar{\boldsymbol{\Omega}}} & \boldsymbol{B}_{2\boldsymbol{\Omega}} \end{bmatrix} \\ \boldsymbol{D_{12}} = \begin{bmatrix} \boldsymbol{D}_{12\bar{\boldsymbol{\Omega}}} & \boldsymbol{D}_{12\boldsymbol{\Omega}} \end{bmatrix} \\ \boldsymbol{D} = \begin{bmatrix} \boldsymbol{D}_{\bar{\boldsymbol{\Omega}}}^{\mathrm{T}} & \boldsymbol{D}_{\boldsymbol{\Omega}}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \\ \bar{\boldsymbol{\delta}} = \mathrm{diag}\begin{bmatrix} \bar{\boldsymbol{\delta}}_{\bar{\boldsymbol{\Omega}}} & \bar{\boldsymbol{\delta}}_{\boldsymbol{\Omega}} \end{bmatrix} \end{cases} \tag{11}$$

Set $\varphi \subseteq \boldsymbol{\Omega}$, which is the subset of failure actuators, and $\boldsymbol{L}_\varphi = \mathrm{diag}\begin{bmatrix}\boldsymbol{I}_{\bar{\varphi}} & 0\end{bmatrix}$, $\boldsymbol{u}_\varphi$ is the corresponding set of failure actuators. Similarly,

$$\begin{cases} \boldsymbol{B}_2 = \begin{bmatrix} \boldsymbol{B}_{2\bar{\varphi}} & \boldsymbol{B}_{2\varphi} \end{bmatrix}, \boldsymbol{D}_{12} = \begin{bmatrix} \boldsymbol{D}_{12\bar{\varphi}} & \boldsymbol{D}_{12\varphi} \end{bmatrix}, \\ \boldsymbol{D} = \begin{bmatrix} \boldsymbol{D}_{\bar{\varphi}}^{\mathrm{T}} & \boldsymbol{D}_{\varphi}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}, \boldsymbol{u} = \begin{bmatrix} \boldsymbol{u}_{\bar{\varphi}} & \boldsymbol{u}_{\varphi} \end{bmatrix}^{\mathrm{T}}, \\ \boldsymbol{k} = \begin{bmatrix} \boldsymbol{k}_{\bar{\varphi}} & \boldsymbol{k}_{\varphi} \end{bmatrix}^{\mathrm{T}}, \boldsymbol{\Delta k} = \begin{bmatrix} \boldsymbol{\Delta k}_{\bar{\varphi}} & \boldsymbol{\Delta k}_{\varphi} \end{bmatrix}^{\mathrm{T}}, \\ \boldsymbol{\delta}_k = \begin{bmatrix} \boldsymbol{\delta}_{k\bar{\varphi}} & 0 \\ 0 & \boldsymbol{\delta}_{k\varphi} \end{bmatrix}, \bar{\boldsymbol{\delta}} = \begin{bmatrix} \bar{\boldsymbol{\delta}}_{\bar{\varphi}} & 0 \\ 0 & \bar{\boldsymbol{\delta}}_{\varphi} \end{bmatrix}, \end{cases} \tag{12}$$

where $\bar{\varphi} = \bar{\boldsymbol{\Omega}} \cup (\bar{\boldsymbol{\Omega}} - \varphi)$. Then, the descriptor's closed-loop system with actuator failures can be written as

$$\begin{cases} \dot{\boldsymbol{x}} = \bar{\boldsymbol{A}}_{\bar{\varphi}}\boldsymbol{x} + \boldsymbol{B}_1\boldsymbol{w} \\ \boldsymbol{z} = \bar{\boldsymbol{C}}_{\bar{\varphi}}\boldsymbol{x} \end{cases} \tag{13}$$

where, $\bar{\boldsymbol{A}}_{\bar{\varphi}} = \boldsymbol{A} + \boldsymbol{B}_{2\bar{\varphi}}(\boldsymbol{k}_{\bar{\varphi}} + \boldsymbol{\delta}_{k\bar{\varphi}}\boldsymbol{k}_{\bar{\varphi}})$, $\bar{\boldsymbol{C}}_{\bar{\varphi}} = \boldsymbol{C}_1 + \boldsymbol{D}_{12\bar{\varphi}}(\boldsymbol{k}_{\bar{\varphi}} + \boldsymbol{\delta}_{k\bar{\varphi}}\boldsymbol{k}_{\bar{\varphi}})$.

Given $\|T(s)\|_\infty < \gamma$ and $\|T_F(s)\|_\infty < \gamma$ ($\gamma > 0$), the systems (10) and (13) are asymptotically stable and admissible by state-feedback gain variation (7), under the actuators' normal working conditions or when parts of them fail. Thus, the state-feedback (7) is called H_∞ robust fault-tolerant controller with gain variation. Where $T(s) = \bar{\boldsymbol{C}}(s\boldsymbol{E} - \bar{\boldsymbol{A}})^{-1}\boldsymbol{B}_1$, $T_F(s) = \bar{\boldsymbol{C}}_{\bar{\varphi}}(s\boldsymbol{E} - \bar{\boldsymbol{A}}_{\bar{\varphi}})^{-1}\boldsymbol{B}_1$.

This paper is to design the H_∞ robust fault-tolerant controller by analyzing state-feedback gain variation for the descriptor system (6), and apply it to the navigation control system for AUV.

3.2 Main result

Our objective is to design the state-feedback controller (7), for all admissible controller gain variations, such that the closed-loop system satisfies the following requirements 1) and 2) simultaneously.

1) $\boldsymbol{D}_{12}^{\mathrm{T}}\begin{bmatrix}\boldsymbol{C}_1 & \boldsymbol{D}_{12}\end{bmatrix} = \begin{bmatrix}\boldsymbol{0} & \boldsymbol{I}\end{bmatrix}$;

2) The descriptor system $(\boldsymbol{A}, \boldsymbol{B}_{2\bar{\Omega}})$ is controllability.

Lemma 1: When parts of actuators are failure toward the descriptor system (6), if there exist a constant ε ($0 < \varepsilon < 1$) and the matrix as Eq.(14), which satisfy LMI (15), the state-feedback controller (16) can make the system (15) admissible, and satisfy $\|T_F(s)\|_\infty < \gamma$.

$$\boldsymbol{Q} = \begin{bmatrix} \boldsymbol{Q}_s & \boldsymbol{0} \\ \boldsymbol{Q}_1 & \boldsymbol{Q}_f \end{bmatrix} \in \boldsymbol{R}^{n\times n} \tag{14}$$

where $\boldsymbol{Q}_s \in \boldsymbol{R}^{r\times r}$, $\boldsymbol{Q}_s = \boldsymbol{Q}_s^T > 0$, $\boldsymbol{Q}_1 \in \boldsymbol{R}^{(n-r)\times r}$, $\boldsymbol{Q}_f \in \boldsymbol{R}^{(n-r)\times(n-r)}$, and $\boldsymbol{Q}_f$ is invertible matrix.

$$\begin{bmatrix} \boldsymbol{Q}^{\mathrm{T}}\boldsymbol{A}^{\mathrm{T}} + \boldsymbol{AQ} + \boldsymbol{B}_{2\bar{\Omega}}\alpha_{0\bar{\Omega}}\boldsymbol{B}_{2\bar{\Omega}}^{\mathrm{T}} & \boldsymbol{B}_1 & \boldsymbol{Q}^{\mathrm{T}}\boldsymbol{C}_1^{\mathrm{T}} \\ \boldsymbol{B}_1^{\mathrm{T}} & -\gamma^2\boldsymbol{I} & 0 \\ \boldsymbol{C}_1\boldsymbol{Q} & 0 & -\boldsymbol{I} \end{bmatrix} < 0 \tag{15}$$

where $\alpha_{0\bar{\Omega}} = (1-\varepsilon)^{-1}(\varepsilon\boldsymbol{I} - (\boldsymbol{I} + \varepsilon^{-1}(1-\varepsilon)\bar{\delta}_{\bar{\Omega}})^{-1}) < 0$. Then

$$\boldsymbol{u}_{\bar{\varphi}} = (\boldsymbol{I} + \boldsymbol{\delta}_{k\bar{\varphi}})\boldsymbol{k}_{\bar{\varphi}}\boldsymbol{x} \tag{16}$$

where $\boldsymbol{k}_{\bar{\varphi}} = -(1-\varepsilon)^{-1}\boldsymbol{R}_{1\bar{\varphi}}^{-1}\boldsymbol{B}_{2\bar{\varphi}}^{T}\boldsymbol{Q}^{-1}$, $\boldsymbol{R}_{1\bar{\varphi}} = \varepsilon^{-1}\bar{\boldsymbol{\delta}}_{\bar{\varphi}} + (1-\varepsilon)^{-1}\boldsymbol{I}$.

Lemma 2: Toward the descriptor system (6), if there exist a constant ε ($0 < \varepsilon < 1$) and matrix $\boldsymbol{Q}$ in Eq.(14) satisfying LMI:

$$\begin{bmatrix} \boldsymbol{Q}^{\mathrm{T}}\boldsymbol{A}^{\mathrm{T}} + \boldsymbol{AQ} + \boldsymbol{B}_2\alpha_0\boldsymbol{B}_2^{\mathrm{T}} & \boldsymbol{B}_1 & \boldsymbol{Q}^{\mathrm{T}}\boldsymbol{C}_1^{\mathrm{T}} \\ \boldsymbol{B}_1^{\mathrm{T}} & -\gamma^2\boldsymbol{I} & 0 \\ \boldsymbol{C}_1\boldsymbol{Q} & 0 & -\boldsymbol{I} \end{bmatrix} < 0 \tag{17}$$

where $\alpha_0 = (1-\varepsilon)^{-1}(\varepsilon\boldsymbol{I} - (\boldsymbol{I} + \varepsilon^{-1}(1-\varepsilon)\delta)^{-1}) < 0$. Then

$$\boldsymbol{u} = (\boldsymbol{I} + \boldsymbol{\delta}_k)\boldsymbol{kx} \tag{18}$$

where $\boldsymbol{k}=-(1-\varepsilon)^{-1}\boldsymbol{R}_1^{-1}\boldsymbol{B}_2^T\boldsymbol{Q}^{-1}$, $\boldsymbol{R}_1=\varepsilon^{-1}\overline{\boldsymbol{\delta}}+(1-\varepsilon)^{-1}\boldsymbol{I}$. The state-feedback controller (18) can make the system (10) admissible, and $\|T(s)\|_\infty<\gamma$.

By Lemmas 1 and 2, if there exist a constant ε ($0<\varepsilon<1$) and matrix $\boldsymbol{Q}$ in Eq.(14) satisfying LMI (15), then Eq.(18) is the H∞ robust fault-tolerant controller with the state-feedback gain variations for the system.

When the system has actuator failures, although controller (18) is still used, the switching matrix is $\boldsymbol{L}_k=\mathrm{diag}\left[\boldsymbol{I}_{\bar{\varphi}}\quad 0\right]$, thus, only parts of the signals $\boldsymbol{u}_{\bar{\varphi}}$ are back to system (13). According to different actuator failures, although not all of the control parameters of $\boldsymbol{u}_{\bar{\varphi}}$ are useful, they are parts of the signals for controller (18), make the systems (10), (13) admissible without recalculating the controller.

3.3 Design navigation controller for the AUV

According to the strategies for navigation control, the controller is designed with the navigation speed u_0=1.0, 2.0 and 3.0 m/s, respectively. Then, the system (4) can be written at the speed u_0=3.0 m/s.

$$\begin{bmatrix}\dot{\psi}_I\\ \dot{\psi}_e\\ \dot{v}\\ \dot{r}\end{bmatrix}=\begin{bmatrix}0&1&0&0\\0&0&0&-1\\0&0&-0.2546&-1.765\\0&0&-0.0958&-0.8571\end{bmatrix}\begin{bmatrix}\psi_I\\ \psi_e\\ v\\ r\end{bmatrix}+\begin{bmatrix}0&0&0\\0&0&0\\0.2664&0.3516&-0.0048\\-0.5517&-0.0048&0.1988\end{bmatrix}\begin{bmatrix}\delta_r\\ Y_{\text{prop}}\\ N_{\text{prop}}\end{bmatrix} \quad (19)$$

Thus,

$$\boldsymbol{A}=\begin{bmatrix}0&1&0&0\\0&0&0&-1\\0&0&-0.2546&-1.765\\0&0&-0.0958&-0.875\end{bmatrix},\quad \boldsymbol{B}_1=\begin{bmatrix}0\\0.1586\\0.3472\\-0.5168\end{bmatrix} \quad (20)$$

$$\boldsymbol{B}_2=\begin{bmatrix}0&0&0\\0&0&0\\0.2664&0.3516&-0.0048\\-0.5517&-0.0048&0.1988\end{bmatrix}. \quad (21)$$

Assume ε =0.75, γ =1.7, $\boldsymbol{C}_1=\begin{bmatrix}0.25&0.25&0.25&0\end{bmatrix}$, $\boldsymbol{D}_{12}=\begin{bmatrix}0&0&0&1\end{bmatrix}$. By the state-feedback gain variation, we can get $\delta_r=u_1=\boldsymbol{k}_1\begin{bmatrix}\psi_I&\psi_e&v&r\end{bmatrix}^{\mathrm{T}}$ for navigation controller based on rudder mode at the speed of 3.0 m/s. By the YALMIP Toolbox with the SeDuMi solver in Lofberg (2004), the control parameters $\boldsymbol{k}_1=\begin{bmatrix}0.2172&1.0576&0.0479&-1.7761\end{bmatrix}$ can be obtained.

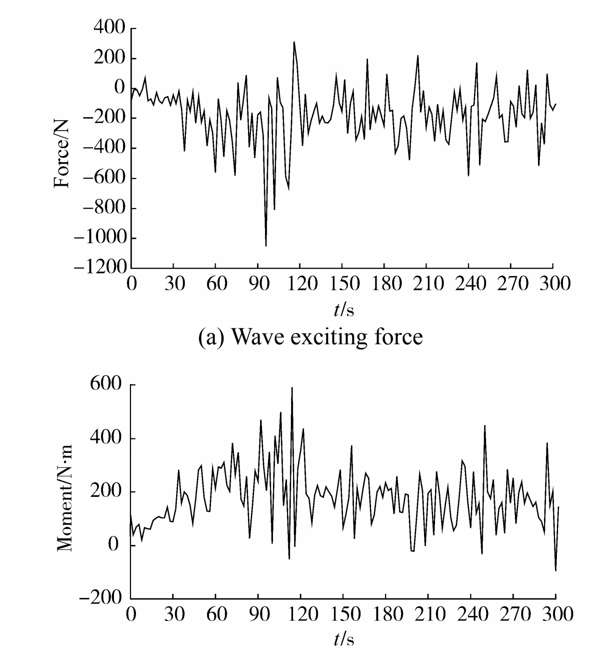
Then, by the similar method, we can get $Y_{\text{prop}}=u_2=\boldsymbol{k}_2\begin{bmatrix}\psi_I&\psi_e&v&r\end{bmatrix}^{\mathrm{T}}$ based on host thrusters in the differential motion mode at the navigation speed of 2.0 m/s, where, $\boldsymbol{k}_2=\begin{bmatrix}26.8&376.8&18.9&-1957.1\end{bmatrix}$; also $N_{\text{prop}}=u_3=\boldsymbol{k}_3\begin{bmatrix}\psi_I&\psi_e&v&r\end{bmatrix}^{\mathrm{T}}$ based on auxiliary thrusters mode at the speed of 1.0m/s, where $\boldsymbol{k}_3$ = [43.6 548.3 27.9 -3097.8].

As to the navigation control system for the AUV, rudders, host thrusters and auxiliary thrusters are actuators. Parts of them will fail at different navigation speeds of AUV. Thus, the H∞ robust fault-tolerant controller can be written as

$$\boldsymbol{u}=(\boldsymbol{I}+\boldsymbol{\delta}_k)\begin{bmatrix}0.2172&1.0576&0.0479&-1.7761\\26.8&376.8&18.9&-1957.1\\43.6&548.3&27.9&-3097.8\end{bmatrix}\begin{bmatrix}\psi_I\\ \psi_e\\ v\\ r\end{bmatrix} \quad (22)$$

4 Simulation for the AUV navigation control system

Dynamic model of the AUV in horizontal plane is used in the design process for navigation controller. In order to demonstrate the controller is useful and valid for the AUV 6-DOF mathematical model, and the robust fault-tolerant controller has been improved in the dynamic characteristics and robustness, the disturbance model of ocean waves are established, as shown in Fig.3.



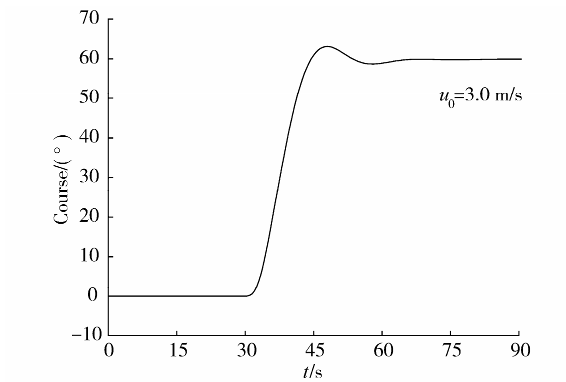
(a) Wave exciting force

(b) Wave exciting moment

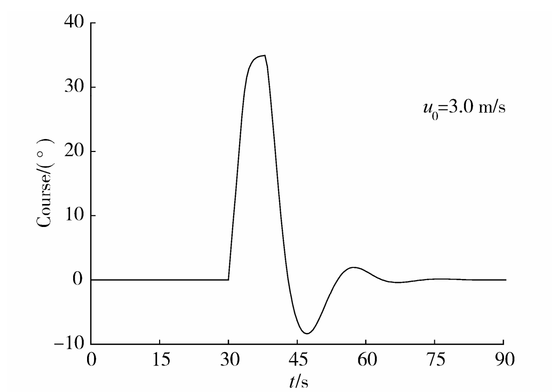
Fig.3 The responses of wave exciting force and moment

In order to improve the precision of state variables, H∞ Filter was used for the AUV navigation control system as in Xiong *et al*. (2005). By the feedback control law (22), the navigation control curves were obtained at different modes of AUV under the conditions of slight sea. Figs.4~6 show the transient process of navigation under these three modes.

The AUV navigates at the speed of 3.0 m/s from time 0 s to 90 s, as shown in Fig.4. The course injunction changes to 60º from 0º at the time 30 s, and the transient process of navigation was shown in Fig.4 under the rudder mode, thus, the feedback controller is $\delta_r = u_1 = \boldsymbol{k}_1[\psi_I \quad \psi_e \quad v \quad r]^{\mathrm{T}}$. Fig.4(a) shows the transient process of navigation control at the speed of 3.0 m/s, whose maximum overshoot is 4.5%, rise time and stabilizing time are 13 s and 30 s, respectively. Fig.4(b) shows the transient process of rudder whose angle range is -35º~35º.



(a) Transient process of navigation control

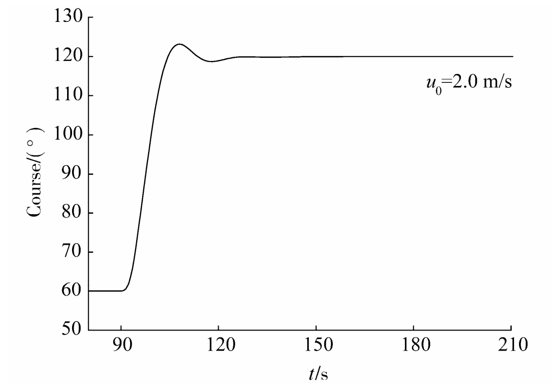


(b) Transient process of rudder

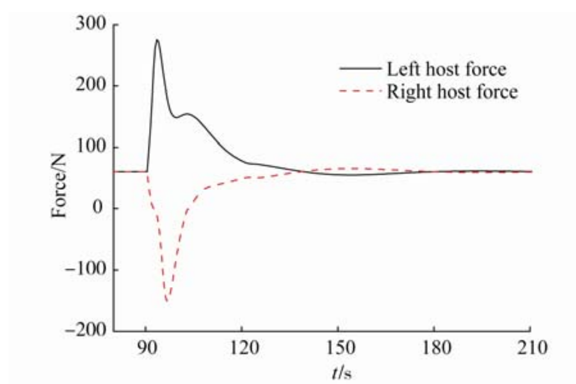
Fig.4 Transient process of navigation control under the rudder

At the time 90 s, the speed of AUV changes to 2.0 m/s, which keeps unchanged until the time 210 s, as shown in Fig.5. And the course injunction changes to 120º from 60º with the actuator switched to host thrusters, also the transient process of navigation is shown in Fig.5 in the host thrusters mode. The feedback controller is $Y_{\mathrm{prop}} = u_2 = \boldsymbol{k}_2[\psi_I \quad \psi_e \quad v \quad r]^{\mathrm{T}}$. Fig.5(a) shows the transient process of navigation control at the speed of 2.0 m/s, whose maximum overshoot is 3.8%, rise time and stabilizing time are 15 s and 31 s, respectively. Fig.5(b) shows the transient process of host thrusters. The force of left host thruster is positive and right host thruster is negative, when AUV is turning right.

Then, at the time 210 s, the speed of the AUV changes to 1.0m/s again, which keeps unchanged until the time 300 s, as in Fig.6. And the course injunction changes to 180º from 120º with the actuator switched to auxiliary thrusters, also the transient process of navigation is shown in Fig.6 in the auxiliary thrusters mode. Thus, the feedback controller is $N_{\mathrm{prop}} = u_3 = k_3[\psi_I \quad \psi_e \quad v \quad r]^{\mathrm{T}}$. Fig.6(a) shows the transient process of navigation control at the speed of 1.0m/s, whose maximum overshoot is 1.5%, rise time and stabilizing time are 20 s and 33 s, respectively. Fig.6(b) shows the transient process of auxiliary thrusters. It is easy to measure the current and voltage of the auxiliary thruster, so we get the force instead of moment of them. The force of left auxiliary thrusters is negative and that of right auxiliary thrusters is positive, when the AUV is turning right.

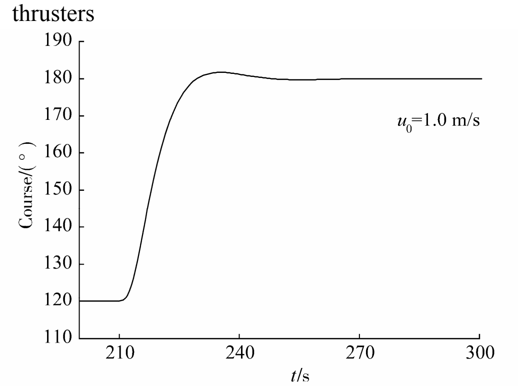


(a) Transient process of navigation control

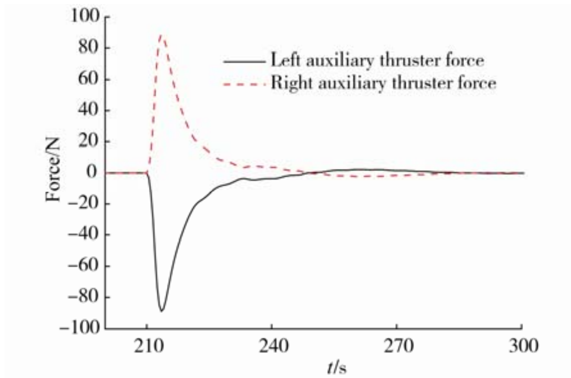


(b) Transient process of host thrusters

Fig.5 Transient process of navigation control under the host thrusters



(a) transient process of navigation control



(b) transient process of auxiliary thrusters

Fig.6 Transient process of navigation control under the auxiliary thrusters

The simulation results show that the navigation control system has a good performance and strong robustness for the AUV under the ocean wave disturbance. And the results demonstrate that designed with controller gain variations which could improve the security for the AUV navigation control system, the H_∞ robust fault-tolerant controller is feasible and reliable.

5 Conclusions

It is all known that the actuators of the AUV, just as rudder and thrusters under the complex ocean environment, would get failure easily. In order to improve the security and reliability of the AUV, the H_∞ robust fault-tolerant controller is designed with controller gain variations for navigation control system. In this paper, the navigation control system based on multiactuators is established, and the method of designing fault-tolerant controller by controller gain variations is given. Using the approach of linera matrix inequalities，the sufficient conditions and the design method are studied to show that there exist H_∞ robust fault-tolerant controller for the AUV navigation system． According to the object mode of the AUV, the controller (22) is applied to the simulations of navigation control system under the wave disturbance. The response curves under the different actuators show that response time and maximum overshoot are low, which demonstrate the H_∞ robust fault-tolerant controller is feasible and reliable for the AUV navigation control. However, the H_∞ robust fault-tolerant controller has to be further demonstrated under the real ocean environment for the AUV navigation control system.

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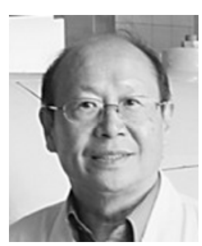

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