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Influence of Wave Breaking on Wave Statistics for Finite-Depth Random Wave Trains

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Abstract: The influence of wave breaking on wave statistics for finite-depth random wave trains is investigated experimentally. This paper is to investigate the influence of wave breaking and water depth on the wave statistics for random waves on water of finite depth. Greater attention is paid to changes in wave statistics due to wave breaking in random wave trains. The results show skewness of surface elevations is independent of wave breaking and kurtosis is suppressed by wave breaking. Finally, the exceedance probabilities for wave heights are also investigated.
Keywords: wave breaking; wave statistics; wave height distribution; skewness; kurtosis
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1 Introduction

Breaking of surface waves is known to play an important role in upper ocean processes. Especially when they occur as large-scale breakers, these waves represent the most hazardous conditions for seafarers and offshore structures. Wave breaking is one of the most significant energy redistribution terms in ocean wave models. It contributes to the exchange of gas, water vapor, energy, and momentum between the atmosphere and the ocean. These exchanges affect the growth of wind waves (Phillips, 1977), the generation of bubbles and sea spray (Johnson and Cooke, 1979), the formation of surface currents (Longuet-Higgins, 1969; Rapp and Melville, 1990), and the generation and distribution of near-surface turbulence (Thorpe, 1995). These numerous and diverse aspects of wave breaking and their importance for upper ocean dynamics and offshore engineering are reviewed in greater detail (Banner and Peregrine, 1993; Thorpe, 1993; Melville, 1996; Duncan, 2001).

Accurate estimates of wave statistics of random waves (e.g. wave height distribution, skewness, kurtosis, wave group, *etc*.), especially the statistical properties of breaking waves, are becoming increasingly important not only for scientific research, but also for ocean structure-design purposes. Studies on breaking waves have been addressed by many authors using analytical methods, numerical models (Tao and Han, 2001; Tang *et al*., 2008; Khayyer *et al*., 2008; Dong *et al*., 2008), experimental measurement (Liu and Hong, 2005; Rapp and Melville, 1990) and field observations (Holthuijsen and Herbers, 1986; Ding and Farmer, 1994). Their main attentions are focused on the breaking wave characters, the breaking criteria (Xu *et al*., 1998; Song and Banner, 2002; Banner and Peirson, 2007), airflow (Reul *et al*., 2008), eddy viscosity (Zhang *et al*., 2007), and energy loss (Kuznetsov and Saprykina, 2004) due to wave breaking. These papers, however, do not address the effect of wave breaking on wave statistics (e.g. wave height distribution, skewness, kurtosis, *etc*.).

The purpose of this study is to investigate the influence of wave breaking and water depth on the wave statistics for random waves on water of finite depth. The effects of wave breaking on wave statistics are evaluated by comparing with nonbreaking waves experimentally in different water depth.

2 Experimental methods

The laboratory experiments were carried out in a 2-D glass-walled wave flume of 50 m long, 3.0 m wide and 1 m deep at the State Key Laboratory of Coastal and Offshore Engineering, Dalian University of Technology, China. The flume was filled to a depth of approximately 0.3 m, 0.4 m, and 0.5 m. The wave flume was equipped with a piston-type wavemaker and a downstream wave-energy absorbing beach. Eight resistance-type wave gauges with 50 Hz scan rate were used to measure water surface elevations. The first wave gauge was located at x=6 m away from the wavemaker. The next 7 gauges with 2 m distance between each other were installed to record the generated waves along the wave flume. Fig.1 is the sketch of the experimental setup. Data recording started when the generated waves passed the last wave gauge and up to 160 s of water-surface oscillations were recorded.

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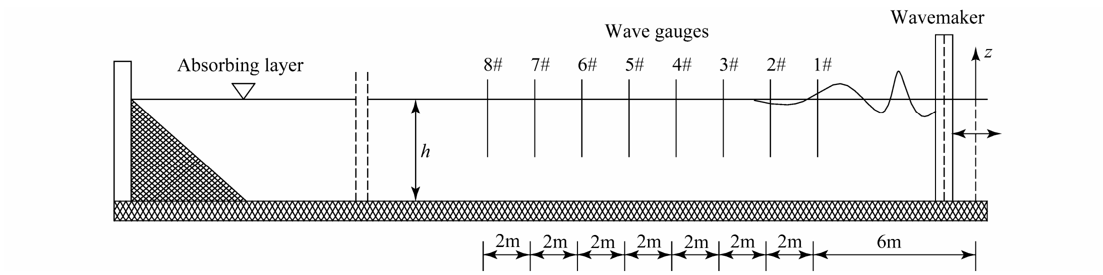


Fig.1 The sketch of experimental layout

The generation of a time-series by superposition was demonstrated by using the JONSWAP type spectra. In linear wave theory, the initial free surface elevation could be represented by linear combinations of sinusoidal waves:

$$\eta(t)=\sum_{i=1}^{\infty}\sqrt{2S(f_i)df}\cos(2\pi f_i t+\tau_i) \quad (1)$$

where $\eta(t)$ is the free surface elevation of the initial waves, and τ_i is the phase constant, which is given by uniformly distributed random numbers. The density distribution was calculated by the formula (Goda, 1999):

$$S(f)=\beta_J H_{1/3}^2 T_p^{-4} f^{-5}\exp\left[-1.25(T_p f)^{-4}\right]\gamma^{\exp\left[-(f/f_p-1)^2/2\sigma^2\right]} \quad (2)$$

where

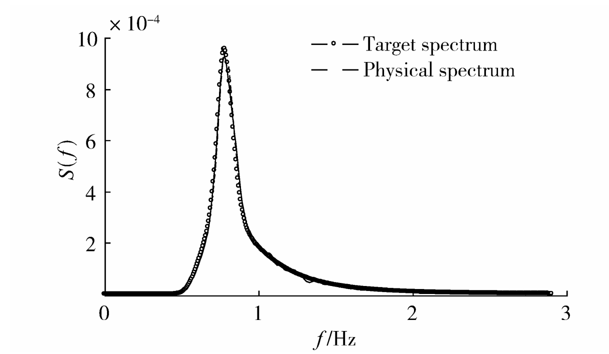
β_J=0.06238/[0.23+0.0336γ-0.185(1.9+γ)$^{-1}$]·(1.094-0.01915γ), γ=3.3, σ=0.07 ($\omega<\omega_p$) σ=0.07 ($\omega\geq\omega_p$), f is the frequency, f_p is the peak frequency of power spectrum, $S(f)$ is the power spectrum. The peak period T_p is equal to 1.2 s.

The waves have been determined by zero-down crossing definitions. The number of down-crossing waves was approximately 140~160. The initial input significant wave height H_s was set at approximately 6 cm, 10 cm. In this study, for simplicity, the case of input H_s=6 cm would be considered non-breaking waves, and the case of input H_s=10cm would be considered breaking waves. The frequencies of wave breaking were controlled by the initial steepness $k_p a_{1/3}$. Table 1 presents the initial wave steepness $\varepsilon=k_p a_{1/3}$. Here, k_p is the spectral peak wave number and $a_{1/3}$ is half the significant wave height H_s of the initial wave. As pointed out by Mori (2003), there were no breaking waves if the value of $k_p a_{1/3}$ was less than approximately 0.12. It can be noticed that there are no breaking waves for the case of H_s=6 cm, while wave breaking appears with low frequencies for the case of H_s=10 cm.

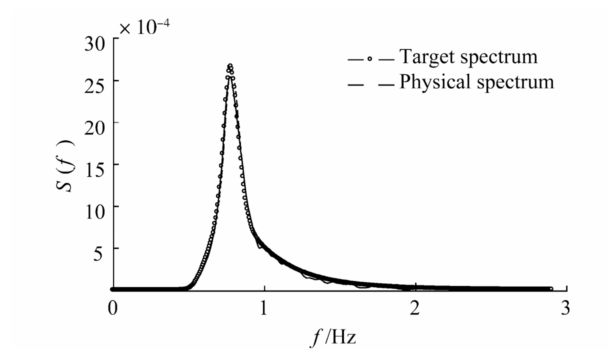
The experimental verification was restricted to the significant wave height H_s and the spectrum $S(f)$ at the first wave gauge. The measured spectral density distribution $S(f)$ and the measured significant wave height H_s are shown in Fig.2 and Fig.3 respectively. Typical measurements of two cases for water depth h=0.5 m are shown in Fig.2 for non-breaking and breaking cases. The measured spectral density distribution $S(f)$ is presented in Fig.2 and is compared with the target spectrum. Fig.2(a) presents the non-breaking case, while Fig.2(b) presents the breaking case. The plots in Fig.2 show a good agreement between the measured spectral density distribution and the target spectrum. The measured significant wave height H_s and corresponding significant wave height input are shown in Fig.3. The comparison is conducted for random wave trains in different water depth corresponding to h=0.3 m, 0.4 m, 0.5 m. The measured significant wave height H_s is stable except at the second and the third wave gauge.

Table 1 Wave steepness

h/m	H_s/cm	ε
0.3	6	0.107
0.3	10	0.178
0.4	6	0.0975
0.4	10	0.163
0.5	6	0.092
0.5	10	0.154

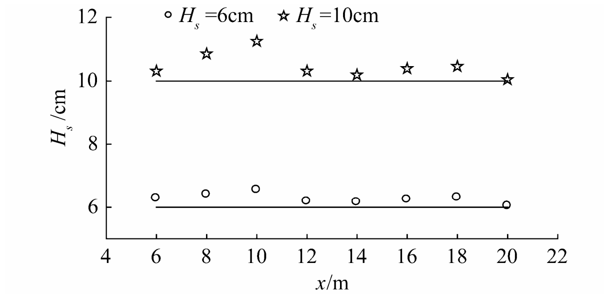


(a) H_s=6 cm

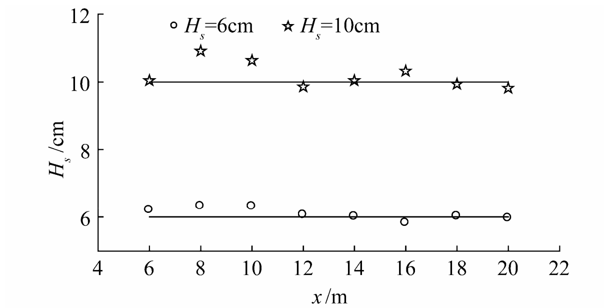


(b) H_s=10 cm

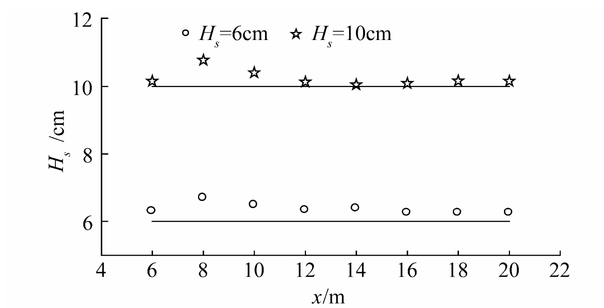
Fig.2 Comparison of measured and target spectrum for h=0.5 m



(a) h=0.3 m



(b) h=0.4 m



(c) h=0.5 m

Fig.3 Comparison of measured and target significant wave height for different water depth

3 Experimental results

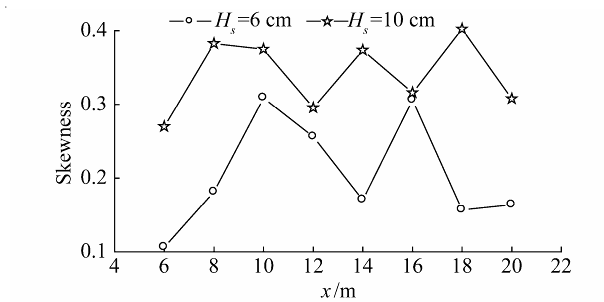
Some of the parameters describing the properties of the random wave train are discussed in this section. The interest is focused on the difference of some basic wave statistics between non-breaking and breaking waves for several cases.

As a measure of nonlinearity in the random wave field, the coefficient of skewness and the coefficient of kurtosis are usually applied. Skewness, μ_3, reflects quantitatively the increased frequency of occurrence of high wave crests, as compared with the linear theory predictions, while kurtosis, μ_4, reflects the frequency of encountering large crest-to-trough excursions. Skewness and kurtosis are defined as:

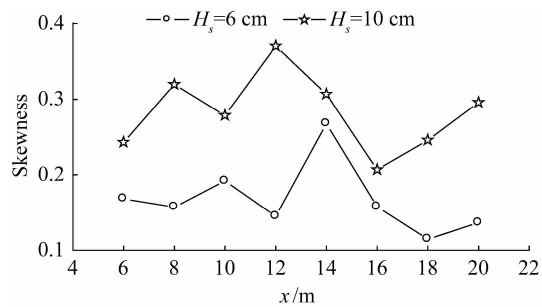
$$\mu_3 = \frac{1}{N}\sum_{n=1}^{N}\frac{\left(\eta_n - \overline{\eta}\right)^3}{\eta_{rms}^3} \tag{3}$$

$$\mu_4 = \frac{1}{N}\sum_{n=1}^{N}\frac{\left(\eta_n - \overline{\eta}\right)^4}{\eta_{rms}^4} \tag{4}$$

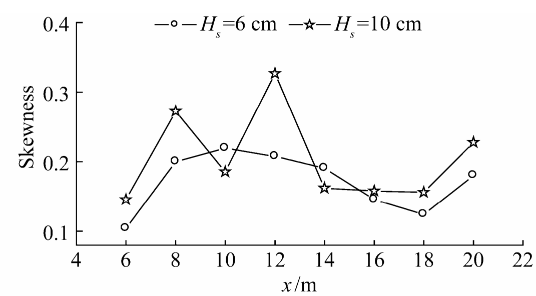
where N is the number of data points, η_n is the free surface elevation, $\overline{\eta}$ is the mean value of η. η_{rms} is the root mean square value of η. It is obvious that for a Gaussian process the value of the skewness μ_3 is 0 and the kurtosis μ_4 is equal to 3.



(a) h=0.3 m

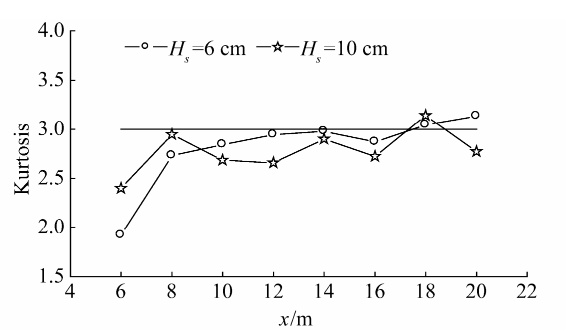


(b) h=0.4 m

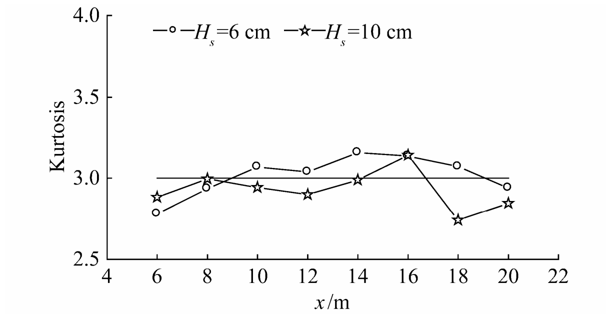


(c) h=0.5 m

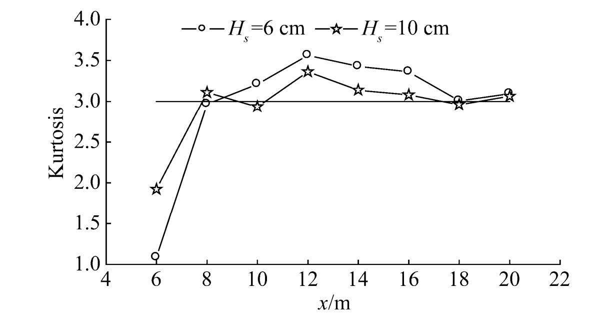
Fig.4 Spatial variations of skewness μ_3 of surface elevations for non-breaking and breaking waves with different water depth



(a) h=0.3 m



(b) h=0.4 m



(c) h=0.5 m

Fig.5 Spatial variations of kurtosis μ_4 of surface elevations for non-breaking and breaking waves with different water depth

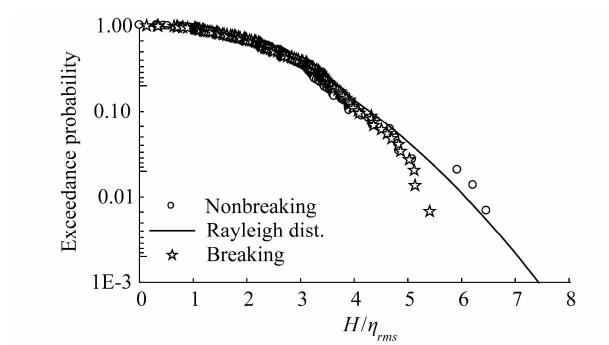
A commonly used model for describing wave heights is the Rayleigh distribution (Longuet-Higgins, 1952). The distribution can be expressed as (Forristall, 1978):

$$P(H > H_0) = \exp\left(-\frac{H_0^2}{8\eta_{rms}^2}\right) \tag{5}$$

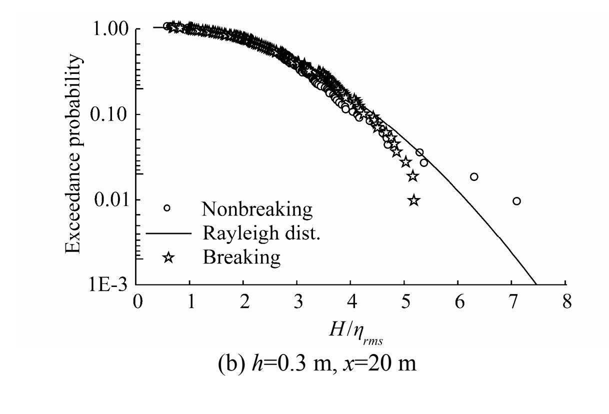
where P is the probability that wave height H exceeds H_0.

The exceedance probabilities of wave heights are plotted in Fig.6 for different water depths and different locations. Fig.6 (a, c, e) display the analysis results at x=12 m, and Fig.6 (b, d, f) display the analysis results at x=20 m. For the cases of non-breaking waves, the experimental data agrees well with the Rayleigh distribution in the range of H/η_{rms}<3; the Rayleigh distribution overpredicts the wave height distribution in the range of 3<H/η_{rms}<5; however, the Rayleigh distribution underpredicts the wave height distribution in the range of H/η_{rms}>5. For the case of breaking waves, with the water depth h=0.3 m, there is a good agreement between the experimental data and the Rayleigh distribution in the range of H/η_{rms}<4, while the Rayleigh distribution overpredicts the wave height distribution with the increased H/η_{rms} because of the occurrence of wave breaking. For deeper water depth (h=0.4m, 0.5 m), the difference between the experimental data and the Rayleigh distribution is not significant except at the tail of the distribution, and there is no significant difference, however, in the shape of the exceedance probability of wave height between breaking and non-breaking waves. Therefore, wave breaking only affects the wave height distribution of shallower water (h=0.3 m), not the deeper water (h=0.4 m, 0.5 m). For all cases, the tail of wave height distribution for the case of breaking waves is lower than the case of non-breaking waves. Wave breaking prevents the extreme waves from occurring, which will be discussed later.

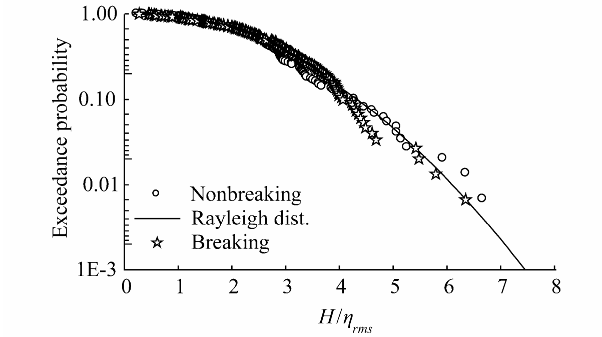
The parameter, abnormal index H_{max}/H_s, is currently used to distinguish the extreme event from the largest wave in a random wave field (Petrova *et al*., 2006; Dean, 1990). This ratio becomes known as the amplification or abnormality index, H_{max}/H_s. The significant wave height H_s is given as four times the standard deviation of the elevation process, $H_s=4m_0^{1/2}$, where m_0 is equal to the variance of the process. Here, the effect of wave breaking on the value of H_{max}/H_s is investigated. Fig.7 displays the spatial variations of H_{max}/H_s in different water depth: (a) h=0.3 m; (b) h=0.4 m; (c) h=0.5m. As shown in Fig.7, one can notice that the values of H_{max}/H_s for breaking waves are smaller than that of non-breaking waves and the values of H_{max}/H_s increase with increased water depth. The reason is the fact that the highest waves are influenced by the wave breaking directly, and the influence of wave breaking on the significant wave height is found negligible, as shown in Fig.3. The dependence of H_{max}/H_s on wave breaking shows a clear trend, which is the same as the kurtosis.



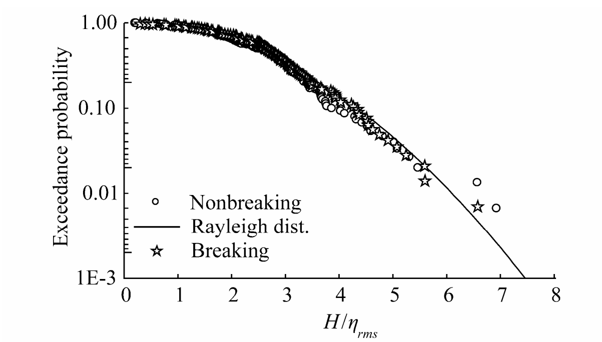
(a) h=0.3 m, x=12 m



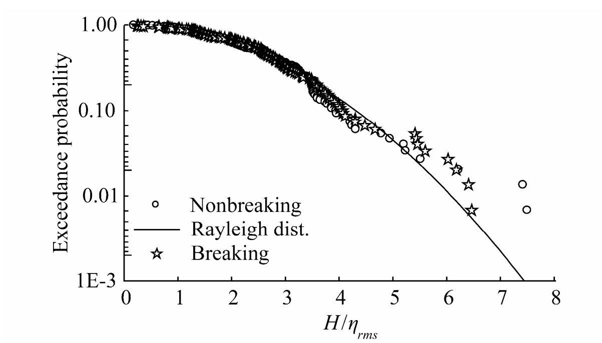
(b) h=0.3 m, x=20 m



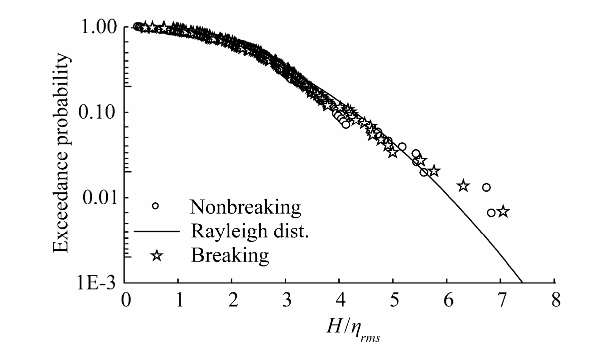
(c) h=0.4 m, x=12 m



(d) h=0.4 m, x=20 m

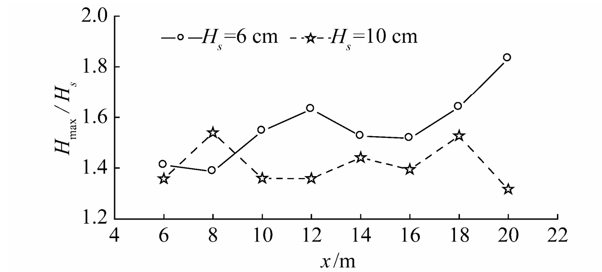


(e) h=0.4 m, x=12 m

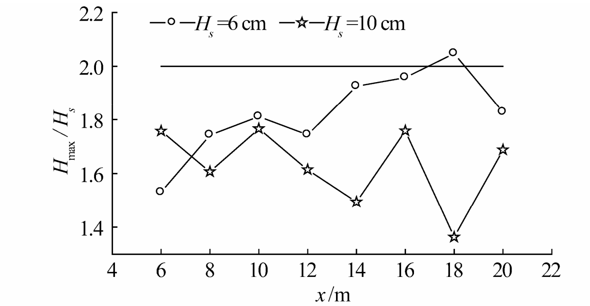


(f) h=0.4 m, x=20 m

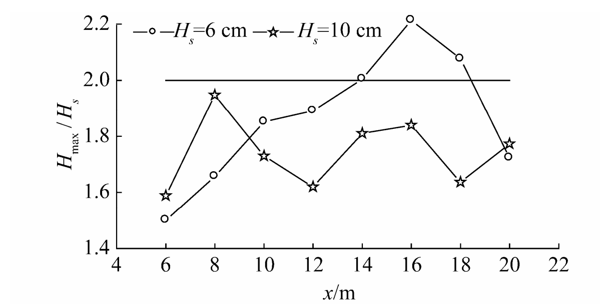
Fig.6 The exceedance probability of wave height in different water depth and locations



(a) h=0.3 m



(b) h=0.4 m



(c) h=0.5 m

Fig.7 The value of the parameter H_{max}/H_s along the flume in different water depth

4 Conclusions

The short-term wave statistics of random wave trains in an experimental wave flume with breaking waves are discussed. They are compared with the statistics observed in non-breaking wave cases. It is found that the value of the skewness μ_3 increases with an increasing input H_s and is independent of wave breaking. For the case of h=0.3 m, the exceedance probabilities of wave height shows large discrepancy in the tail of the distribution between breaking case and non-breaking case. For the case of h=0.4 m, 0.5 m, there is no significant difference, however, in the shape of the wave height distribution between breaking and non-breaking waves. Kurtosis and AI are strongly related to wave breaking and their values are suppressed by wave breaking. The effects of wave breaking on the generation of extreme waves in random wave trains should be further investigated.

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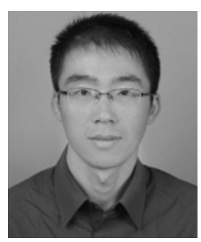

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