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Maximizing the probability an aerial anti-submarine torpedo detects its target

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Abstract: As a result of the high speed of anti-submarine patrol aircraft as well as their wide range, high efficiency and other characteristics, aerial torpedoes released by anti-submarine patrol aircraft have become the key anti submarine tool. In order to improve operational efficiency, a deep study was made of the target detection probabilities for aerial torpedoes released by anti-submarine patrol aircraft. The operational modes of aerial torpedoes were analyzed and mathematical-simulation models were then established. The detection probabilities of three attacking modes were then calculated. Measures were developed for improving low probabilities of detection when attacking a probable target position. This study provides an important frame of reference for the operation of aerial torpedo released by anti-submarine patrol aircraft.
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1 Introduction

With increasing threats from submarines, the importance of anti-submarine measures has increased. A submarine makes little noise, is well concealed, and has high maneuverability, making it difficult to find, while in contrast, it can easily identify attacking surface vehicles and torpedoes. Therefore, in order to attack submarines, the torpedo must suddenly appear in the vicinity of the submarine, so that the submarine has hardly any opportunity to take countermeasures. This is the reason why aerial torpedoes are the most effective defense against submarines.

At present, anti-submarine aircrafts are classified into fixed-wing aircraft (anti-submarine patrol aircraft) and helicopters. With their characteristics of high speed, long range, high efficiency, anti-submarine patrol aircrafts are operationally superior to helicopters. Therefore torpedoes air-dropped from an anti-submarine patrol aircraft are the most important component of a modern anti-submarine force.

The probability of a torpedo air dropped from an anti-submarine patrol aircraft detecting its target depends on the operational modes, air drop precision, attack strategies, and position. So it was necessary to make a deep study of the detection probabilities for torpedoes air dropped from anti-submarine patrol aircraft. This article starts with an analysis of operational modes of employment for anti-submarine patrol aircraft. Research on the probability a torpedo will detect its target was made with two cases; either a anti-submarine patrol aircraft indicated a target's probable location, or it determined the target's definite location. Attacking a definite target location was analyzed according to two circumstances: considering target's motion parameters; not considering target's motion parameters.

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2 Operation modes of anti-submarine patrol aircraft

2.1 Typical operation mode of anti-submarine patrol aircraft

The anti-submarine warfare unit of anti-submarine patrol aircraft is mainly used to carry out the medium and short range detection and attack missions. Anti-submarine patrol aircrafts are generally equipped with radar, infrared detectors, magnetic detection instrument, as well as sonobuoy and other submarine searching equipments. The operation modes are usually as follows.

1) Carry on the escort with anti-submarine patrol aircraft to surface fleet. In this mode, the searching submarine equipments equipped on the local aircraft are used to search in the designated anti-submarine area. After detecting the target, the aircraft will release torpedo or other antisubmarine weapons to carry out attack.

2) Anti-submarine on calling. In this mode, once

receiving the target parameters, the anti-submarine patrol aircraft will fly over the target to air drop torpedoes and other anti-submarine weapons to attack the target.

During the anti-submarine patrols, the main anti-submarine detection equipment is sonobuoy. Sonobuoy equipped on anti-submarine patrol aircraft is classified into two major categories of active and passive. Active sonobuoy remotely controlled by anti-submarine patrol aircraft can transmit sound wave to water by the acoustic transmitter, then receive the echo from submarine target by hydrophone, and then send the information to the anti-submarine patrol aircraft by radio transmitter. Passive sonobuoy released from anti-submarine patrol aircraft has two kinds of directional and non-directional.

2.2 Attacking mode of aerial torpedo

According to the obtained target information and operational situation, the aircraft's attack to submarine is classified into two attacking modes: one is to attack a target's probable location according to indication and another is to attack a definite target location. The attack to a definite target location is also classified into two modes: not considering target motion parameters and considering target motion parameters[1].

2.2.1 The attacking to a probable location of submarine

When the equipment for searching a submarine could not be acquired or the conditions at the time do not allow the submarine to determine precise course and speed, and it needs to use aerial torpedo in the shortest possible time, we usually adopt the method of attacking to a probable location of submarines.

The attacking method for a probable location point of submarine is used when the searching equipments need to determine the existence of submarine in a certain region, but do not know the exact location of it, so the center of submarine's probable location is regarded as the target point for attack.

When attacking with the aid of target indication by sonobuoy, the aircraft directly drops aerial torpedoes to passive non-directional buoys which are sending signals. This kind of buoy could not determine the distance of itself from aircraft. Usually at the minimum allowable height, the anti-submarine patrol aircraft would fly toward the buoy which is sending signals and drop the aerial torpedoes by manual when flying above the buoy.

2.2.2 The attacking method without considering the target's motion parameters

This method is usually used in the following circumstances: maybe there's no time to determine the target's motion parameters, or the anti-submarine patrol aircraft is threatened in the air by the enemy; in order to reduce the disadvantageous acoustic effects, the enemy submarine needs to be damaged in the shortest possible time; when receiving the command of damaging enemy submarine in the possible shortest time; to obstruct the attacking actions by enemy submarine; when the enemy submarine has sharply changed its course and speed, there's no time to continuously track it. No matter which circumstance above occurred, the current point should be attacked. Attacking without considering the target's motion parameters is to implement attack by regarding the current point of the discovered target as the target point. The current target position could be given before releasing the air-dropped torpedo. And then attack the final given submarine's location without considering the motion parameters of enemy submarine.

2.2.3 The attacking method with consideration of the target's motion parameters

The attack with consideration of the target's motion parameters is to regard the calculated target advance point as the target point where to carry on the aircraft attack and release the aerial torpedo, on the premise of assuming the enemy submarine move at a uniform speed and in a straight line during attacking.

When using passive non-directional buoys to detect submarine and indicate the target to be attacked, the anti-submarine patrol aircraft should release two buoy arrays in succession. The essential motion factors of enemy submarine are determined by buoy arrays. Input the essential motion factors to the search–scan system. The anti-submarine patrol aircraft flying to the releasing aerial torpedo point is controlled by search–scan system. When passive directional buoy attacks submarine, if previously laid non-directional passive sonobuoy did not detect the course of submarine, lay the passive directional buoy nearby the area of non-directional passive sonobuoy which sent out the signals. If passive non-directional buoy has detected the course of submarine, the passive sonobuoy will be laid on the head of submarine course. Finally, passive directional buoys array is used to detect the submarine and give the target parameters.

1) The process of flying attack by releasing aerial torpedo

The flying attack by releasing aerial torpedo is a method adopted on the basis of aircraft's characteristic of fast speed. The aircraft approaches the target rapidly, and then releases the torpedo.

The main course of the initial aircraft flight may be not consistent with the aircraft's optimum main course calculated by advance angle. At this time the aircraft begins turning from point b of Fig.1 until to the optimum main course of aircraft. The turning angle is solved by iterative method. The ideas are as follows: since the precise location of point O is not known, first of all assuming that point b and point O are coincided, based on above principle, solve the turning angle and obtain the initial value of point O, and then starting from point O, solve the second turning angle and value of point O until the turning angle of two times solved is less than a certain small constant.

When the aircraft flies to drop torpedoes, the running direction of torpedo in the water is just the course of flight. The torpedo does not need to turn after it has entered water. In this airdropping mode, after torpedo has experienced the delaying periods of gravitational falling in the air and water without driving power, the homing system initiates and enters the searching state directly. Geometry relational graphics for flight attack and releasing aerial torpedo is shown in Fig.1.

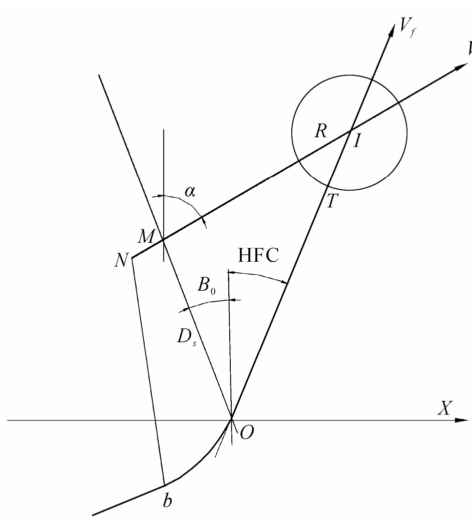

Fig.1 Geometric relations for flight releasing aerial torpedo

In Fig.1, O is the initial point of aircraft releasing torpedo; M is target's initial position; D is target's initial distance/m; B_0 is target's initial azimuth/(°); α is target's course/(°); V_m is target speed/(m·s^{-1}); I is pre-meeting point; R is radius of torpedo releasing; T is point of torpedo releasing; HFC is the planned course of flight; V_f is flight speed of aircraft /(m·s^{-1}).

The parameters demanded for aircraft's pre-flying torpedo releasing are calculated according to relative position between the starting point and target, and then calculate the pre-meeting point-I, the radius of torpedo releasing-R, the planned distance of flight-HFR(OT) and the planned course of flight-HFC. According to HFC, the torpedo will be dropped when arriving at T after flying for a distance of HFR.

2) Parameters calculation of aircraft attack by dropping aerial torpedo

The circle drawn taking the pre-meeting point I as the center and the advance R as the radius is called torpedo releasing circle.

After torpedo goes into the water, the course will be dispersed, so the calculation of radius R of torpedo releasing only takes account of the gravitational falling period in the air, not including the underwater period. By calculating the procedures of aerial torpedo in the air period, we can get the falling time t_{02} in the air, flight distance S_1 in the air, diving speed to the water, incident angle and other parameters. Torpedo releasing radius R=S_1. The corresponding time t_R=t_{02}.

Establish the coordinate system, taking O as the origin, i.e. the point of the best main course at which the anti-submarine patrol aircraft arrives, and due east as X-axis.

Given that the target's initial point is M at which detection stops, I is the point of pre-meeting. After the aircraft has flied for t_f in the OI direction, it arrives at the target point and releases torpedo, which will meet the target at point I after t_R. In the triangle ΔOMI, the following equations can be obtained according to the sine law:

$$\begin{cases} \dfrac{D_s}{\sin(\pi-\alpha+\text{HFC})} = \dfrac{V_f t_f + R}{\sin(\pi-\alpha-B_0)}; \\ \dfrac{D_s}{\sin(\pi-\alpha+\text{HFC})} = \dfrac{(t_f+t_R)V_m}{\sin(B_0+\text{HFC})}. \end{cases}$$

t_f and HFC can be derived easily from the above equations, and also we can get HFR=$V_f t_f$.

3 The trajectory mathematic model of torpedo parachute system in the air period

To deduce the trajectory equations of torpedo parachute system in the air period: (a) Suppose the torpedo parachute system has a rigid body with constant mass, and has the characteristic of revolving symmetry; (b) Suppose the ground coordinate system is an inertial reference system; (c) Because the running time of torpedo parachute system is very short in the air, neglecting the earth curvature and regarding the ground as a plane; (d) Suppose the aerodynamic coefficient of the parachute increases with time linearly during the process of gas-filling.

After releasing the aerial torpedo, the parachute is stuffed with gas, the motion of aerial torpedo and parachute can be considered as a complicated system which consists of two rigid bodies, this is the so-called torpedo parachute system. The torpedo parachute system motion is a non-stationary motion with a constant mass. The motion is influenced not only by the geometrical size, the mass and the aerodynamic characteristic of torpedo and parachute, but also by the coupling aerodynamic performance between torpedo and parachute. So the forces imposing on the torpedo parachute system include gravity, aerodynamic force on torpedo, and aerodynamic force on parachute. The moments imposed on the system include gravity moment, aerodynamic force moment of parachute and aerodynamic force moment of torpedo. Fig.2 shows the stresses of torpedo parachute system. The external forces and external force moments imposed on torpedo parachute system are respectively:

$$\begin{aligned}
&F_x(t)=G_x(t)+Q(t)+Q_{sx}(t),\\
&F_y(t)=G_y(t)+Y(t)+Q_{sy}(t)+R_y(t),\\
&F_z(t)=G_z(t)+Z(t)+Q_{sz}(t);\\
&M_x(t)=M_{tx}(t),\\
&M_y(t)=M_{ty}(t)+M_{sy}(t),\\
&M_z(t)=M_{tz}(t)+M_{sz}(t).
\end{aligned}$$

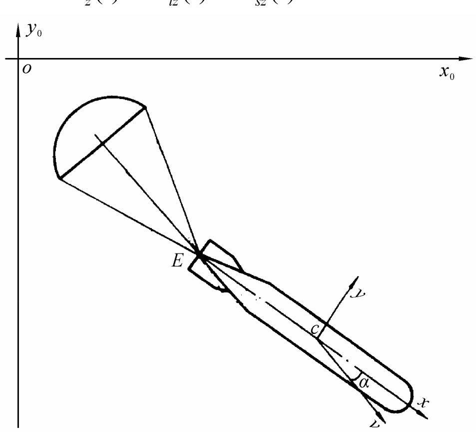

Fig.2 The stresses of torpedo parachute system

Among the formulas, F_x is a component of external force acting on torpedo parachute system in the x-axis/N; F_y is a component of external force acting on torpedo parachute system in the y-axis/N; F_z is a component of external force acting on torpedo parachute system in the z-axis/N; Q is a component of aerodynamic force acting on torpedo in the x-axis/N; Y is a component of aerodynamic force acting on torpedo in the y-axis/N; Z is a component of aerodynamic force acting on torpedo in the z-axis/N; M_x is a component of external force moment acting on torpedo parachute system in the x-axis/N·m; M_y is a component of external force moment acting on torpedo parachute system in the y-axis/N·m; M_z is a component of external force moment acting on torpedo parachute system in the z-axis/N·m; M_{sy} is a component of aerodynamic force moment acting on parachute in the y-axis/N·m; Q_{sx} is a component of aerodynamic force acting on parachute in the x-axis/N; Q_{sy} is a component of aerodynamic force acting on parachute in the y-axis/N; Q_{sz} is a component of aerodynamic force acting on parachute in the z-axis/N; M_{tx} is a component of aerodynamic force moment acting on torpedo in the x-axis/N·m; M_{ty} is a component of aerodynamic force moment acting on torpedo in the y-axis/N·m; M_{tz} is a component of aerodynamic force moment acting on torpedo in the z-axis/N·m; M_{sy} is a component of aerodynamic force moment acting on parachute in the y-axis/N·m; M_{sz} is a component of aerodynamic force moment acting on parachute in the z-axis/N·m.

According to the theorem of momentum and momentum moment, establish the dynamics equations and kinematics equations, taking the centroid of torpedo parachute system as the origin of the coordinate.

$$\begin{bmatrix}\dot V_{x1}\\ \dot V_{y1}\\ \dot V_{z1}\\ \dot\omega_{x1}\\ \dot\omega_{y1}\\ \dot\omega_{z1}\end{bmatrix}=\begin{bmatrix}F_x/m\\ F_y/m\\ F_z/m\\ M_x/J_x\\ M_y/J_y\\ M_z/J_z\end{bmatrix}+\begin{bmatrix}V_{y1}\omega_{z1}-V_{z1}\omega_{y1}\\ V_{z1}\omega_{x1}-V_{x1}\omega_{z1}\\ V_{x1}\omega_{y1}-V_{y1}\omega_{x1}\\ \omega_{y1}\omega_{z1}(J_y-J_z)/J_x\\ \omega_{x1}\omega_{z1}(J_z-J_x)/J_y\\ \omega_{y1}\omega_{x1}(J_x-J_y)/J_z\end{bmatrix}.$$

$$\begin{bmatrix}\dot\gamma\\ \dot\psi\\ \dot\theta\end{bmatrix}=\begin{bmatrix}1 & -\cos\gamma\tan\theta & \sin\gamma\tan\theta\\ 0 & \cos\gamma/\cos\theta & -\sin\gamma/\cos\theta\\ 0 & \sin\gamma & \cos\gamma\end{bmatrix}\begin{bmatrix}\omega_{x1}\\ \omega_{y1}\\ \omega_{z1}\end{bmatrix}.$$

$$V=\sqrt{V_{x1}^2+V_{y1}^2+V_{z1}^2}\ .$$

$$\alpha=\arctan(-V_{y1}/V_{x1})\ ,$$

$$\beta=\arcsin(V_{z1}/V)\ .$$

$$\dot x_g=V\cos\Theta\cos\Psi,$$

$$\dot y_g=V\sin\Theta,$$

$$\dot z_g=-V\cos\Theta\sin\Psi.$$

$$\begin{aligned}
&\sin\Theta=\cos\alpha\cos\beta\sin\theta-(\sin\alpha\cos\beta\cos\gamma+\sin\beta\sin\gamma)\cos\theta,\\
&\sin\Psi\cos\Theta=\cos\alpha\cos\beta\sin\psi\cos\theta+\sin\alpha\cos\beta(\cos\psi\sin\gamma+\\
&\quad\sin\psi\sin\theta\cos\gamma)-\sin\beta(\cos\psi\cos\gamma-\sin\psi\sin\theta\sin\gamma).
\end{aligned}$$

Among the formulas, θ is a pitching angle; ψ is a yawing angle; ϕ is a rolling angle; Θ is a trajectory inclination angle; J_x, J_y, J_z are moment of inertia relative to X、Y、Z

axis of torpedo/ $Kg \cdot m^2$; α is a attack angle; β is a sideslip angle; V is a motion velocity of torpedo—parachute system; V_x, V_y, V_z are the components on motion speed of torpedo—parachute system in the direction of X, Y, Z ; X_g , Y_g , Z_g are gravity center coordinate position of torpedo in ground-based/ m.

4 Simulation conditions and results

4.1 Simulation conditions

The detecting probability of aerial torpedo for anti-submarine patrol aircrafts can usually be calculated by two methods of theoretical analysis (analytical method) [2] and Monte Carlo (statistical simulation). The calculation is always difficult because the trajectory involves many factors and two periods in the air and water. As to such a complicated system, using analytical method to calculate the shooting efficiency will be a heavy task, and sometimes it cannot be solved. Therefore, the Monte Carlo method is used here to analyze the detecting probability. The simulation conditions are as follows:

1) Aerial torpedo releasing conditions.

Aerial torpedo releasing conditions are shown in Table 1.

Table 1 Torpedo releasing conditions

Items	Pre-flying torpedo releasing
Releasing speed/$km \cdot h^{-1}$	300～500
Releasing height/m	100～200
Parallel flight time/s	No less than 5

Wind velocity can be measured because the anti-submarine patrol aircraft is equipped with meteorological data system, and the dispersion of parachute trajectory on torpedo can be compensated.

2) The tactical situation.

(a) The initial distance D_S between the anti-submarine patrol aircraft (torpedo) and the target: 2 500~5 500 m, distributed evenly; (b) The initial target's bow bearing q_{m0}: 20°~160°, distributed evenly; (c) Torpedo's initial search depth y_{t0}: 80 m; (d) Target's running depth y_m: 50~200 m.

3) The torpedo's characteristics

(a) Speed: 42 kn; (b) Range: 9 700 m; (c) Homing range: N(1 100, 200) /m.

Suppose that the target starts to evade from the beginning of simulation to a certain time (distributed evenly at 0~40s). The turning angular velocity of avoidance during the evasion is 1.4 (˚)/s, and linear acceleration of maneuver is 0.043 m/s^2, from 10 kn up to 32 kn.

4.2 Simulation results

4.2.1 Attacking method to a definite target position

Suppose that the sonobuoy measured movement direction error of the target submarine is 5° and the target's movement velocity error is 4 m/s. The two attacking methods of considering target's motion parameters and not considering target's motion parameters (i.e. attacking the submarine by aiming at the currently discovered point of the target) were calculated for 100 times, using the Monte Carlo method. The simulation results are shown in Table 2.

It can be seen from Table 2 that because the sonobuoy can measure the target position, then the submarine position's dispersion error is far less than the torpedo's homing operation distance. Therefore, the probability of target submarine's being detected is higher after the aerial torpedo enters the water.

Table 2 Simulation results of the attacking method to a definite target position

Attacking method	Attacking method of considering the target's motion parameters	Attacking method of not considering the target's motion parameters
Detecting probability	1.00	0.99

4.2.2 The attacking method to a probable position of submarine

The submarine should be evenly distributed within the tactics operating distance of buoy, and the initial course of the submarine should also be evenly distributed within 0°~360°, then the results of simulation are shown in Table 3.

Table 3 Simulation results of the attacking method to a probable position of submarine

Tactics operating distance of buoy /km	1	1.5	2	2.5	3	3.6
Detecting probability	0.86	0.75	0.54	0.48	0.42	0.40

4.3 Analysis of the simulation results

It can be seen from the results of simulation that among the three attacking methods of anti-submarine patrol aircraft, the detecting probabilities of above two methods of considering and not considering the parameters of target motion are higher. Moreover, as to the attacking method to a probable position of submarine, if the tactics operating distance of buoy is longer, then the detection probability will be lower. We can make a brief analysis of Fig3.

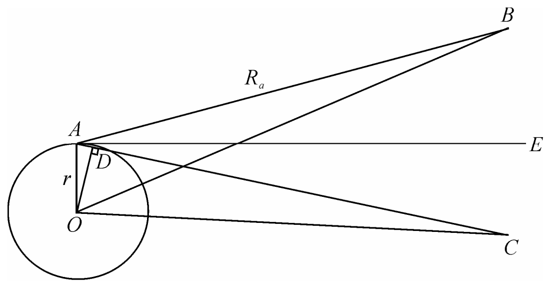

Fig.3 Sketch map of torpedo's circle searching

In Fig.3, r is the turning radius of torpedo's circle searching, R_a is torpedo's homing operation distance, $\angle BAE$ is torpedo's homing sector half-angle θ, OD is the line perpendicular to AC, then it can be proven that the torpedo searching area is an annular area encircled by the circle drawn by taking OD as the radius and the circle by OB as the radius. Then it can be derived as follows:

$$OD = r\cos\theta ,$$

$$OB = \sqrt{r^2 + R_a^2 - 2rR_a\cos(\pi/2+\theta)} ,$$

$$OB - OD = \sqrt{r^2 + R_a^2 + 2rR_a\sin\theta} - r\cos\theta ,$$

$$\pi(OB^2 - OD^2) = \pi(r\sin\theta + R_a)^2 .$$

The annular area $\pi(OB^2 - OD^2)$ is an increasing function, either for R_a, or for θ at $[0, \pi/2]$, or for r.

Therefore, the torpedo detecting probability can be improved by the following methods: (a) To increase the homing operation distance R_a; (b) To widen the homing sector width; (c) To increase the turning radius r of circle searching (by the methods of spiral trajectory, etc.).

Table 4 The relation between detecting probability and tactics operating distance of buoy at different homing operating distances

Operating distance of buoy/km	Homing operating distance of torpedo /km	Detecting probability
1	1.1	0.86
	2	1.0
	3	1.0
2	1.1	0.54
	2	0.85
	3	1.0
3.6	1.1	0.40
	2	0.61
	3	0.83

By the simulation calculation, when the anti-submarine patrol aircraft adopts the attacking method toward a probable position of submarine, if the tactics operation distance of buoy is longer, the detecting probability can be improved obviously by only increasing the homing operation distance R_a. See Table 4 for more details. Moreover, the influence on detecting probability is little by only widening the homing sector width and increasing the turning radius r of circle search. See Table 5 and Table 6 for more details. In fact, due to that r and θ are submerged by R_a when $R_a >> r\sin\theta$, so improving r and θ has less influence on the detecting probability.

Table 5 The relation between detecting probability and tactics operating distance of buoy at different homing sector half-angles

Operating distance of buoy/km	Torpedo homing sector half-angle/(°)	Detecting probability
1	15	0.86
	20	0.86
	30	0.88
2	15	0.54
	20	0.55
	30	0.56
3.6	15	0.40
	20	0.41
	30	0.43

Table 6 The relation between detecting probability and tactics operating distance of buoy at different search turning angle speeds

Operating distance of buoy/km	Torpedo search turning angle speed/(°)·s^{-1}	Detecting probability
1	10	0.86
	5	0.88
	2	0.89
2	10	0.54
	5	0.56
	2	0.58
3.6	10	0.40
	5	0.43
	2	0.47

5 Conclusions

According to the feature of anti-submarine patrol aircraft, three attacking modes of anti-submarine patrol aircraft by releasing the aerial torpedo are presented in this paper, and the detecting probabilities of these three attacking modes are analyzed and calculated. Combined with the simulation results, the improvement measures are proposed on the lower detecting probability when attacking a probable position of submarine.

Due to the complexity of anti-submarine operation, the simulation conditions and the adaptability of calculation models proposed in this article need to be further researched by combining the evasion mode of the target submarine.

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反潜巡逻机空投鱼雷发现概率仿真研究

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摘　要：由于反潜巡逻机具有速度快、范围大、效率高、能力强等特点，从而使得反潜巡逻机投放的空投鱼雷成为现代反潜力量的最重要的组成部分。为了提高作战效能，有必要对反潜巡逻机投放空投鱼雷发现概率进行深入的研究，该文分析研究了反潜巡逻机投放空投鱼雷的作战使用方式，并建立了仿真数学模型，对其三种攻击方式下的发现概率进行了计算分析，对向潜艇大概位置点攻击方法造成的发现概率偏低的问题提出了改进措施，可为反潜巡逻机空投鱼雷作战使用提供重要的参考依据。

关键词：空投鱼雷仿真；发现概率；反潜鱼雷