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A simplified model for extreme-wave kinematics in deep sea

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Abstract: Based on the fifth-order Stokes regular wave theory, a simplified model for extreme-wave kinematics in deep sea was developed. In this model, from the wave records the average of two neighboring wave periods for the extreme crest or trough was defined as the period of the Stokes wave by the up and down zero-crossing methods. Then the input wave amplitude was deduced by substituting the wave period and extreme crest or trough into the expression for the fifth-order Stokes wave elevation. Thus the corresponding formula for the wave velocity can be used to describe kinematics beneath the extreme wave. By comparison with the published numerical models and experimental data, the proposed model is validated to be able to calculate the extreme wave velocity rather easily and accurately.
Keywords: extreme wave; deep sea; fifth-order Stokes regular wave; kinematics; velocity field
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1 Introduction

Because of the effects of hurricanes and earthquakes, there often occur some extreme waves to devastate ocean structures, such as offshore platforms, risers and ships etc., in the complicated and severe deep sea. The velocities of extreme waves are required for related calculation and analysis of loads on structures. It will be directly related to the economics and safety of ocean structure designing. Because of the complexity of deep sea environments, fast and accurate description of the distribution of extreme-wave velocity is still a challenge at present. Many researches on extreme wave kinematics have been carried out numerically and experimentally in the past two decades. For example, Baldock et al[1], Johannessen et al[2], She et al[3] and Grue et al[4] physically studied the kinematics of extreme waves in deep water, and Fenton et al[5], Johannessen et al[6] and Bateman et al[7] researched the extreme wave kinematics using various numerical models. Although all above models can describe the wave kinematics well, it is difficult to apply them directly to engineering practice because many given conditions are needed before physical or numerical simulation, such as frequency range and amplitude distribution of wave components, etc.

A simple and accurate model was developed based on the fifth-order Stokes wave theory for fast calculating the extreme-wave velocity field in deep sea. The only known condition was the time series of extreme wave surface, which could be easily recorded in practice. The key problems are to define proper wave period and appropriate input wave amplitude for the Stokes wave which can represent extreme waves above or below the mean water surface. In the proposed model, the up and down zero-crossing method is adopted to define the average of two neighboring wave periods containing the extreme crest or trough as the period of the Stokes wave. Thus the input wave amplitude can be deduced by substituting the value of extreme wave crest or trough into the formula for the fifth-order Stokes wave elevation. The proposed model is validated by comparison with the published numerical results and experimental data. It is also compared with the linear and second-order irregular analytical solutions.

2 Mathematical model

Fig.1 gives the sketches of extreme wave crest and trough. They are generally generated by the method of wave focusing, in which all wave components are considered. Some researchers attempted to use the Stokes wave to describe extreme waves for convenience. The fifth-order Stokes wave is a much higher nonlinear regular wave solution at present. Although it just describes the periodic wave, it can accurately predict extreme wave crest or trough value and the corresponding velocity field if proper wave period and input amplitude are chosen. The fifth-order Stokes wave velocity for deep water can be expressed as follows[8]:

$$v(x,y,z,t)=\frac{1}{k}[(k\varepsilon-\frac{1}{2}k\varepsilon^3-\frac{37}{24}k\varepsilon^5)\mathrm{e}^{kz}\cos(kx-\omega t)+ \\ k\varepsilon^4\mathrm{e}^{2kz}\cos2(kx-\omega t)+\frac{1}{4}k\varepsilon^5\mathrm{e}^{3kz}\cos3(kx-\omega t)]\cdot\sqrt{g/k}. \tag{1}$$

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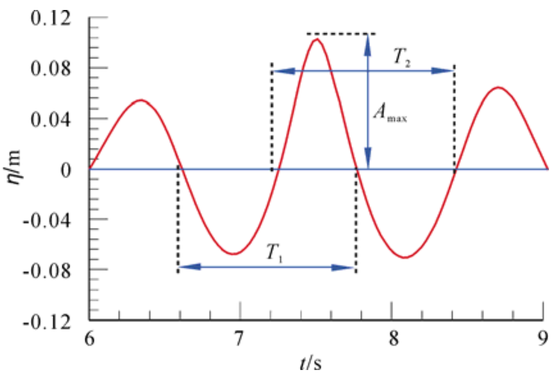
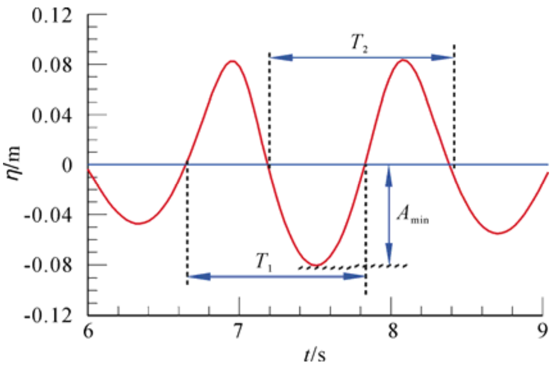
The corresponding wave elevation expression is



(a) Crest



(b) Trough

Fig.1 Sketch of extreme wave elevation histories

$$\eta=\frac{1}{k}[(\varepsilon-\frac{3}{8}\varepsilon^3-\frac{422}{384}\varepsilon^5)\cos(kx-\omega t)+$$
$$(\frac{1}{2}\varepsilon^2+\frac{1}{3}\varepsilon^4)\cos 2(kx-\omega t)+(\frac{3}{8}\varepsilon^3+\frac{297}{384}\varepsilon^5)\cos 3(kx-\omega t)+$$
$$\frac{1}{3}\varepsilon^4\cos 4(kx-\omega t)+\frac{125}{384}\varepsilon^5\cos 5(kx-\omega t)] \tag{2}$$

where wave slope $\varepsilon=kA$, k is wave number, A input wave amplitude and ω angular frequency. The following dispersion relation is satisfied:

$$1+\frac{\varepsilon^2}{2}+\frac{\varepsilon^4}{8}=\frac{\omega}{\sqrt{gk}}$$

As a comparison, the linear theoretical wave velocity and elevation in deep sea are also given as follows:

$$v(x,y,z,t)=A\omega e^{kz}\cos(kx-\omega t) \tag{3}$$

and

$$\eta(x,y,z,t)=A\cos(kx-\omega t) \tag{4}$$

where k and ω satisfy the linear dispersion relation ω 2=gk.

From Eqs.(1)~(4), it can be seen that the time series of wave elevation and velocity and their extreme values can be obtained once the wave period T and input wave amplitude A are given. Similarly, the linear fifth-order input wave amplitudes can also be obtained from Eqs.(2) and (4) if the wave period T and extreme surface values A_{max} or A_{min} are known.

To describe the extreme waves as shown in Fig.1 using the Stokes regular wave theory, two wave periods T_1 and T_2, both containing the period of extreme wave crest/trough, are defined by the up and down zero-crossing methods respectively and their average is taken as the Stokes wave period. From the studies of Ning et al[9]and Bateman et al[10], the shape of extreme wave is symmetrical about the extreme crest or trough if the wave nonlinearity is rather weaker and the case is close to the linear one, i.e. T_1=T_2. However, such symmetry will be deteriorated and T_1 will not equal to T_2 again with the increase of nonlinearity. Therefore, the average of T_1 and T_2 is taken as the Stokes wave period in this paper to consider the nonlinear effects. The proposed method is similar to that mentioned by Grue et al[4], using the time interval of two neighboring troughs of extreme crest as the wave period. But the latter's disadvantage is that the minimal location can not be found easily because the trough will be much flatter owing to stronger wave nonlinearity. Therefore, it is not suggested here. Thus the input wave amplitude can be calculated by substituting the wave period T and the extreme wave amplitude A_{max} or A_{min} into Eqs.(2) and (4) respectively.

3 Numerical results and discussions

To validate the proposed model, the input parameters of extreme waves in an infinitely deep water used by Ning et al[10] are taken as an example and shown in Fig.2 and Table 1. The fully nonlinear numerical wave tank based on a time-domain higher-order boundary element method [9] is adopted to simulate the corresponding extreme wave time series and velocity field beneath the extreme crest or trough. By comparison with it, the effect of the present regular wave model for predicting extreme wave can be validated.

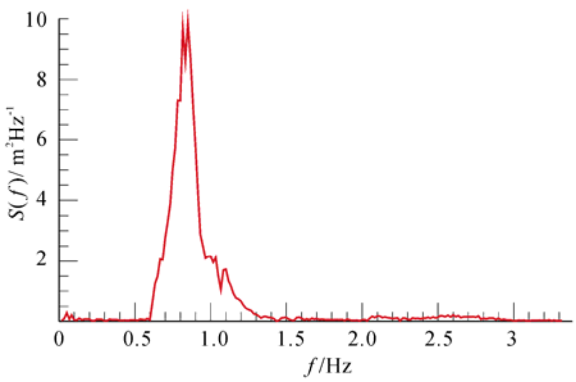


Fig.2 Input wave spectra & frequencies

Table 1 Parameters of input wave group

Frequency range f (Hz)	$0.6 \le f \le 1.5$
Peak frequency f_p	0.80
Sum of input amplitudes A_I (m)	0.043 75, 0.087 5, 0.120
Number of wave components	30
Wave slopes ε_i	0.130, 0.283, 0.356

Figs.3 and 4 give the time series of extreme wave crest and the corresponding velocity field beneath the highest crest with A_I= 0.043 75 m, respectively, in which the results obtained by the fully nonlinear numerical wave tank technique are marked as 'FN-NWT'. The comparisons with the fifth-order Stokes regular wave results resolved by the present model are also shown in the figures. The linear regular wave velocity field is also given in Fig.4 and marked as 'linear-RWT'. From the fully nonlinear crest wave elevation history in Fig.3, it can be deduced that the period and input amplitude corresponding to the fifth-order Stokes regular wave are T=1.186 s and A_5=0.043 15 m, respectively. And the linear input amplitude is A_1=A_{max}=0.045 87 m. From Fig.4, it can be seen that there are good agreements between the fully nonlinear results and the fifth-order solutions, and the linear solution is a little greater than the two formers.

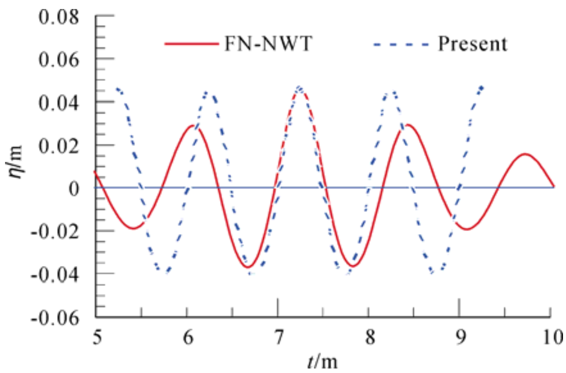


Fig.3 Time series of extreme wave crest with A_I =0.043 75 m

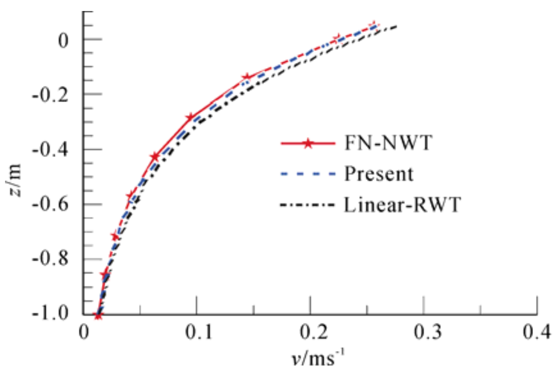


Fig.4 Distribution of the extreme velocity field beneath the highest crest with A_I =0.043 75 m

Figs.5 and 6 describe the extreme wave trough case with A_I=-0.043 75 m. In this case, the period and input amplitude of the fifth-order Stokes wave are T=1.197 s, A_5=-0.044 03 m respectively. The corresponding linear input amplitude is A_1=A_{min}=-0.041 23 m. Similar conclusions with those in Figs. 3 and 4 can be obtained except that the linear kinematic result is a little smaller than the others. The proposed model shows a good description for extreme trough below the mean water level and its velocity field.

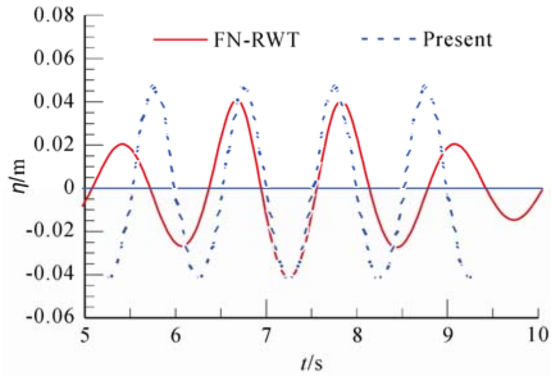


Fig.5 Time series of extreme wave trough with A_I= −0.043 75 m

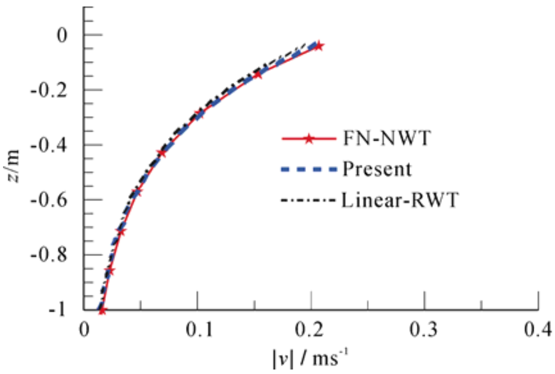


Fig.6 Distribution of the extreme velocity field beneath the lowest trough with A_I= −0.043 75 m

Figs.7 and 8 show the extreme wave crest case as the input amplitude sum A_I is increased to 0.087 5 m. From the fully nonlinear numerical results in Fig.7, the wave period and input amplitude corresponding to the fifth-order Stokes wave are T=1.135 s and A_5=0.089 11 m respectively, and the linear input wave amplitude is A_1=A_{max}=0.102 8 m. From Fig.8, it can be seen that the linear kinematics is much greater than the others with the increasing of wave nonlinearity, especially near the instantaneous free surface. However, the three results agree well with each another when the water depth exceeds about 0.6m, which indicates that the nonlinearity is decreased to the linear case below the depth -0.6 m.

Figs.9 and 10 are the extreme wave trough case as the input amplitude sum A_I is decreased to -0.087 5 m. From the fully nonlinear numerical results in Fig.9, the wave period and input amplitude corresponding to the fifth-order Stokes wave are T=1.195 s and A_5=-0.094 31 m respectively, and the linear input wave amplitude is A_1=A_{max}=0.102 8 m. From the figure, it can be seen that the difference between linear kinematics and the others is not as great as that in the extreme crest case. The reason is that the instantaneous free surface is lower than the static free surface and the nonlinearity is decreased by exponential index with decreasing of water level.

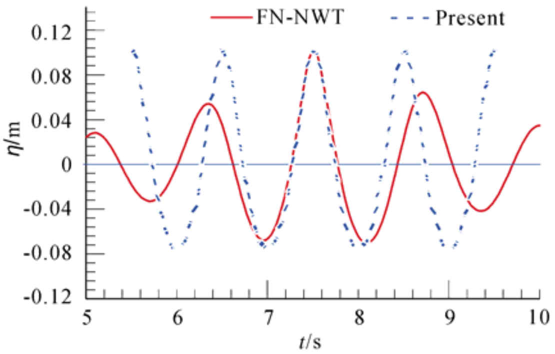


Fig.7 Time series of extreme wave crest with A_I =0.087 5 m

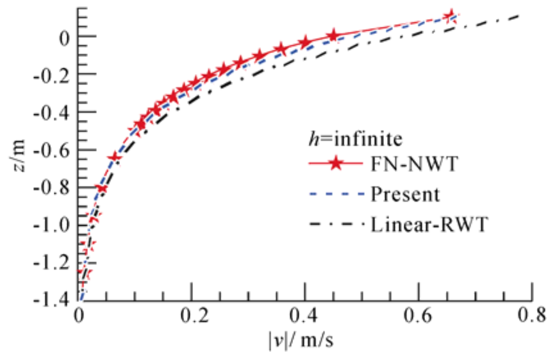


Fig.8 Distribution of the extreme velocity field beneath the highest crest with A_I =0.087 5 m

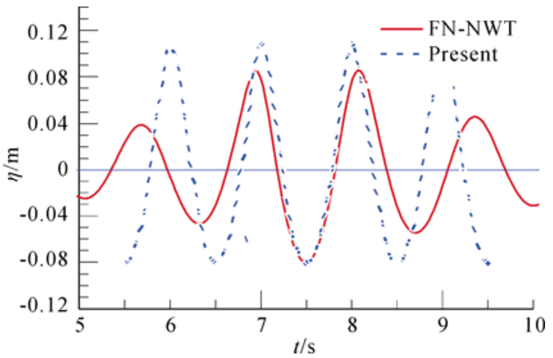


Fig.9 Time series of extreme wave trough with A_I = -0.087 5 m

Figs.11 and 12 are the extreme crest case as the input amplitude is further increased to A_I = 0.12 m. In this case,

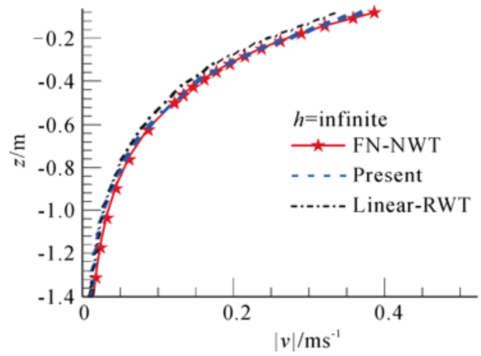


Fig.10 Distribution of the extreme velocity field beneath the lowest trough with A_I= −0.087 5 m

the period and input amplitude of the fifth-order Stokes regular wave are T = 1.165 5 s and A_5 = 0.122 1 m respectively, and the linear input amplitude is A_1= A_{max} = 0.148 m. From Fig.11, it can be seen that the present model describes the extreme crest very well. Fig.12 gives the comparisons of the velocity field beneath the greatest crest to the fully nonlinear numerical results, the present, linear and third-order solutions. The third-order Stokes solution is obtained from the method of Grue et al[4] and marked as '3rd-SRWT' in the figure, in which the period is corresponding to 1.14 s. Except the linear solutions, the others agree well with one another and only a little difference exists between the third-order one and the fully nonlinear results in the scope 0.2 ms^{-1}< v <0.5 ms^{-1}.

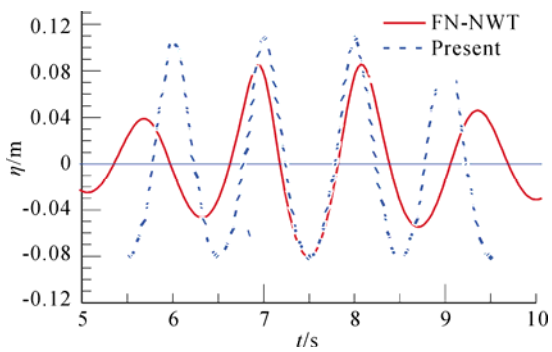


Fig.11 Time series of extreme wave crest with A_I = 0.12 m

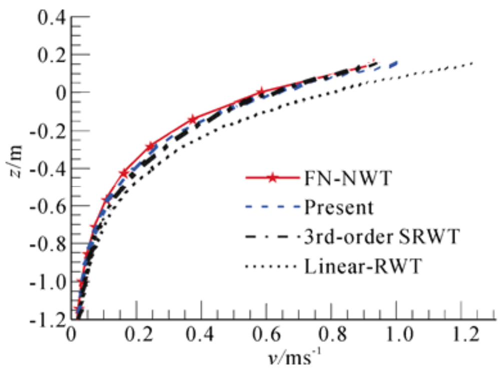


Fig.12 Distribution of the extreme velocity field beneath the highest crest with A_I = 0.12 m

Further work is carried out to validate the proposed model by comparison with experimental data [1]. The physical model was made in a flume with water depth 0.7 m, input wave periods varying from 0.8 s to 1.2 s; the wave group is composed of 29 wave components. Two cases (kh>3.0) with input amplitude A_1 = 0.038 m and 0.055 m respectively are considered. From the time series of experimental wave elevations (in Figs.6(b) and (c) from Ref.[1]), the corresponding periods and input amplitudes of the fifth-order Stokes wave are (T=0.952 1 s, A_5= 0.042 68 m) and (T=0.943 s, A_5=0.062 57 m) respectively. The experimental results were ever used to validate the linear and second-order irregular analytical solutions by Baldock et al[1]. The irregular solutions are also used to compare with the proposed model in the present study. Figs.13~16 give the comparisons of wave elevation history and velocity field beneath the extreme crest for these two cases. From Figs.13 and 15, it can be seen that the proposed model describes the extreme wave crest above mean water level very well. In Fig.14, the results on the proposed model and two irregular wave theories all agree well with experimental results because of smaller wave nonlinearity. In Fig.16, the irregular wave theories can not predict extreme velocity field well, especially near the free surface, with stronger nonlinearity. However, the proposed model still shows a good agreement with experiment.

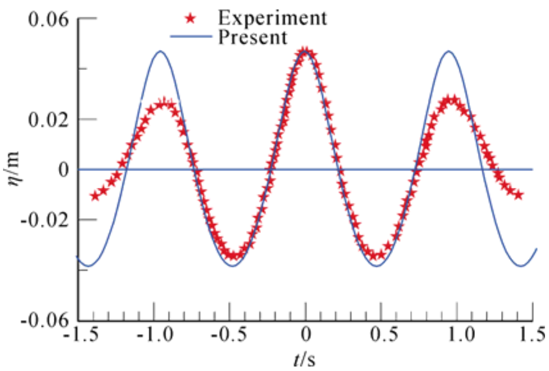


Fig.13 Time series of extreme wave crest with A_I =0.038 m

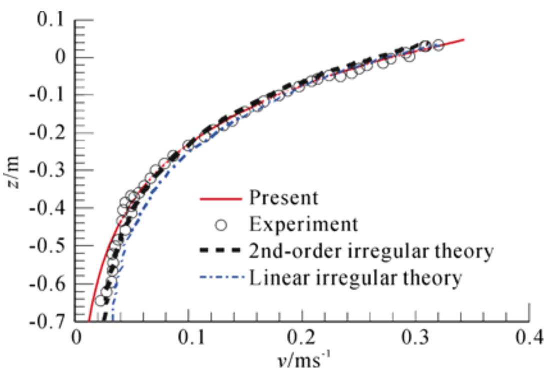


Fig.14 Distribution of the extreme velocity field beneath the highest crest with A_I =0.038 m

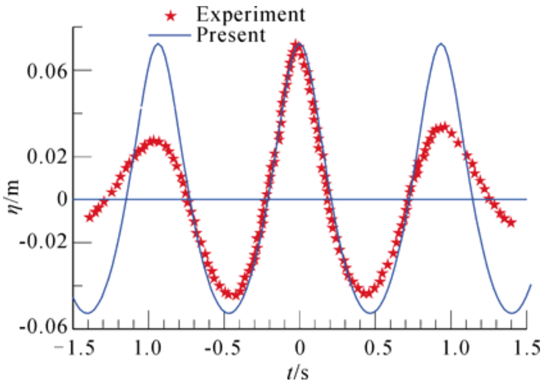


Fig.15 Time series of extreme wave crest with A_I=0.055 m

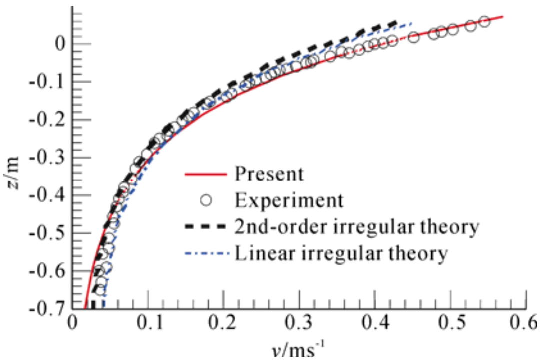


Fig.16 Distribution of the extreme velocity field beneath the highest crest with A_I =0.055 m

4 Conclusions

This paper proposes a simplified model which is developed based on the fifth-order Stokes wave theory, for predicting the extreme-wave velocity field in deep sea. In the model, up and down zero-crossing methods are used to obtain two neighboring wave periods containing extreme wave crest or trough and the average of them is taken as the regular wave period representing the extreme wave. Thus the corresponding input wave amplitude can be deduced by substituting the measured extreme-wave value into the fifth-order Stokes wave elevation formula so that the velocity field beneath the extreme wave can be directly calculated from the fifth-order Stokes wave velocity expression. The comparisons with the fully nonlinear numerical results, the experimental data and some other analytical solutions show that the proposed model can describe extreme wave very well and has a good prediction for the extreme wave velocity field. The proposed model resorts only to the wave record, which makes it more easily applicable to ocean engineering practice.

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深海极限波浪运动特性的简便算法

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摘　要：本文基于五阶 Stokes 规则波理论，提出了一种快速求解深海极限波浪运动特性的数学模型. 研究中，按照上跨零点和下跨零点的方法由计算或实测的极限波浪波面时间历程确定包含极限波峰或波谷的相邻两个周期的平均值为五阶 Stokes 规则波的波浪周期，然后根据极限波峰或波谷值反推确定波浪入射波幅. 通过与已有的数值结果和实验数据对比，验证了本文所建立的数值模型可以快速准确的计算出极限波浪下的速度场，相比其他模型，更适合于工程应用.

关键词：极限波浪；深海；五阶斯托克斯规则波；运动特性；速度场