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Dynamic anti-plane interaction between an impermeable crack and a circular cavity in an infinite piezoelectric medium

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Abstract: Wave propagation in an infinite elastic piezoelectric medium with a circular cavity and an impermeable crack subjected to steady-state anti-plane shearing was studied based on Green's function and the crack-division technique. Theoretical solutions were derived for the whole elastic displacement and electric potential field in the interaction between the circular cavity and the impermeable crack. Expressions were obtained on the dynamic stress concentration factor (DSCF) at the cavity's edge, the dynamic stress intensity factor (DSIF) and the dynamic electric displacement intensity factor (DEDIF) at the crack tip. Numerical solutions were performed and plotted with different incident wave numbers, parameters of piezoelectric materials and geometries of the structure. Finally, some of the calculation results were compared with the case of dynamic anti-plane interaction of a permeable crack and a circular cavity in an infinite piezoelectric medium. This paper can provide a valuable reference for the design of piezoelectric actuators and sensors widely used in marine structures.

Keywords: piezoelectric medium; impermeable crack; dynamic stress intensity factor; dynamic stress concentration factor; dynamic electric displacement intensity factor; Green's function

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1 Introduction

Due to their intrinsic electromechanical coupling behavior, piezoelectric materials have recently been widely used in smart materials and structures technology. Especially, piezoelectric actuators and piezoelectric sensors have played important roles in the design and inspection of marine structures. When subjected to mechanical and electrical loads in service, these piezoelectric materials may fail prematurely due to their electro-mechanical coupling behavior. Failure or fracture often occurs in the newly developed piezoelectric materials because of their brittleness and presence of faults or flaws during the manufacturing and the polling process. So the investigation of the failure behaviors caused by stress concentrations has become more important. It should be noted that most of the published papers about piezoelectric materials with flaws are static. In recent years, WANG and MEGUID have investigated the interaction of two cracks in an infinite piezoelectric medium under dynamic anti-plane shearing[1-2]. CHEN, et al. obtained Green's functions for anti-plane problems in piezoelectric media with a finite crack[3]. GU, et al. investigated transient response of an interface crack between dissimilar piezoelectric layers under mechanical impacts[4]. WANG, et al. provided some analytical methods to study the dynamic response of a multi-layer piezoelectric plate containing some non-collinear cracks[5]. WANG and YU studied the scattering of SH-wave about an arc-crack between an inclusion and piezoelectric material under remote uniform loads[6]. These results showed how the loading state and material's non-homogeneity influence elastic stress fields and electric displacement fields ahead of the crack tip. Studies about piezoelectric materials with defects under static electromechanical loading have been more reported, but few dynamic cases have been reported.

The objective of present paper is to provide a theoretical study on dynamic anti-plane interaction between an impermeable crack and a circular cavity in an infinite piezoelectric medium. The emphases are placed on dynamic stress intensity factors at the

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cavity's edge and dynamic stress concentration factors at the crack tip. The boundary conditions of the crack are assumed to be traction free and electrically impermeable, while the circular cavity is assumed to be traction free and electrically permeable.

2 The governing equations

In the absence of body forces and free charges, while the piezoelectric medium is assumed to have been poled along the z-axis normal to the xy-plane (see Fig.1).

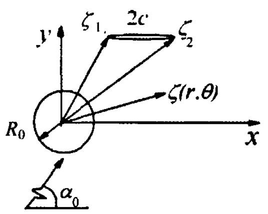

Fig.1 Interacting circular cavity and impermeable crack

The governing equations of linear piezoelectricity for time harmonic dynamic anti-plane shearing are given by

$$\begin{aligned} &c_{44}\nabla^2 w + e_{15}\nabla^2\phi + \rho\omega^2 w = 0, \\ &e_{15}\nabla^2 w - \kappa_{11}\nabla^2\phi = 0, \end{aligned} \tag{1}$$

In which, exponential time factor exp (-iωt) has been omitted; c_{44}, e_{15} and κ_{11} are shear elastic modulus, piezoelectric constant and dielectric constant of the medium, respectively; w, ϕ, ρ and ω are anti-plane displacement, electric potential, mass density and frequency of the incident wave, respectively. Eq.(1) can be simplified further:

$$\begin{cases} \nabla^2 w + k^2 w = 0, \\ \phi = \dfrac{e_{15}}{\kappa_{11}} w + g, \\ \nabla^2 g = 0, \end{cases} \tag{2}$$

where $k^2 = \dfrac{\rho\omega^2}{c^*}$ with $c^* = c_{44} + \dfrac{e_{15}^2}{\kappa_{11}}$. The constitutive relations about the piezoelectric material can be written in column coordinate system (r,θ), $\zeta = re^{i\theta}$.

$$\begin{cases} \tau_{rz} = c_{44}\dfrac{\partial w}{\partial\theta} + e_{15}\dfrac{1}{r}\dfrac{\partial\phi}{\partial\theta}, \\ \tau_{\theta z} = c_{44}\dfrac{1}{r}\dfrac{\partial w}{\partial\theta} + e_{15}\dfrac{1}{r}\dfrac{\partial\phi}{\partial\theta}, \\ D_r = e_{15}\dfrac{\partial w}{\partial r} - \kappa_{11}\dfrac{\partial\phi}{\partial r}, \\ D_\theta = e_{15}\dfrac{1}{r}\dfrac{\partial w}{\partial\theta} - \kappa_{11}\dfrac{1}{r}\dfrac{\partial\phi}{\partial\theta}, \end{cases} \tag{3}$$

where τ_{rz}, $\tau_{\theta z}$, D_r and D_θ are two shear stress components and two electric displacement components, respectively.

3 Green's functions and boundary value problem

First, consider the problem of single circular cavity (The radius is R_0) in an infinite piezoelectric medium subjected to an anti-plane steady-state wave directed at an incident orientation α_0 with the positive x-axis as shown in Fig.1. This problem has the solutions of the following form according to Ref.[7].

$$\begin{cases} w = w_0\sum\limits_{n=0}^{\infty}\varepsilon_n i^n J_n(kr)\cos n(\theta-\alpha_0) + \\ \qquad w_0\sum\limits_{n=0}^{\infty}A_n H_n^{(1)}(kr)\cos n(\theta-\alpha_0), \\ \tau_{\theta z} = -\dfrac{c^* k w_0}{kr}\sum\limits_{n=0}^{\infty} n\sin n(\theta-\alpha_0)\times \\ \qquad \left[\varepsilon_n i^n J_n(kr) + A_n H_n^{(1)}(kr) + \dfrac{\lambda}{1+\lambda}B_n(kr)^{-n}\right], \\ \phi = \dfrac{e_{15}}{\kappa_{11}}w^{(t)} + w_0\dfrac{e_{15}}{\kappa_{11}}\sum\limits_{n=0}^{\infty}B_n(kr)^{-n}\cos n(\theta-\alpha_0), \\ D_\theta = \dfrac{e_{15}kw_0}{kr}\sum\limits_{n=0}^{\infty}B_n(kr)^{-n} n\sin n(\theta-\alpha_0), \end{cases} \tag{4}$$

where $\varepsilon_0 = 1$, $\varepsilon_n = 2$, $n \ge 1$; $\lambda = \dfrac{e_{15}^2}{c_{44}\kappa_{11}}$, J_n and $H_n^{(1)}$ express Bessel function and Hankel function of the first kind, respectively; unknown constants A_n and B_n can be obtained by combination with the boundary conditions given below:

$$\tau_{rz} = 0,\ D_r = D_r^c,\ \phi = \phi^c,\quad |\varsigma| = R_0 \tag{5}$$

in which superscript "c" denotes the variable in the cavity, same in the following.

Second, construct a couple of Green's functions G_w and G_ϕ, which are fundamental solutions of elastic displacement and electric potential respectively for an infinite piezoelectric medium with a circular cavity subjected to a couple of time harmonic line-force and line-charge perpendicular to the xy-plane at an arbitrary point $\varsigma_0 = r_0 e^{i\theta_0}$ in an infinite piezoelectric medium. Therefore, a couple of Green's functions of present paper should satisfy the boundary conditions (5) and the following equations:

$$\begin{cases} \nabla^2 G_w + k^2 G_w = 0, \\ \tau_{\theta z} = \dfrac{\delta(\varsigma - \varsigma_0)}{1+\lambda}, \\ G_\phi = \dfrac{e_{15}}{c_{44}} G_w + g_\phi, \nabla^2 g_\phi = 0, \\ D_\theta = \dfrac{e_{15}}{c_{44}} \dfrac{\delta(\varsigma - \varsigma_0)}{1+\lambda}, \end{cases} \tag{6}$$

in which δ indicates Dirac-δ function. G_w and G_ϕ can be solved as follows:

$$\begin{cases} G_w = \dfrac{i}{4c_{44}(1+\lambda)} \sum_{m=0}^{\infty} D_m H_m^{(1)}(kr) \cos m(\theta - \theta_0) + \\ \dfrac{i}{4c_{44}(1+\lambda)} \sum_{m=0}^{\infty} \varepsilon_m \cos m(\theta - \theta_0) \times \\ \begin{cases} J_m(kr_0) H_m^{(1)}(kr), r > r_0, \\ J_m(kr) H_m^{(1)}(kr_0), r < r_0, \end{cases} \\ G_\phi = \dfrac{e_{15}}{\kappa_{11}} G_w - \dfrac{e_{15}}{2\pi \kappa_{11} c_{44}(1+\lambda)} \times \\ \left[\ln k|\varsigma - \varsigma_0| + \sum_{m=0}^{\infty} E_m (kr)^{-m} \cos m(\theta - \theta_0) \right], \\ G_\phi^c = -\dfrac{i e_{15}}{2\pi \kappa_{11} c_{44}(1+\lambda)} \sum_{m=0}^{\infty} F_m (kr)^m \cos m(\theta - \theta_0), \end{cases} \tag{7}$$

where unknown constants D_m, E_m and F_m can be obtained by solving Eq.6 combined with boundary conditions (5).

Finally, negative shear stress$-\tau_{\theta z}$ and negative electric displacement$-D_\theta$ based on the second and the fourth expressions of Eq.4 are applied to the crack's location and along its surface, respectively. This may result in $\tau_{\theta z}$=0 as well as D_θ=0 at the above location, namely, the traction free and an electrically impermeable crack is constructed. Thus the total electromechanical field can be given by

$$\begin{cases} w^{(t)} = w - \int_{\xi_1}^{\xi_2} \tau_{\theta z}\big|_{\xi=\xi_0} \times G_w(\varsigma, \varsigma_0) d\varsigma_0, \\ \phi^{(t)} = \phi - \dfrac{e_{15}}{\kappa_{11}} \int_{\xi_1}^{\xi_2} D_\theta\big|_{\xi=\xi_0} \times G_\phi(\varsigma, \varsigma_0) d\varsigma_0. \end{cases} \tag{8}$$

Dynamic stress concentration factor (DSCF) at the cavity's edge can be expressed as

$$\tau_{\theta z}^* = \frac{\tau_{\theta z}^{(t)}}{\tau_0} = \frac{1}{\tau_0} c_{44} \frac{\partial w^{(t)}}{r\partial\theta} + e_{15} \frac{\partial \phi^{(t)}}{r\partial\theta}, \ |\varsigma| = R_0, \tag{9}$$

where $\tau_0 = kc_{44}w_0$. While dynamic stress intensity factor (DSIF) K_{III}^τ and dynamic electric displacement intensity factor (DEDIF) K_{III}^D at the crack's tip are given as follows, respectively:

$$K_{III}^\tau = \frac{\tau_{rz}^{(t)}\big|_{\xi\to\xi_1}}{\tau_0(1+\lambda)L} = \frac{1}{\tau_0(1+\lambda)L} (c_{44} \frac{\partial w^{(t)}}{\partial r} + e_{15} \frac{\partial \phi^{(t)}}{\partial r})\Big|_{\xi\to\xi_1}, \tag{10}$$

$$K_{III}^D = \frac{D_r^{(t)}\big|_{\xi\to\xi_1}}{D_0 L} = \frac{1}{D_0 L} (e_{15} \frac{\partial w^{(t)}}{\partial r} - \kappa_{11} \frac{\partial \phi^{(t)}}{\partial r})\Big|_{\xi\to\xi_1}, \tag{11}$$

in which L denotes the crack's characteristic length, $D_0 = \frac{\kappa_{11}}{e_{15}} \tau_0$.

4 Computational cases and their discussions

Some computational cases based on Eqs (9) up to (11) are plotted to show the sensitivity of piezoelectric medium to material constants. Figs.2~4 show DSCFs at the cavity's edge to the case of horizontal crack under vertically incident wave, which vary linearly with dimensionless piezoelectric parameter λ. It can be seen through the asymmetry of Figs.2 and 3 that the influence of an impermeable crack on DSCFs at the cavity's edge at higher frequencies is more obvious than that at lower frequencies, but the maximal DSCFs always appear at lower frequency kR_0=0.4 nearby (see Fig.4).

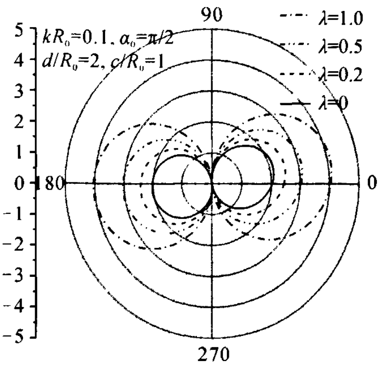

Fig.2 DSCFs at the cavity's edge under vertical incident wave

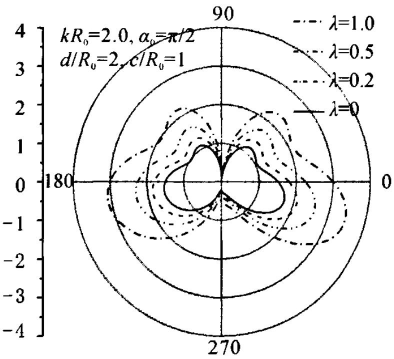

Fig.3 DSCFs at the cavity's edge under vertical incident wave

Fig.5 shows DSIFs at the crack's left tip. Due to the coupling of electromechanical behavior, both of dynamic overshoot and undershoot phenomena are considerably increased comparing with both of traditional elastic material (solid line λ=0.0) and piezoelectric medium with permeable crack at the same situation[8].

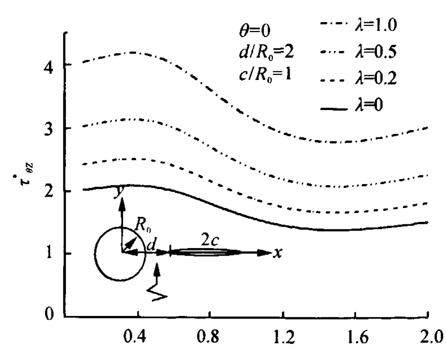

Fig.4 DSCFs at θ=0 of the cavity's edge

While the computational results shown in Fig.6 indicate that DEDIFs increase linearly with the increment of λ at the same frequency and the oscillating phenomena are more obvious when λ is larger. In addition, the computational results also indicate that the great changes of dielectric constants' ratio κ_{11}/κ_0 from 500 to 1 500 have little influence on DSCF, DSIF and DEDIF, in which κ_0 denotes dielectric constant of the cavity.

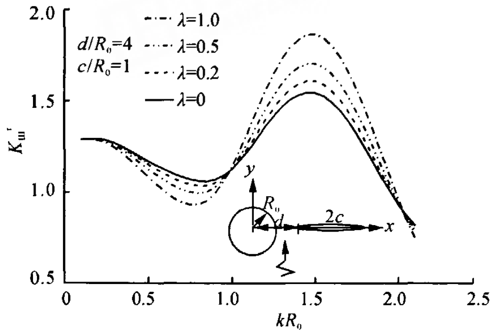

Fig.5 DSIFs at left tip of the crack

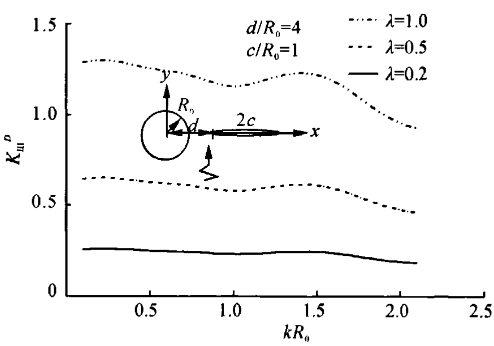

Fig.6 DEDIFs at left tip of the crack

5 Conclusions

The interaction of an impermeable crack and a circular cavity are more obvious than that with only impermeable crack both at lower and higher frequencies. Dynamic analyses are very important for an infinite piezoelectric medium with a circular cavity and an impermeable crack at lower frequencies and larger piezoelectric characteristic parameters. The changes of dielectric constant ratio of the piezoelectric medium to the cavity have little influence on DSCF, DSIF and DEDIF. This paper can provide a valuable reference to the design of piezoelectric actuators and sensors widely served in marine structures.

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相似文献(4条)

cracks under electrically permeable condition, as well as under semi-permeable conditions by using all iterative approach. Two important crack problems in fracture mechanics, the center cracks and the edge cracks in piezoelectric strips, are analyzed by the proposed method. The stress intensity factor and the electric displacement intensity factor are calculated. Meanwhile the effects of strip size: and the electric boundary conditions on these intensity factors are studied.

3.外文期刊 B. Rogowski Fundamental solutions in piezoelectricity. Penny-shaped crack solution

The problem of electroelasticity for piezoelectric materials is considered. For axially symmetric states three potentials are introduced, which determine the displacements, the electric potentials, the stresses, the components of the electric field vector and the electric displacements in a piezoelectric body. These fundamental solutions are utilized to solve the penny-shaped crack problem. Two cases of boundary--value problems are considered, namely the permeable and impermeable crack boundary conditions. Exact solutions are obtained for elastic and electric fields. The main results are the stress intensity factor for singular stress and the electric displacement intensity factor. The numerical results are presented graphically to show the influence of applied mechanical and electrical loading on the analyzed quantities and to clarify the effect of anisotropy of piezoelectric materials. It is show that the influence of anisotropy of the materials on these fields is significant.

4.外文期刊 Rajapakse. RKND Angular distribution of energy release rates and fracture of piezoelectric solids

Applicability of energy release rate criteria to piezoelectric materials is examined in this paper. The angular distribution of strain and total energy release rates at a crack tip in a plane piezoelectric medium is first formulated by using a branched crack model. By relaxing the commonly held assumption of self-similar crack propagation and incorporating in the fracture toughness anisotropy, the criteria of modified strain energy release rate and modified total energy release rate are applied to discuss the propagation path of an impermeable crack. Numerical results indicate that the predictions based on the criterion of modified total energy release rate do not show qualitative agreement with experimental observations, whereas the predictions based on the criterion of modified strain energy release rate appear more promising. Based on the criterion of modified strain energy release rate, a crack tends to branch off from a straight path under a positive electric loading, regardless of crack orientation. An applied electric field can either promote or retard crack propagation depending on the direction of electric field, crack orientation and crack extension direction. The predictions made by the energy-based criteria are compared with those corresponding to a recently reported stress-based criterion.