# SPSM and its application in cylindrical shells

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**Abstract:** In naval architectures, the structure of prismatic shell is used widely. But there is no suitable method to analyze this kind of structure. Stiffened prismatic shell method (SPSM) presented in this paper, is one of the harmonic semi-analytic methods. Theoretically, strong stiffened structure can be analyzed economically and accurately. SPSM is based on the analytical solution of the governing differential equations for orthotropic cylindrical shells. In these differential equations, the torsional stiffness, bending stiffness and the exact position of each stiffener are taken into account with the Heaviside singular function. An algorithm is introduced, in which the actions of stiffeners on shells are replaced by external loads at each stiffener position. Stiffened shells can be computed as non-stiffened shells. Eventually, the displacement solution of the equations is acquired by the introduction of Green function. The stresses in a corrugated transverse bulkhead without pier base of an oil tanker are computed by using SPSM.
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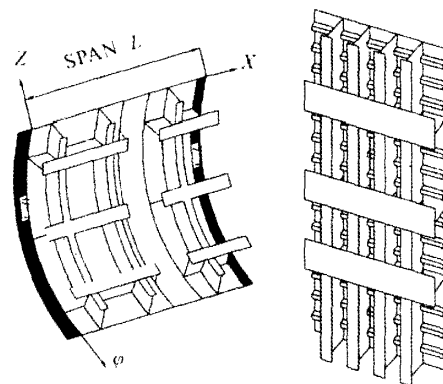
## 1 Introduction

The requirements of longer spans, lighter structures, higher accuracy, etc., imposed on steel structures, have deeply changed their conception and design techniques.

Rapid and accurate verification of the stresses and displacements at every point of a structure is the aim of this study. In spite of different stiffening schemes (ring, stringer, crossbar, as shown in Fig.1), a computing tool (SPSM) has been established without making simplification and approximation about structural geometry. Calculations are not based on smeared rib properties; rather, the torsional stiffness, the lateral (tangent to the shell) bending stiffness, and the exact position of each stiffener are taken into account. By this analytical method, results can be obtained at all points of the structure (sheathing, web, flange, web-flange junction, web sheathing junction).

Naval structure, such as an orthotropic sheathing (see Fig.1), a navigation-dam gate, or a tidal surge barrier, frequently used is made of box girders, which are stiffened by longitudinal and transversal stiffeners. In these cases, the number of stiffeners and crossbars is so large that processing them by usual stress-analysis

methods may become very expensive.



(a) Stiffened cylindrical shell      (b) Orthotropic plate
    (rings and stringers)                with crossbars
Fig.1 Stiffened cylindrical shell (rings and stringers) and orthotropic plate with crossbars

The strongest advantage of this method is the discretization. Each stiffened shell or plate is considered as one panel (Fig.1); "panel" has the significance of an "element" in the finite-element method. However, a panel (shell or plate) includes all the stiffeners that are present; it is not necessary to split up the stiffener into smaller elements. For each panel of a structure, the analytical relationship of the solution (stresses and displacements) is obtained. It takes into account the compatibility conditions and the equilibrium equations that must be satisfied at the

junction between two or more panels. Thus, it is, for instance, possible to study a stiffened square-box girder that is 100 m long, with only four of our elements (panels).

When the shells are cylindrical, which is the case of steel structures in the naval field, the harmonic analysis methods are among the efficient ones (Chung[1], Golberg[2]). In these methods, loads and displacements, as well as stresses, are decomposed by Fourier series along a parallel direction to the cylinder-generating lines.

## 2 Theoretical statements

All the theoretical statements involved in the stiffened sheathing method (Rigo[3]) cannot be included in this paper. Therefore, in presenting this method, only some of its originalities have been outlined.

For linear fields, designers could economically use the stiffened sheathing method (SSM) in order to design orthotropic steel structures composed of shells and plates. By this method, giving the displacements and the stresses around a small critical region of a structure (for instance, a hole in the sheathing or an unstiffened thin-skinned plate), the stress analysis in this region can be studied by FEM. Such a fruitful partnership might help in computing hydraulic steel structure for which design budgets are usually small and for which nonlinear problems are rare and always confined.

Within the linear DKJ method (Dehousse[4]), no additional restrictions have been placed on the orthotropic shell with smeared-stiffener theory on the torsional stiffness, or on the lateral bending stiffness. The basic element is a cylindrical orthotropic shell where $L$ is the length, $q$ is the radius, $\delta$ is the thickness, and $\varphi_0$ is the shell degree of opening. The coordinate system is presented in Fig.2, showing the $x$-axis being along the cylindrical generator, the $\varphi$-axis along the circumference, and the $z$-axis perpendicular to the shell. Displacements $u$, $v$, and $w$ are associated with the $x$-axis, $\varphi$-axis, and $z$-axis. The reference shell is situated between the external and the internal surfaces (Fig.2).



(a) Shell element

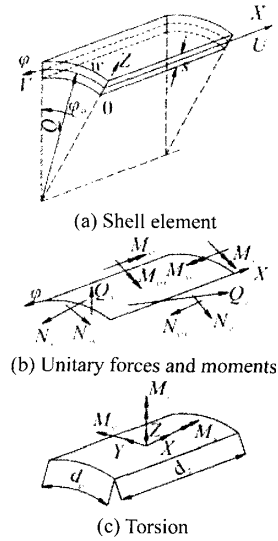(b) Unitary forces and moments

(c) Torsion

Fig.2 Cylindrical shell element without stiffening and unitary forces and moments

The stiffener includes three types of ribs (stringers, rings and crossbars) as shown in Fig.1. These longitudinal and transversal ribs have, respectively, a spacing of $\varepsilon_X$ and $\varepsilon_\varphi$. There are also the unitary forces ($N_x$, $N_\varphi$, $N_{x\varphi}$, $N_{\varphi x}$, $Q_x$, $Q_\varphi$) and the unitary moments ($M_x$, $M_\varphi$, $M_{x\varphi}$, and $M_{\varphi x}$), which are calculated in relation to the middle surface (see Fig.2).

The middle surface (see Fig.2) receives the external loads which can be subdivided into $X$, $Y$, and $Z$, for specific pressures (N/m$^2$), and $\bar{M}_x$, $\bar{M}_\varphi$, and $\bar{M}_z$, for the specific moments (N·m/m$^2$). These loads make it possible to apply dead load, hydrostatic pressure, and pressure that can be varied along $x$-axis and $\varphi$-axis, as well as moments that are applied, for instance, at the extremities.

The six equilibrium equations containing the ten aforementioned variables can be determined.

$$N_x' + \frac{N_{\varphi x}^0}{q} + X = 0, \tag{1}$$

$$\frac{N_\varphi^0}{q} + N_{x\varphi}' - \frac{Q_\varphi}{q} + Y = 0, \tag{2}$$

$$\frac{N_\varphi}{q} + \frac{Q_\varphi^0}{q} + Q_x' = Z, \tag{3}$$

$$\frac{M_\varphi^0}{q} + M'_{x\varphi} - Q_\varphi + \bar{M}_x = 0 , \tag{4}$$

$$M'_x + \frac{M_{\varphi x}^0}{q} - Q_x + \bar{M}_\varphi = 0 , \tag{5}$$

$$N_{x\varphi} - N_{\varphi x} + \frac{M_{\varphi x}}{q} + \bar{M}_z = 0 , \tag{6}$$

Where $f'$ is the derivative of $f$ relative to $\chi$ ($\mathrm{d}f / \mathrm{d}x$) and $f^0$ is the derivative of $f$ relative to $\varphi(\mathrm{d}f / \mathrm{d}\varphi)$.

At the moment, no hypothesis has been assumed. However, to solve the problem, it is necessary to introduce the following assumptions; the validity field of these developments is the elastic range of the DKJ simplification, which is the thin-shell assumptions (i.e., thickness<<radius), the small deformations and the Love-Kirchgoff hypothesis[5]. These hypotheses are sufficient to maintain a linear stress variation along the shell thickness.

In order to write the unitary expression of the forces and moments (see Fig.2) as functions of the $u, v$, and $w$ displacements, and as functions of the two constants $D$ and $K$, which are dependent on the material properties (modulus of elasticity $E$, Poisson ratio $v$), the procedure is as follows.

The equilibrium of an infinitesimal element of two dimensions must be considered. The third dimension is not considered, since only the middle surface is being considered. In order to write the equilibrium of this element, unitary forces and moments calculated in relation to the middle surface have been introduced. For instance:

$$N_x = \int_{-\delta/2}^{+\delta/2} \sigma_x (1 + \frac{z}{q}) \mathrm{d}z, \tag{7}$$

$$M_x = -\int_{-\delta/2}^{+\delta/2} \sigma_x (1 + \frac{z}{q}) z \mathrm{d}z. \tag{8}$$

Taking into account the stress-deformation relationships

$$\sigma_x = \frac{E}{1 - v^2} \left[ u' + v \frac{v^0}{q} + v \frac{w}{q} - z \left( w'' + v \frac{w^{00}}{q^2} \right) \right], \tag{9}$$

$$\sigma_\varphi = \frac{E}{1 - v^2} \left[ \frac{v^0}{q} + \frac{w}{q} + v \left( u' - z w'' \right) - z \frac{w^{00}}{q^2} \right], \tag{10}$$

$$\tau_{x\varphi} = G \left[ \frac{u_0}{q} + v' - 2z \frac{w^0}{q} \right]. \tag{11}$$

Eqs.(12) and (13) for an unstiffened shell are obtained.

$$N_x = \frac{D}{q} \left( u' q + v v^0 + v w \right), \tag{12}$$

$$M_x = \frac{K}{q_2} \left( q^2 w'' + v w^{00} \right). \tag{13}$$

With stringers (see Fig.3), Eqs.(7) and (8) are no longer valid and must be modified as follows:

$$N_x = \int_{-\delta/2}^{+\delta/2} \sigma_x \left( 1 + \frac{z}{q} \right) \mathrm{d}z + f(\varphi) \int_{wx} \sigma_x \frac{e_x}{d_x} \mathrm{d}z , \tag{14}$$

$$M_x = -\int_{-\delta/2}^{+\delta/2} \sigma_x \left( 1 + \frac{z}{q} \right) z \mathrm{d}z + f(\varphi) \int_{wx} \sigma_x \frac{e_x}{d_x} z \mathrm{d}z , \tag{15}$$

where $e_x$ is the stringer width according to the $z$ coordinate, $e_x = fct(z)$; $d_x$ is the maximum value of the $e_x$ stringer width; $w_x$ is the stringer transversal surface area; and $f(\varphi)$ is a function that is 1 under the stringer, and 0 elsewhere.



(b) Sketch map of stiffeners 's shape

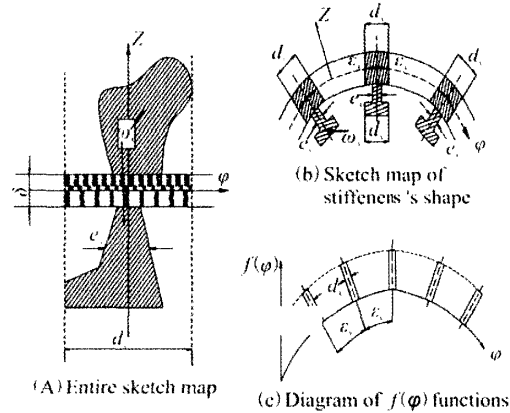(A) Entire sketch map

(c) Diagram of $f(\varphi)$ functions

Fig.3 Diagram of $f(\varphi)$ functions, which permit consideration of exact location of stiffeners

The $f(\varphi)$ functions are composed of Heaviside ones, which enable one to obtain functions perfectly compatible with the structure geometry (see Fig.3). They are equal to unity and cancel out each other's spaces ($d_x$ and $d_\varphi$) where the ribs act. These functions permit the consideration of the exact location of each stringer. Hence, the geometric dimensions of two stringers (symmetric or eccentric) fixed on the same shell can be different. This is one of the most important particularities of this method. Many other methods use the smeared stiffener theory

(Bares and Massonnet[5]; Hutchinson and Amazigo).

After integration of (14) and (15), $N_x$ and $M_x$ become

$$N_x = \frac{D}{q}\left(u'q + vv^0 + vw\right) + f(\varphi)\left(u'\Omega_x - w''H_x\right), \quad (16)$$

$$M_x = \frac{K}{q^2}\left(q^2 w'' + vw^{00}\right) - f(\varphi)\left(u'H_x - w''R_x\right), \quad (17)$$

where $\Omega_x$ is the modified stringer's transversal surface area ($\Omega_x = w_x \cdot E / d_x$); $H_x$ is the static moment related to the middle surface; and $R_x$ is the moment of inertia with respect to the middle surface.

Eqs.(16) and (17) show the precise position of each stringer is accurately considered. In fact, $f(\varphi)$ equals zero between two stringers (see Fig.3) so Eqs. (12) and (13), which are valid for unstiffened shells, are again found to be valid.

The relationship between $N_{x\varphi}$ and $M_{x\varphi}$ can also be obtained as

$$N_{x\varphi} = D\frac{1-v}{2}\left[v' + \frac{u^0}{q}\right] + f(\varphi)S_x\left[v' + \frac{u^0}{q}\right], \quad (18)$$

$$M_{x\varphi} = K(1-v)\frac{w^{0'}}{q} + f(\varphi)\left[T_x\frac{w^{0'}}{q} + L_x\left(v' + \frac{u^0}{q}\right)\right], (19)$$

where $S_x$ and $L_x$ are two coefficients considering the lateral (tangent to the shell) bending stiffness; and $T_x$ is a coefficient taking into account the torsional stiffness. These $S_x$ and $L_x$ coefficients must be carefully determined because they are very important for the transverse shear deformations (that can be seen hereinafter in the numerical verifications.

For the rings, $N_\varphi$ , $N_{\varphi x}$ , $M_\varphi$ and $M_{\varphi x}$ are to be obtained. The $Q_x$ and $Q_\varphi$ unitary forces are determined from the moment Eqs. (4) and (5) by replacing the already-computed unitary forces and moments.

Next, by replacing the unitary forces and moments with their analytical expressions in Eqs.(1), (2), and (3), a system of three differential equations with constant coefficients is deduced (see Eqs.(20), (21), and(22)). These equations, expressing the stiffened cylindrical shell, are only the functions of $u, v$ and $w$ displacements and their spatial derivatives.

$$D\left(qu'' + vv^0 + vw'\right) + \frac{D(1-v)}{2q^2}\left(u^{00} + qv^{0'}\right) + $$
$$f(x)\left[\frac{S_\varphi}{q}\left(v^{0'} + \frac{u^{00}}{q}\right)\right] + \frac{f^0(\varphi)}{q}\left[S_x\left(v' + \frac{u^0}{q}\right)\right] \cdot$$
$$f(\varphi)\left[\Omega_x u'' - H_x W'' + \frac{S_x}{q}\left(v^{0'} + \frac{u^{00}}{q}\right)\right] + X = 0,$$
$$(20)$$

$$\frac{D}{q^2}\left(v^{00} + w^0 + vqu^{0'}\right) + \frac{D(1-v)}{2}\left(u^{0'} + qv''\right) + $$
$$f(x)\left[\frac{\Omega_\varphi}{q^2}\left(v^{00} + w^0\right) - \frac{H_\varphi}{q^3}w^{000} + S_\varphi\left(v'' + \frac{u^{0'}}{q}\right)\right] + $$
$$\frac{f'(x)}{q}\left[S_\varphi\left(v' + \frac{u^0}{q}\right)\right] + f(\varphi)\left[S_x\left(v'' + \frac{u^{0'}}{q}\right)\right] + Y = 0,$$
$$(21)$$

$$\frac{D}{q^2}\left(v^{0'} + w + vqu'\right) + \frac{K}{q^4}w^{0000} + \frac{2K}{q^2}w^{00''} + Kw'''' + $$
$$f(x)\left[\begin{array}{c}\frac{\Omega_\varphi}{q^2}\left(v^{0'} + w\right) - \frac{2H_\varphi}{q^3}w^{00} - \frac{H_\varphi}{q^3}v^{000} + \frac{R_\varphi}{q^4}w^{0000} \\ + \frac{T_\varphi}{q^2}w^{00''} + \frac{L_\varphi}{q}\left(v^{0''} + u^{00'}\right)\end{array}\right] + $$
$$f(\varphi)\left[\frac{T_x}{q^2}w^{00''} + \frac{L_x}{q}\left(v^{0'} + \frac{u^{00}}{q}\right) - H_x u'' + R_x w''\right] + $$
$$\frac{f^0(\varphi)}{q}\left[\frac{T_x}{q}w^{0'} + \frac{L_x}{q}\left(v'' + \frac{u^{0'}}{q}\right)\right] + $$
$$f'(x)\left[\frac{T_\varphi}{q}w^{0'} + \frac{L_\varphi}{q}\left(v'' + \frac{u^{0'}}{q}\right)\right] = -\frac{M_x^0}{q} + M_\varphi' + Z,$$
$$(22)$$

where $\Omega_x, H_x, T_x, L_x, S_x$ and $R_x$ are parameters depending on the stringer geometry, and $\Omega_\varphi, H_\varphi, T_\varphi, L_\varphi, S_\varphi$ and $R_\varphi$ are parameters depending on the ring geometry. The $f^0(\varphi) = df(x)/dx$ functions are derivatives of the Heaviside ones. Thus, these are Dirac functions, which are always zero except under the ribs where their integrals equal unity.

The principle of solving this system (see Eqs.(20), (21), and (22)) is based on the transformation of the three differential equations by a classical system of three equations and three unknowns. The coefficients are the differential operators, and the unknowns are $u, v$, and $w$ displacements. After calculation, only one eighth-order differential equation of $w$ with two variables ($x$ and $\varphi$) is obtained (see Eq.(23)):

$$\bar{A} \cdot w'''''' + \bar{B} \cdot w'''' + \bar{C} \cdot w''''^{00} + \bar{D} \cdot w''' + \bar{E} \cdot w''^{00} +$$
$$\bar{F} \cdot w''^{0000} + \bar{G} \cdot w''^{00} + \bar{I} \cdot w^{0000''} + \bar{J} \cdot w^{0000''00} + \quad (23)$$
$$\bar{K} \cdot w^{00000000} = 0,$$

where $\bar{A}, \bar{B}, \bar{C}, \cdots\cdots,$ and $\bar{K}$ coefficients are constants that only depend on the geometric characteristics of the shell and its stiffeners. They also depend on the mechanical properties of the shell and its stiffeners, and the material ($E, v$).

To obtain an equation with two independent variables, the displacements are developed using Fourier series expansion:

$$w(x, \varphi) = \sum_{n=1}^{\infty} w(\varphi) \sin \frac{n\pi x}{L}, \quad (24)$$

$$v(x, \varphi) = \sum_{n=1}^{\infty} v(\varphi) \sin \frac{n\pi x}{L}, \quad (25)$$

$$u(x, \varphi) = \sum_{n=1}^{\infty} u(\varphi) \cos \frac{n\pi x}{L}. \quad (26)$$

There is no right-hand side in Eq.(23) because only the load lines are considered. They are applied on a $q\mathrm{d}\varphi$ infinitesimal space across a line along the $x$-axis. The analytical relationship of $w$ is obtained after integration of the general solution (see Eq.(23)) along the $\varphi$-axis for the real load distribution.

The real actions of the stringers on the shell are determined by the solution to a system of integral equations that are well known as the Volterra-Fredholm equations of the second kind. The kernels of these integral equations come from the introduction of the Green function in the analytical relationships of the three unitary force lines and the two unitary moment lines. So, the stiffened shell can be computed as an unstiffened shell, where, not only the external loads act, but also five load lines at each stringer (or ring) position act.

After solving the integral equations for the displacements, the first results are the analytical relationships of the $u, v, w$ displacement according to the $x-, \varphi-,$ and $z-$ coordinates. By replacing these in Eqs.(9), (10) and (11), the analytical expressions for the stresses may be obtained. For $z = 0$, stresses are determined on the middle surface of the shell. If the $x, \varphi,$ and $z$ stiffener coordinates are

known, the stresses at every point of a stiffener can also be computed (for instance, in the web, in the flange, etc.). The analytical expressions resulted from the aforementioned principles will not be developed here. These computations are too long and not useful to the reader. For more details, refer to the main work of the author[3].

## 3 Structure with particular end conditions

The sine-series decomposition of $w(x, \varphi)$ (see Eq.(24)) implies that the other displacement expressions are $v(x, \varphi)$ as $\sin \lambda x$ (see Eq.(25)), and $u(x, \varphi)$ as $\cos \lambda x$ (see Eq.(26)). So at the boundaries ($x = 0$ and $x = L$), the $w$ and $v$ displacements vanish, but the $u$ longitudinal displacement and the $w' = \mathrm{d}w/\mathrm{d}x$ rotation do not. Hence, theoretically, the $x = 0$ and $x = L$ extremities must always be simply supported; the other two zones ($\varphi = 0$ and $\varphi = \varphi_0$) can be free, fixed, simply supported, or share with other panels.

As far as the naval structures are concerned, many box girders and stiffened sheathings are simply supported. Nevertheless, many also have other boundary conditions. For instance, box girders of radial gates are elastically supported by both of their arms (at the box-girder ends, $u$ and $w'$ are not free), and some orthotropic steel decks must be computed as fixed (at the ends, $u = w' = 0$).

Later, a general overview of our present investigation will show that such $N_b$ forces and $M_b$ moments are consistent with harmonic analysis of the stiffened shell.

Among the $X$, $Y$, and $Z$ external forces (see Fig.2), end forces $N_b$ and end moments $M_b$ are included and applied at both ends of the plates and the shells of the structure (see Fig.4). These end forces and end moments permit the simulation, for example, of axial force as well as bending and torsion moments that the supporting arms of a radial gate transmit to the main box girder.

As far as the end forces $N_b$ are concerned, two asymmetrical end loads $F$ are applied, and so the

analytical solution of a shell submitted to two asymmetrical longitudinal loads $F$ applied at both ends is obtained. The analytical expression of the $N_b$ end forces is:

$$N_b(x) = \sum_{n=1}^{\infty} \begin{bmatrix} \dfrac{4}{(2n-1)\pi d^*} F(-1)^{n+1} \cdot \\[2mm] \cos \dfrac{(2n-1)\pi(L-2d^*)}{L} \\[2mm] \sin \dfrac{(2n-1)\pi x}{L} \end{bmatrix}, \qquad (27)$$

where $F = F(\varphi)$ is the applied force at unit length along $\varphi_0$; $F$ is a function of $\varphi$ (see Eq.(28)), and $d^*$ is the length of end segments where the force $F$ is applied.

For the end moments, the same method is used. However, there is an important difference as only the derivative of the external load $\bar{M}_\varphi$ exists in the differential Eqs. (20), (21), and (22), so $dM_b(x)/dx$, the derivative of the $M_b$ end moments, must be considered.
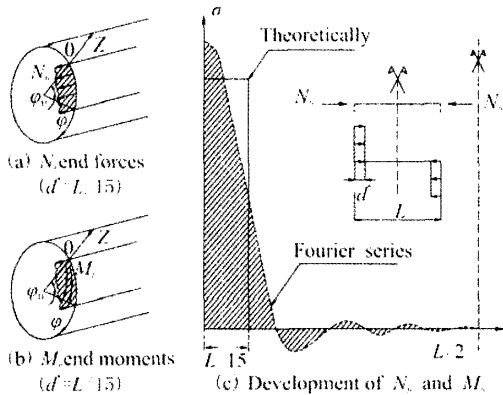


Fig.4   Development of asymmetrical $N_b$ end forces ($d^* = L/15$)· $N_b$ end forces and $M_b$ end moments

# 4 Fourier series expansion of end forces and moments

If an exact load decomposition must be obtained, the development of the end moments requires a very large number of terms (50~100 terms). In practice, concentrated loads do not exist. They are always more or less applied on $d^*$ length segments. So, $N_b$ end forces and $M_b$ end moments are always applied on such $d^*$ segments near the ends. Satisfactory results with only the first seven terms of the Fourier series are obtained with an acceptable accuracy. For $d^* = L/15$, Fig.4 shows the theoretical end force (solid line) and the practical one (dotted line), which has been applied by using the Fourier series expansion.

The $d^*$ length is one of the most important parameters to determine the accuracy of the results. However, in practice, this is influenced by the structure geometry and some other practical considerations. For instance, the junction between the box girder of a radial gate and the arms must be very strong, so there is a minimum (practical) width for arms and there are also many strengthening pieces (plates, gussets, etc.). Therefore, the results in the end regions will always be approximate. After many trials, it has been seen that the most economical solution is to take $d^*$ in the range of $L/15$ ($L$=span), which assures an acceptable level of accuracy.

# 5 An example of SPSM application

Computation of an oil tanker structures:
Principle dimensions: Length: 228.5 m, Width: 32.2 m, Draft: 9.0 m. Profile of the oil tanker is shown in Fig.5.
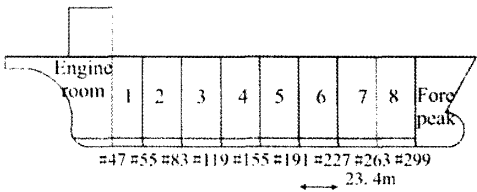


Fig.5 The profile of the oil tanker

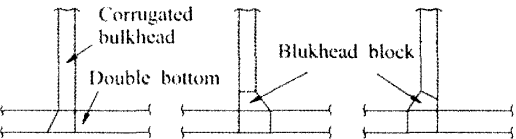The tanker with a corrugated longitudinal bulkhead is shown in Fig.6.



Fig.6 The tanker with a corrugated longitudinal bulkhead

Its loads are calculated by using DNV code. Then the banding moments at $y = 5\,460$ is shown in Fig.7.
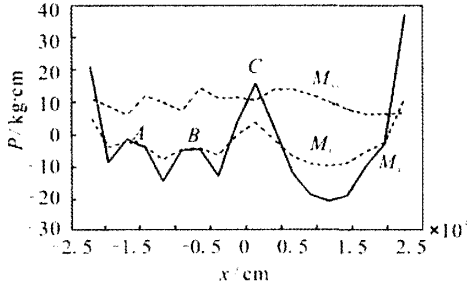
Fig.7 The banding moments at $y = 5\,460$

# 6 Conclusions

The Fourier series expansion for cylindrical shell analysis is a well-known method, but the stiffened sheathing method presents several improvements. It allows the computing of orthotropic cylindrical shells while considering the real position and geometry of each rib. The spacing and the dimensions could change; it is not a smeared-stiffener theory.

One other particularity of the method is that it is possible to compute structures that are composed of many stiffened shells and plates, developments that establish an efficient computation tool, the LBR-3 software, have been presented. The main advantages and qualities of this software are speed, simplicity and accuracy, and, of course, ease. LBR-3[7] enables one to compute very complex structure, such as curved stiffened sheathing, navigation-dam gate, tidal surge barrier, lock gate, canal bridge, etc. The stiffened sheathing method has been confirmed as an efficient alternative to other numerical techniques for analyzing structures composed of stiffened plates and cylindrical shells.

To broaden the application field of the stiffened sheathing method, $N_b$ end forces and $M_b$ end moments that are applied at both ends of the shells, have been added to the classical external forces. Thus, it is now possible to compute many structures with many kinds of boundary conditions using the Fourier series.

# Appendix notations

The following symbols are used in this paper:

$\bar{A}, \bar{B}, \bar{C}, \ldots, \bar{K}$ are the constants defined in (23);

$a, b, c,$ and $d$ are the parameters of end force function $F(\varphi)$;

$D = E \cdot \delta /(1 - v^2)$;

$d_x$ and $d_\varphi$ are the maximal values of $e_x$ stringer width and $e_\varphi$ ring width;

$d^*$ is the length of end segments ( $x = 0, x = L$ ) on which loads are applied;

$E$ is the modulus of elasticity;

$e, f, g,$ and $h$ are the parameters of end moments functions $G(\varphi)$;

$e_x$ or $e_x(z), e_\varphi$ or $e_\varphi(z)$ are the stringer and ring widths according to $z$-coordinate;

$F$ or $F(\varphi)$ = analytical relationship of end forces;

$f(\varphi), f(x)$ are Heaviside functions;

$G(\varphi)$ is the analytical relationship of end moments;

$H_x$ and $H_\varphi$ are the stringer and ring static moments related to middle surface of shell;

$K = E\delta^3 /12(1 - v^2)$;

$L$ is the span of shell along $x$-axis;

$L_x$ and $L_\varphi$ are the coefficients considering stringer (ring)'s lateral bending stiffness;

$M_b$ is the theoretical end moments;

$M_x, M_\varphi, M_{\varphi x},$ and $M_{x\varphi}$ are the unitary moments;

$M_x, M_\varphi,$ and $M_z$ are the specific moments ( $N \cdot m/m^2$ );

$N_b$ is the theoretical end forces;

$N_x, N_\varphi, N_{x\varphi},$ and $N_{\varphi x}$ are the unitary forces;

$n$ is the term number of Fourier series;

$Q_x$ and $Q_\varphi$ are the unitary forces;

$q$ is the radius of shell;

$R_x$ and $R_\varphi$ are the stringer and ring inertia moments with respect to middle surface of shell;

$S_x$ and $S_\varphi$ are the coefficients considering stringer (ring)'s lateral bending stiffness;

$T_x$ and $T_\varphi$ are the coefficients considering stringer (ring)'s torsional stiffness;

$u$ is the displacement along $x$-axis;

$u'$ and $u^0$ are the derivative of $u$ relative to $x$, derivative of $u$ relative to $\varphi$;

$v$ is the displacement along $\varphi$ axis;

$v'$ and $v^0$ are the derivative of $v$ relative to $x$, derivative of $v$ relative to $\varphi$;

$w$ is the displacement along $z$-axis;

$w^{0'}$ is the derivative of $w$ relative to $\varphi$ and $x$;

$x, \varphi$, and $z$ are the axes of system;

$X, Y$, and $Z$ are the specific pressures ($N/m^2$);

$\delta$ is the thickness of shell;

$\varepsilon_x$ and $\varepsilon_\varphi$ are the spacing of stringer and ring;

$\lambda = n\pi / L$;

$v$ is Poisson coefficient;

$\varphi_0$ is the shell degree of opening;

$\sigma_x$ and $\sigma_\varphi$ are the stresses in shell along $x$ and $\varphi$ axis;

$\tau_{x\varphi}$ is the shearing stress in $x\varphi$ plane (middle surface of shell);

$\Omega_x$ and $\Omega_\varphi$ are the modified stringer (ring) transversal surface area;

$\Omega_x = \omega_x E / d_x ; \Omega_\varphi = \omega_\varphi E / d_\varphi$;

$\omega_x$ and $\omega_\varphi$ are the stringer (ring) transversal surface area.

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# SPSM and its application in cylindrical shells

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